Pricing Games in Networks

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Many Computer Science Games

- Routing:
  routers choose path for packets though the Internet
- Bandwidth Sharing:
  routers decide how to share limited bandwidth between many processes
- Load Balancing
  Balancing load on servers (e.g. Web servers)
- Network Design:
  Independent service providers building the Internet

Typical Objectives:

Minimize Delay

- Routing:
  routers choose path for packets though the Internet
- Load Balancing:
  Balancing load on servers (e.g. Web servers)

Minimize Cost

- Bandwidth Sharing:
  routers decide how to share limited bandwidth between many processes
- Network Design:
  Independent service providers building the Internet

Combine Cost and Delay

Prices in Market Models

Exchange market:

- buyers and sellers bring goods
- Market sets prices

Where do prices come from?

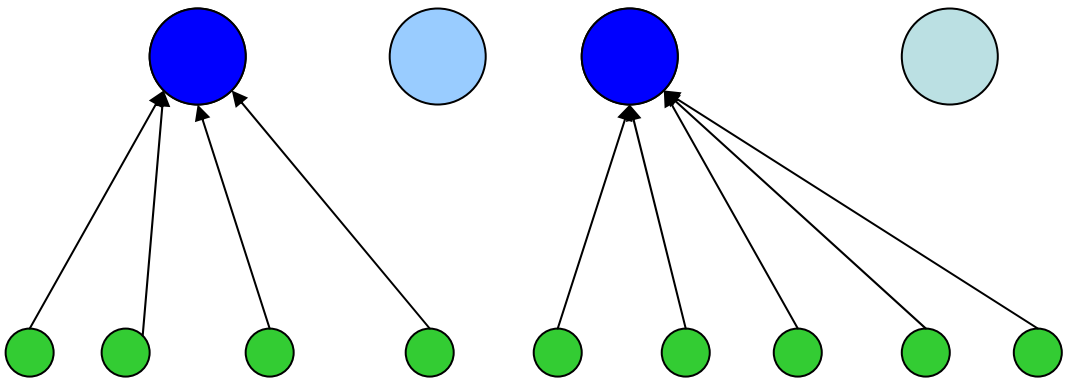
- Efficient algorithms for finding prices
  - Vazirani
- Tatonnement process
  - Cole-Fleischer

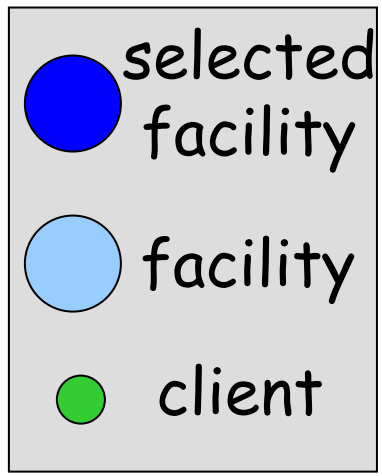
Is setting prices a game?

Price setting as part of a game

Facility location game [Vetta'02]

- Service providers choose locations
- and then select prices
- and users select location based on a combination of price + distance to selected location





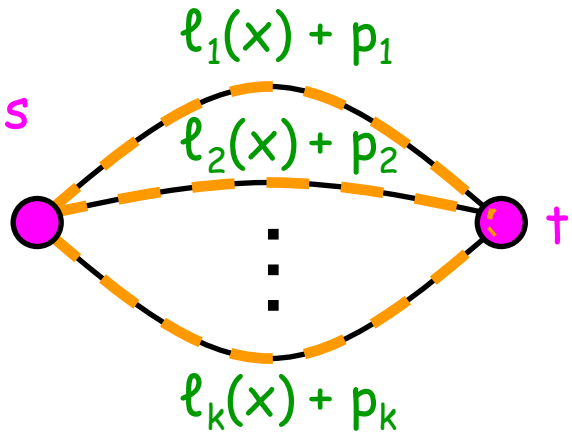
Price of Anarchy: 2

Price setting as part of a game (2)

Pricing Game for Selfish Traffic

[Acemoglu & Ozdaglar], [Hayrapetyan & T & Wexler]

- Service provides choose prices p_i
- users select providers minimizing price + delay (congestion based)



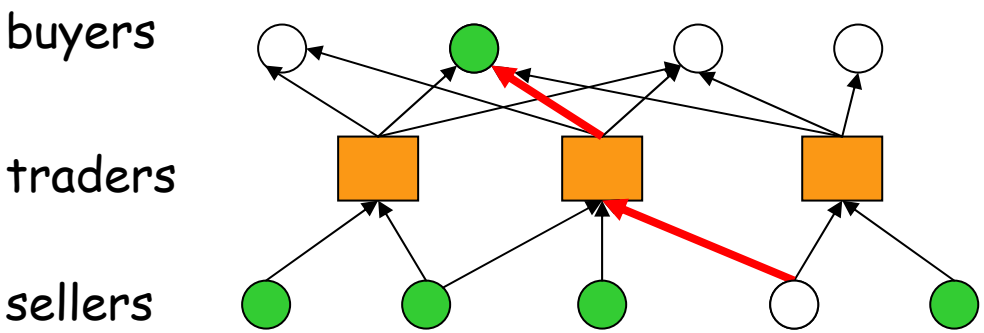
Price of Anarchy bound 3/2 for concave demand

