Unsupervised Learning of Shape Manifolds

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1. Overview

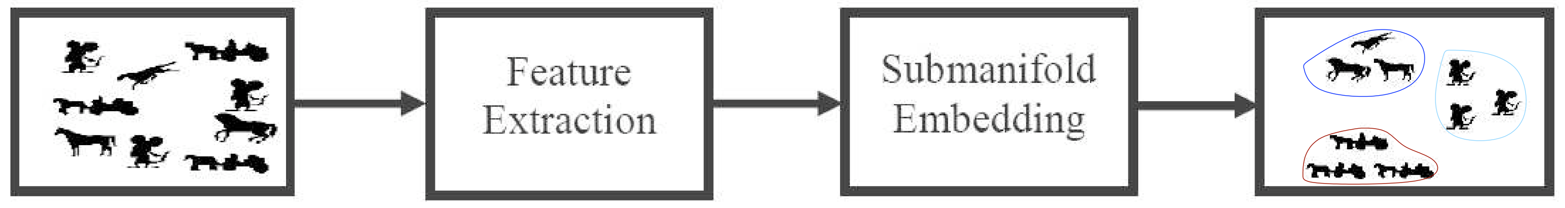
- Representation of the shapes of closed objects ought to be invariant to illumination, rotation, and scale.
- However, this may result in a high-dimensional feature vector for the shapes, resulting in the so-called *curse of dimensionality*.
- Classical shape analysis methods use principal component analysis (PCA) for dimensionality reduction.
- The underlying assumption is that the subspace corresponding to the major modes of variation is linearised, which may not necessarily be the case.
- A non-linear dimensionality reduction method is, therefore, required which preserves the geodesic distance of similar shapes on their submanifolds.
- A novel method for extraction of intrinsic parameters of multiple shape classes, using non-linear embedding of the shape manifolds, is presented.

2. Shape Representation

✓ For a given shape, its corresponding contour $\mathcal{C}$ consisting of N (where N is fixed using cubic spline interpolation) boundary points (x_i, y_i) for $i=1,2,\ldots,N$ is extracted.

✓ A centroidal distance function $r = \{r_1, r_2, \ldots, r_N\}$ of all the boundary points from the centroid of $\mathcal{C}$ is computed. The feature vector $\mathbf{f}$ is then given by,

$$\mathbf{f} = \left(\frac{|F_1|}{|F_0|}, \frac{|F_2|}{|F_0|}, \ldots, \frac{|F_{N/2}|}{|F_0|}\right)^T$$



The Concept Diagram

3. Learning the Shape Manifolds

✓ The shape distance $w(\mathbf{f}_i, \mathbf{f}_j)$, where $i, j = 1, 2, \ldots, n$ and n denotes the total number of shape images in the data set, between two feature vectors $\mathbf{f}_i$ and $\mathbf{f}_j$ is given by,

$$w(\mathbf{f}_i, \mathbf{f}_j) = \exp\left(-\frac{||\mathbf{f}_i - \mathbf{f}_j||^2}{2\varepsilon}\right)$$

✓ A state-transition (or Markov) matrix P is defined on a graph, whose nodes are the shape feature vectors, as follows

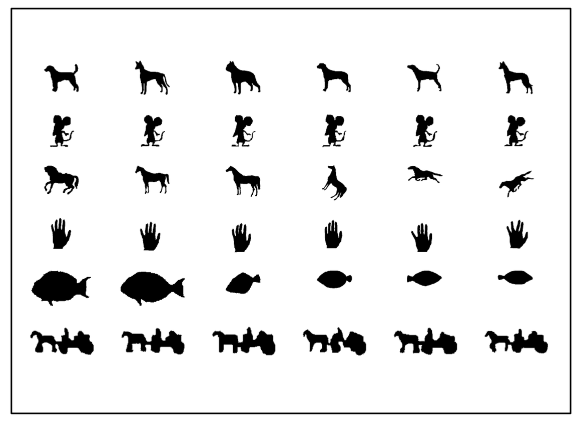
$$p_{ij} = \frac{w(\mathbf{f}_i, \mathbf{f}_j)}{d(\mathbf{f}_i)}$$

where $d(\mathbf{f}_i)$ denotes the degree of node $\mathbf{f}_i$ in the graph. The i th row of matrix P gives the probabilities of transitions from node i to all other nodes in one time step.