Price Setting in Markets as a Game

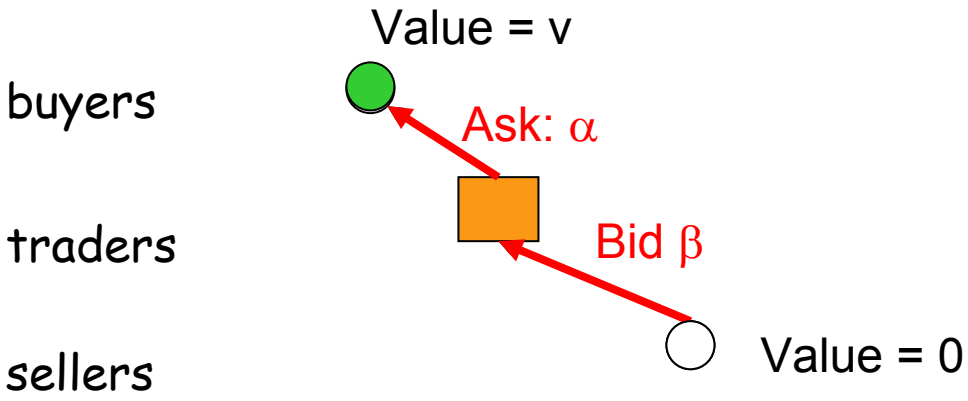
[Larry Blume, David Easley, Jon Kleinberg, T] in EC'07

Example: financial markets

- buyers and sellers come to market
- Market makers (intermediaries) connect them
- Market makers set prices (asks and bids)
- Trade occurs based on prices



Trade though Agents



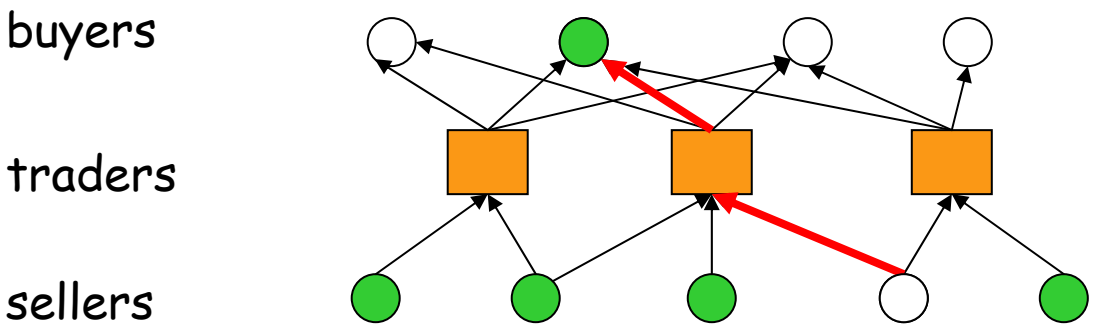
Traders connects buyers and sellers

Traders offer price to sell (α) and buy (β)

Sellers and buyers choose best offers

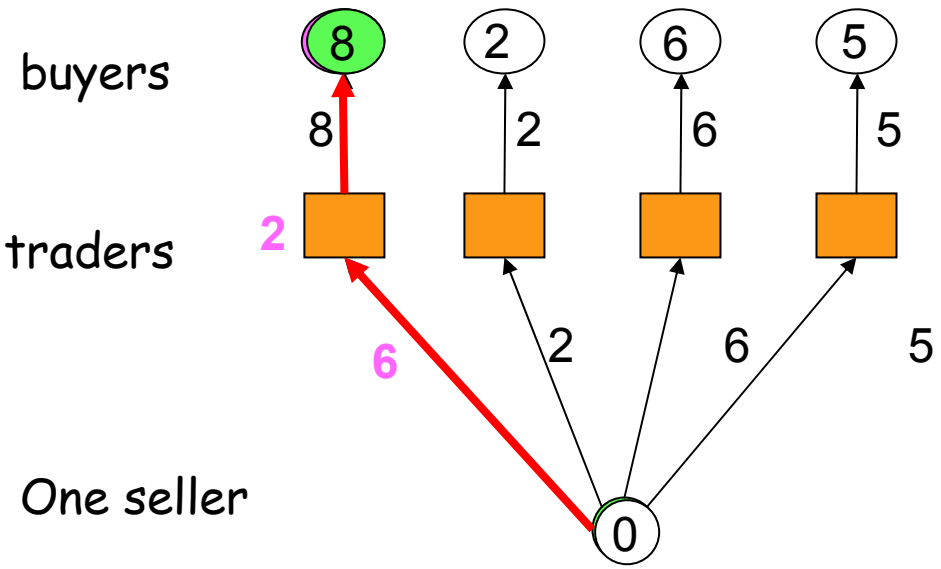
Trade occurs

Networks of Sellers and Buyers



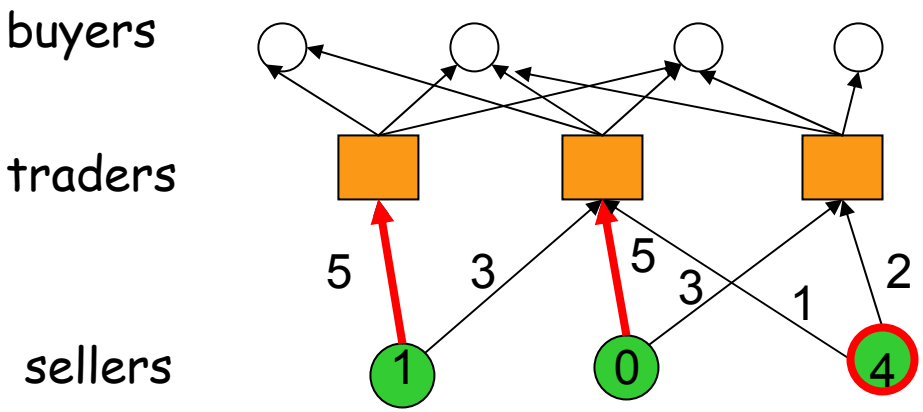
- Traders connect different buyers and sellers
- Traders make price offers to sell and buy
  - Offered prices may differ
- Sellers and buyers choose best offer
  - Sellers choose max
  - Buyers choose min
- and trade occurs

Example: Auction



Buyer with maximal value: 8
Trader offers to buy: monopoly
Trader offers to sell: competition for the seller
Transaction at second best price
trader makes profit

Game Definition

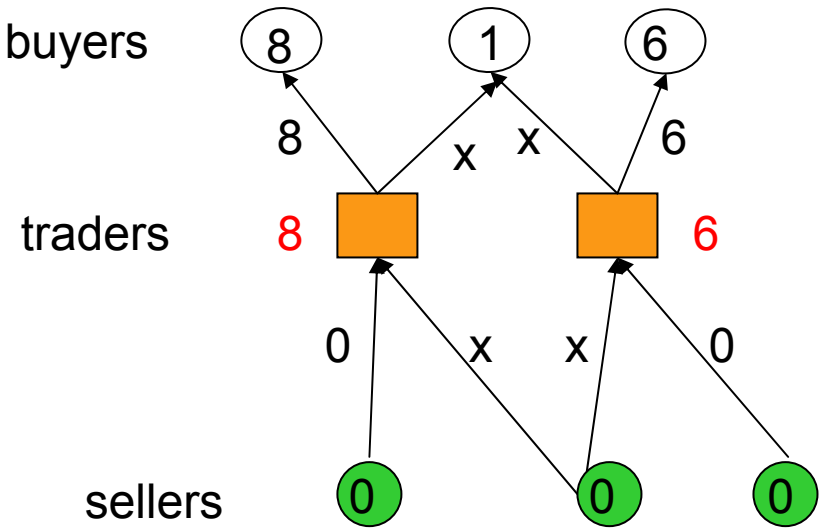


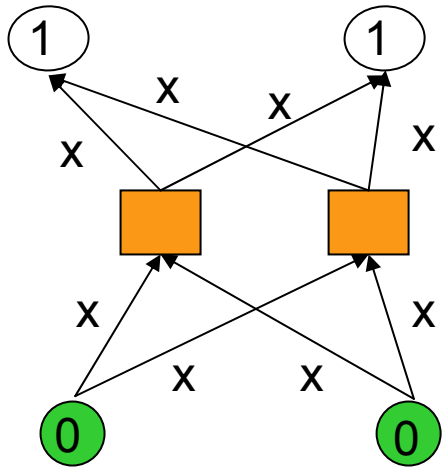
Buyers and sellers valuation public knowledge

The Game:

- Traders make price offers to sell and buy
- Sellers and buyers choose best offer
- Solution concept: subgame perfect equilibrium

Example: competitions





Monopoly prices

Any value $0 \leq x \leq 1$ is subgame perfect equilibrium

- perfect competition

traders only make profit from monopoly

Questions About Market Game

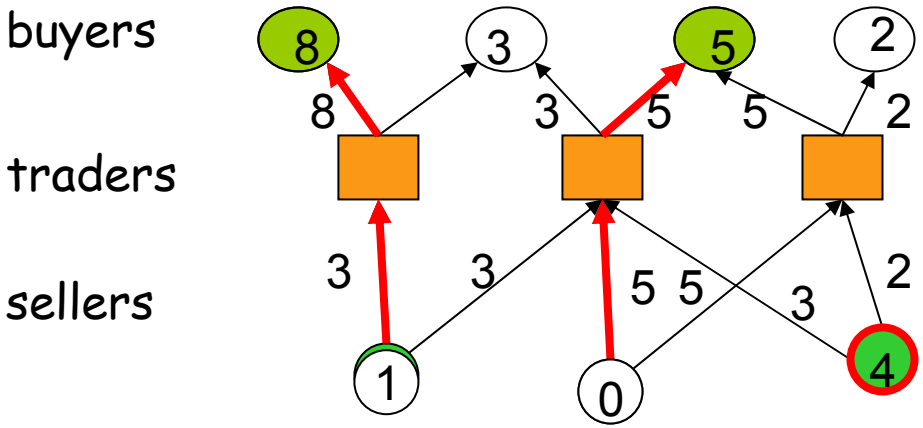
Questions:

- Is there a subgame perfect equilibrium?
- how good is this outcome?
- Who ends up with the profit?