✓ Let $|\lambda_0| \geq |\lambda_1| \geq \cdots$ denote the eigenvalues of P and $\{\psi_i\}$ are the corresponding eigenvectors. Then a diffusion map from the shape feature space to a lower-dimensional Euclidean space $\mathcal{R}^m$, where $m << N$, in t steps is given as follows,

$$\Psi^{(t)} : \mathbf{f} \longmapsto \left((\lambda_1)^t \psi_1(\mathbf{f}), (\lambda_2)^t \psi_2(\mathbf{f}), \ldots, (\lambda_m)^t \psi_m(\mathbf{f})\right)$$

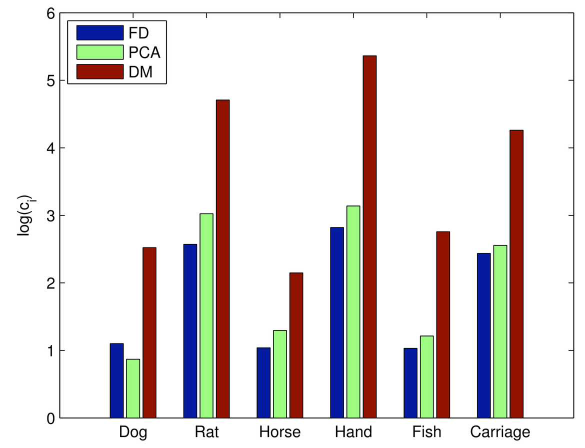
If m axes of the lower-dimensional space can be associated with the intrinsic shape parameters, the above can be regarded as a mapping from a low-level feature space to a high-level semantic space.



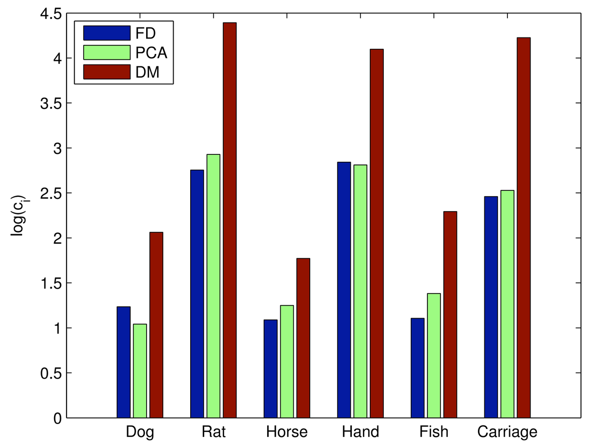
The Kimia Dataset:
6 classes; 157 samples

Clustering Separability Index (CSI):