Extensions to distinguishable goods

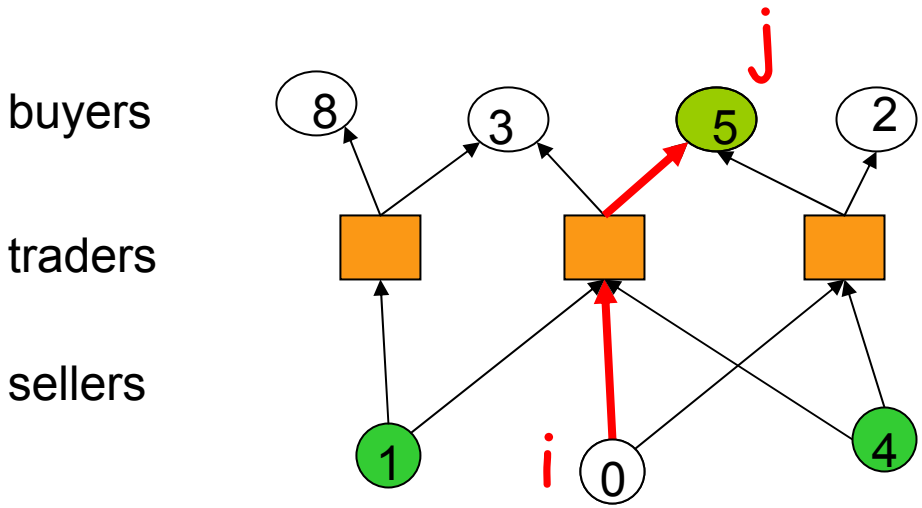
- Example: Job market
  - Seller = job seeker
  - Buyer = hiring company
  - Both have preferences over the others

Results I



- Subgame perfect equilibrium exists
  - In pure strategies
- Outcome socially optimal
  - = Total valuation of those with goods is maximized
    - Note prices do not directly effect social welfare
    - Only buyers and sellers who end up with the good

What is Socially Optimal?



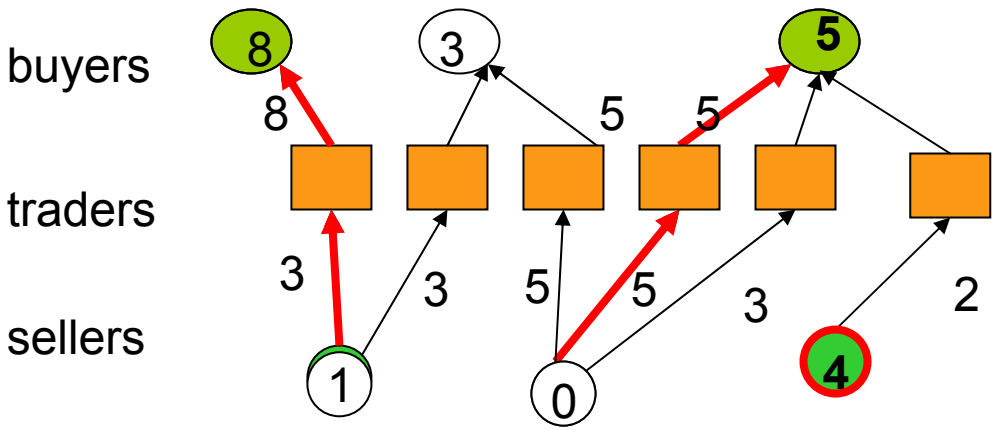
Max Value Matching problem

- Value of connecting seller i - buyer j = $=v_j - v_i = 5-0 = v(i,j)$
- Maximum social value = maximum value matching in the induced bipartite graph

Socially optimal: proof

Simple special case: pair traders

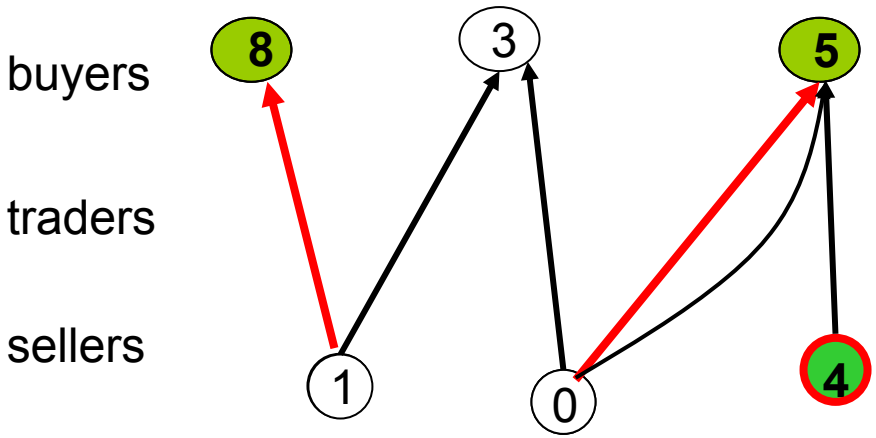
- Each traders connect one buyer and one seller



Max value matching problem:

Value of edge = value of matching buyer to seller

Proof for Pair Traders



v(i.j)= value of matching buyer j to seller i

Matching problem as linear program

LP

$$\text{Max } \Sigma_{ij}\, v(i,j)\, x_{ij}$$

$$\Sigma_j\, x_{ij} \leq 1 \text{ for all } i$$

$$\Sigma_j\, x_{ij} \leq 1 \text{ for all } j$$

$$x \geq 0$$

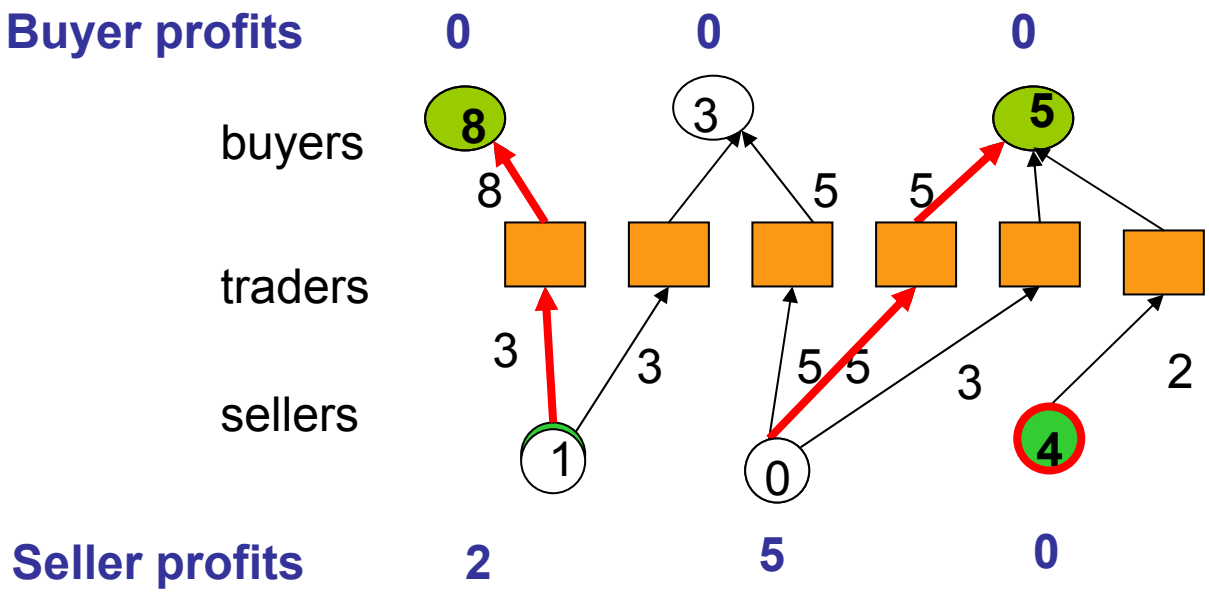
Dual-LP

$$\min\ \Sigma_i\, y_i + \Sigma_j\, y_j$$

$$y_i + y_j \geq v(i,j) \text{ for edge } (i,j)$$

$$y \geq 0$$

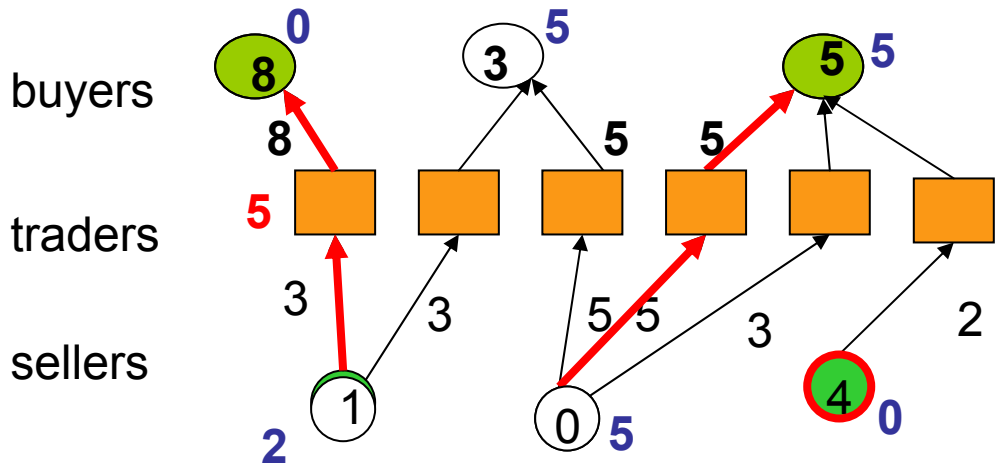
Proof for Pair Traders



Theorem: Seller and buyer profits form linear programming dual variables with complementary slackness

$\Rightarrow$ solution is of maximum value

Complementary Slackness?