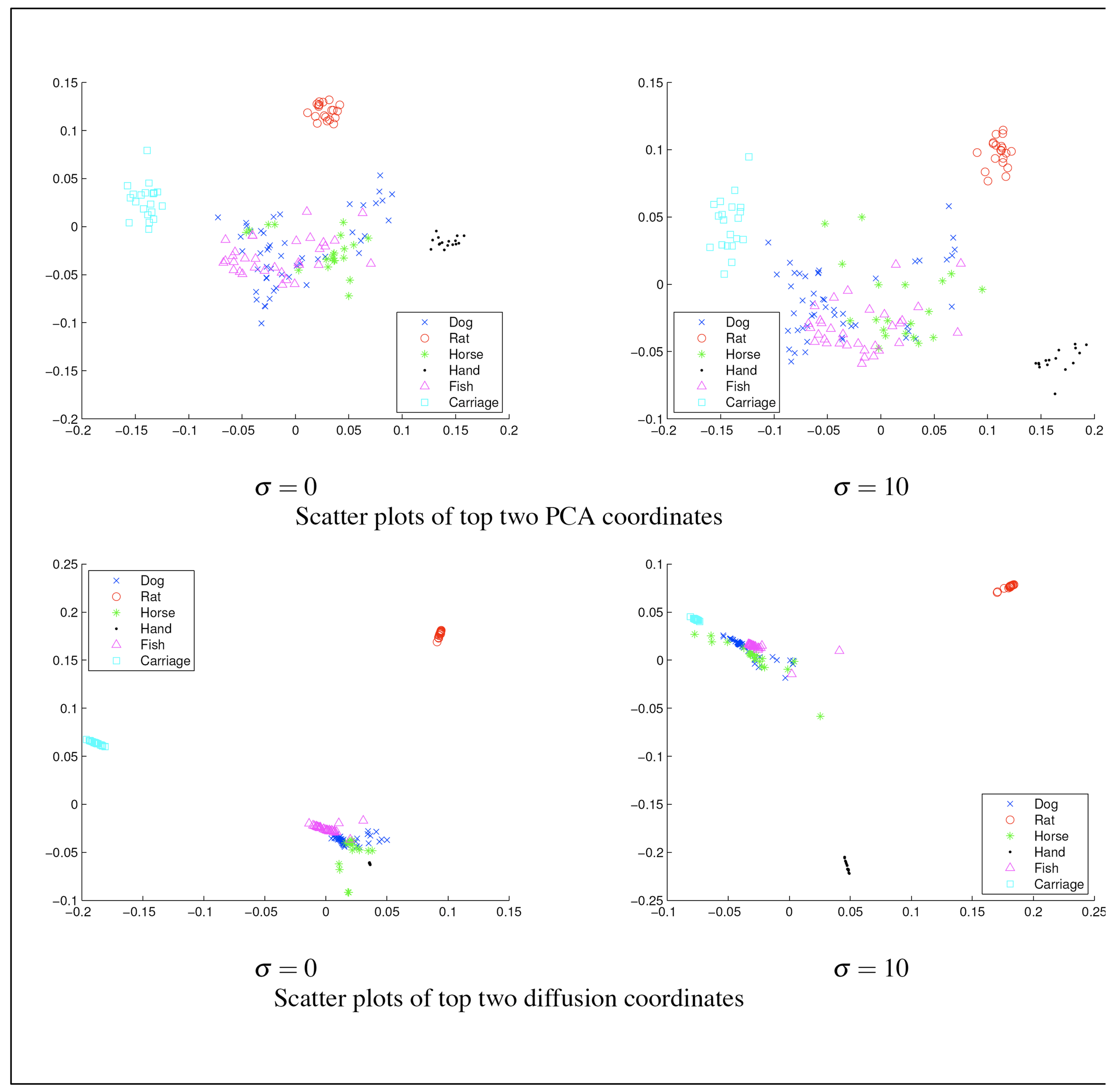
$$c_i = \frac{\bar{d}_i}{\sqrt{\bar{\sigma}_i^2}}$$