Theorem: Seller and buyer profits satisfy complementary slackness

- Seller or buyer makes money $\Rightarrow$ involved in sale
  - $y_i > 0$ implies than i is matched $\Sigma_j x_{ij} = 1$
- Trader makes money $\Rightarrow$ involved in sale
  - $y_i + y_j < v(i,j)$ for edge (i,j) than (i,j) in matching
- Trader is not in use $\Rightarrow$ no trade opportunity
  - Edge (i,j) not used then $y_i + y_j \geq v(i,j)$

Equilibrium exists and socially optimal

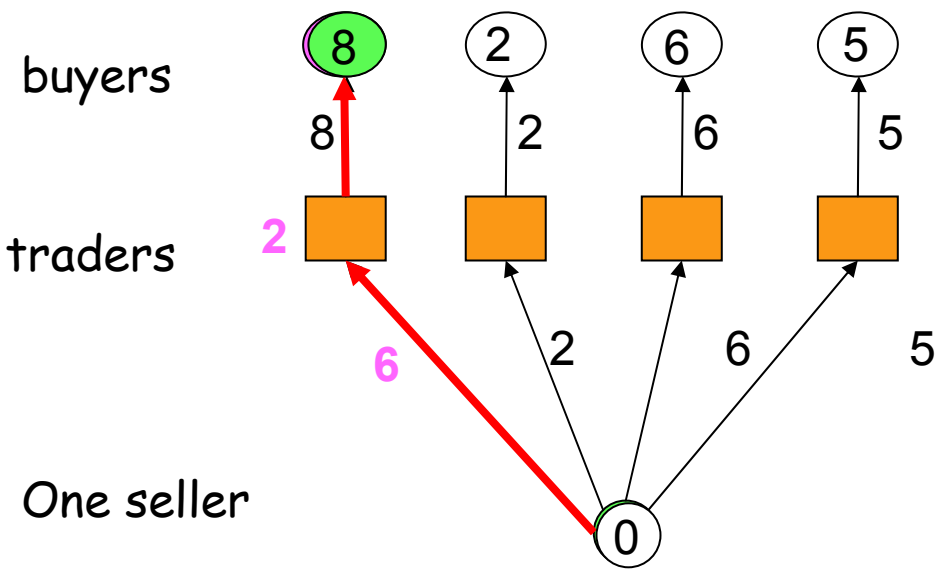
Theorem:

1. Seller and buyer profits satisfy complementary slackness, hence trade maximizes social value
2. Optimal dual solution can be used to create (pure) subgame perfect equilibrium

Extends also to

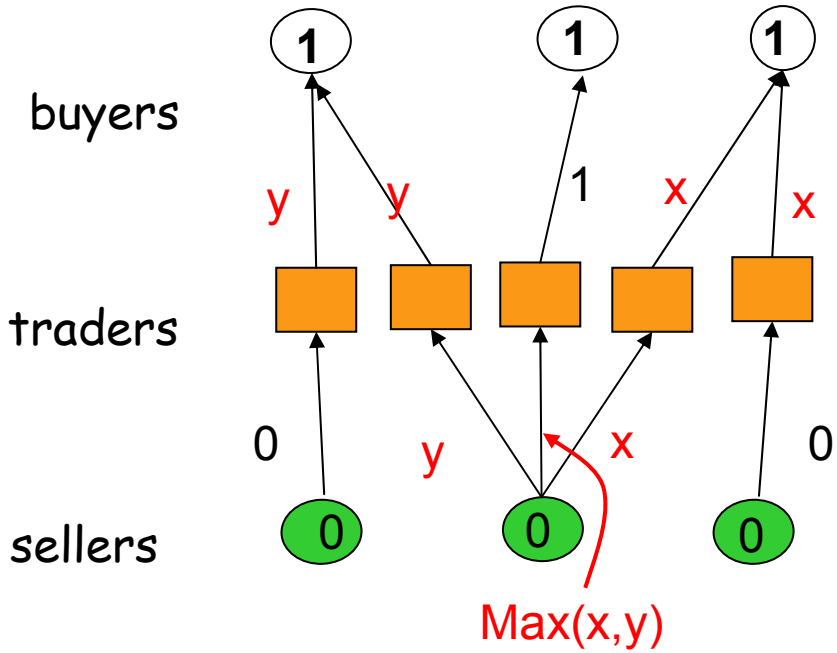
- general traders and
- distinguishable goods (job-market)

Who ends up with the profit?



Range of Trader Profit?

Trader profit can vary:

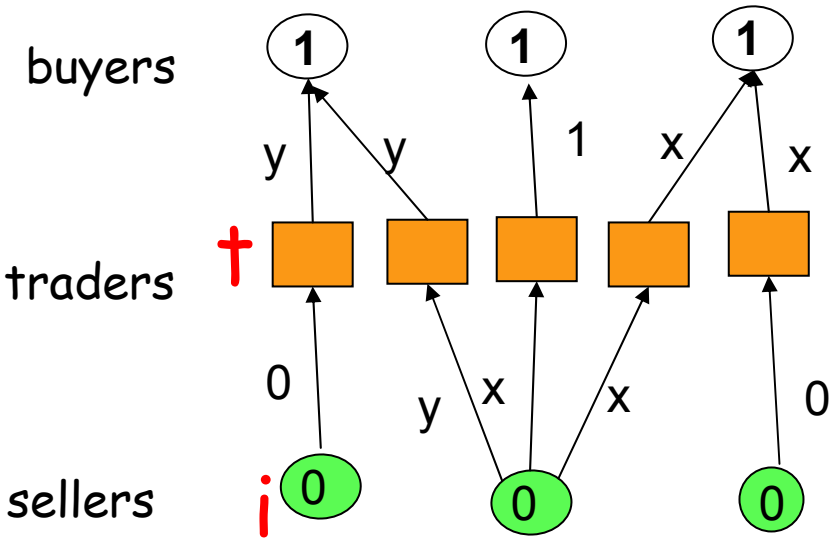


Monopoly ask and buy values

Subgame perfect equilibrium for any bid value $y,x \in [0,1]$

Trader profit is $x+y+(1-x) = 1+y$ between 1 and 2

Results II



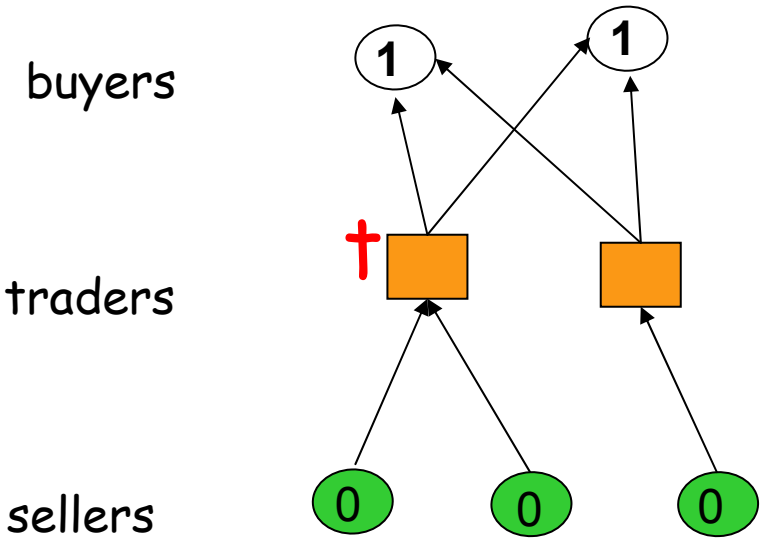
Theorem 1: we can get max. and min. possible profit in poly time

Theorem 2: trader t can make profit if and only if its connection to a seller of buyer i is essential for social welfare.

Analogous to VCG,

- but it's "budget balanced"
- and ….

Maximum possible profit?

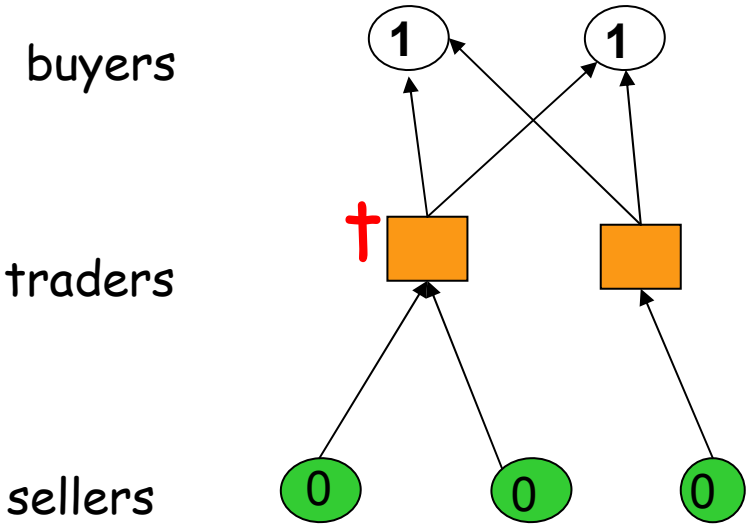


Theorem: trader t can make profit if and only if its connection to a seller of buyer i is essential for social welfare

Note: trader t <u>cannot</u> make profit!