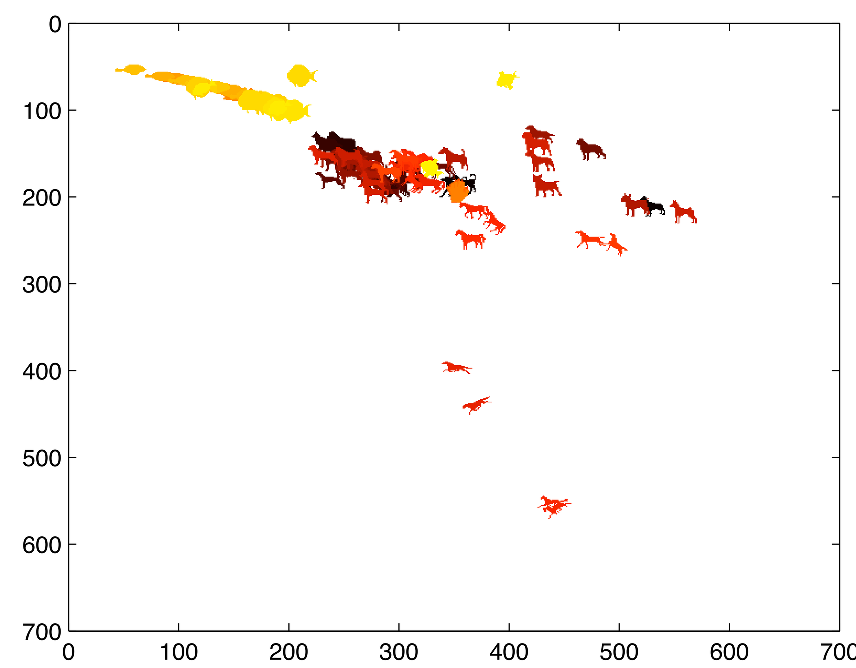


$\sigma = 0$

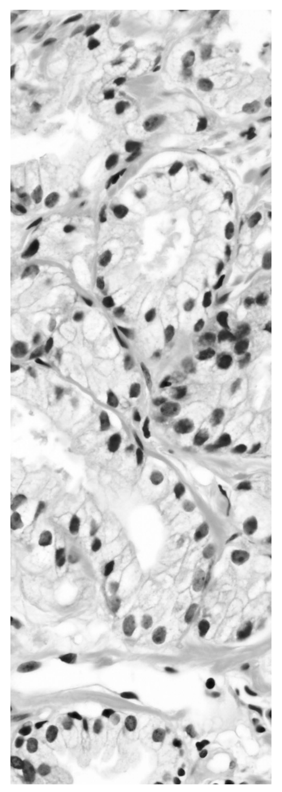


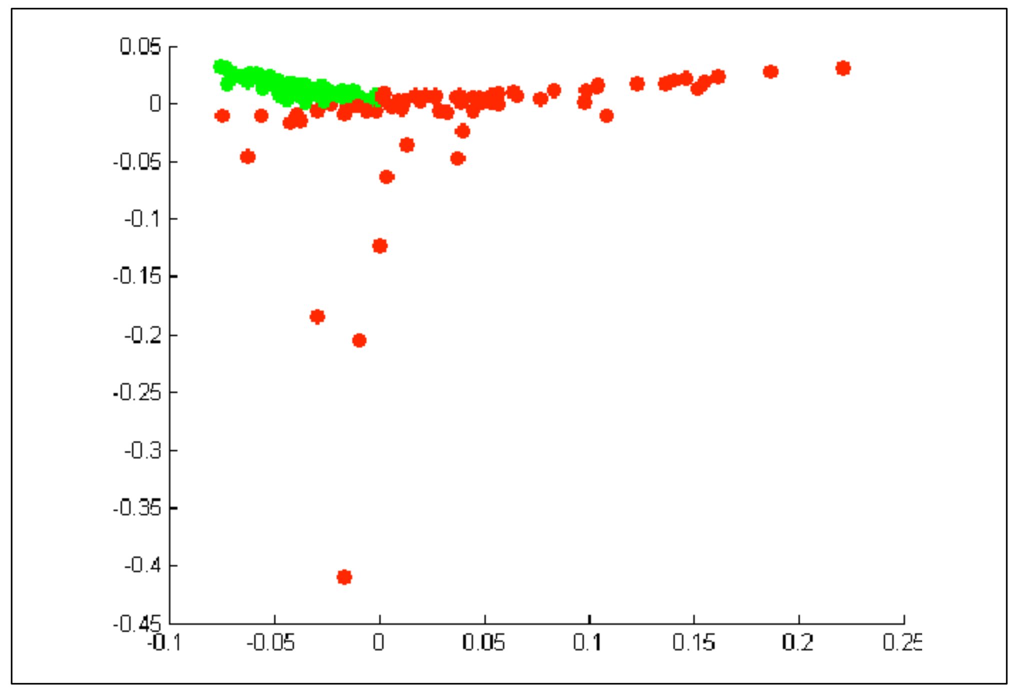
$\sigma = 10$

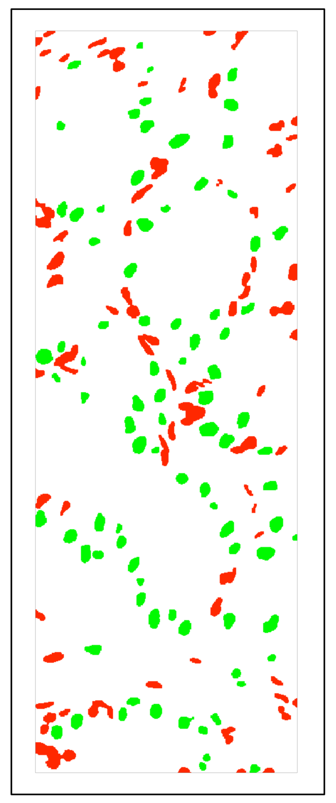




4. Experimental Results







5. Conclusions

✓ A non-linear embedding method is proposed to extract the intrinsic parameters of the shape manifolds, yielding promising preliminary results.

✓ Future directions include application of this framework to classification and modelling of shapes.

Key References

1. R. Coifman and S. Lafon. Diffusion maps. *Applied and Computational Harmonic Analysis, Special Issue on Diffusion Maps and Wavelets*, 21:5–30, July 2006.
2. L.J.P. van der Maaten, E.O. Postma, and H.J. van den Herik. Dimensionality reduction: A comparative review. Preprint, 2007.