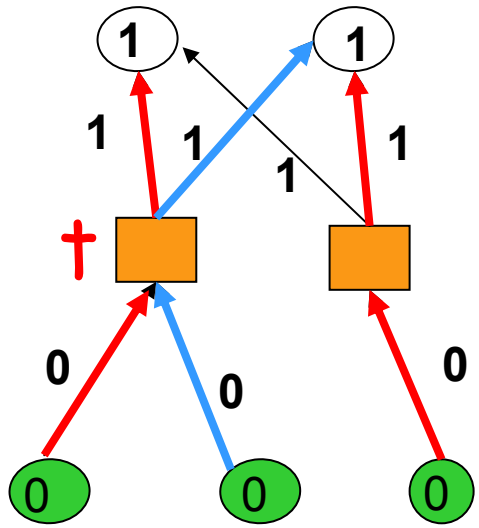
- Trader is essential (without t maximum social value is only 1)
- But no single connection to a seller or buyer is essential

Trader t cannot make profit?



- Trader is essential (without t social value =1)
- But no single connection to a seller or buyer is essential

One example



This is not a Nash

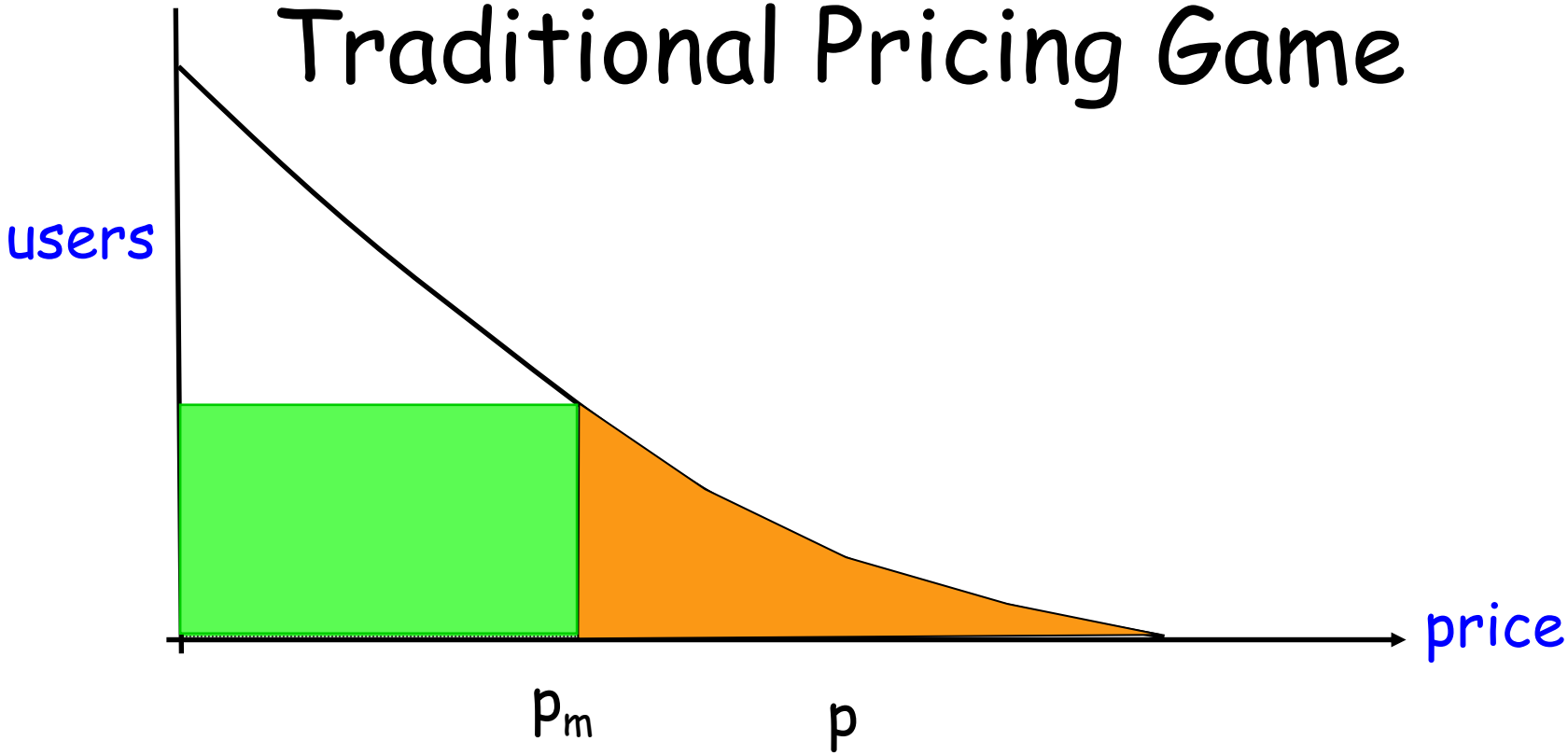
Summary of Market Pricing Game

Price-setting as a strategic game

- Subgame perfect equilibrium as solution
- Pure equilibrium exists
- And is always socially optimal

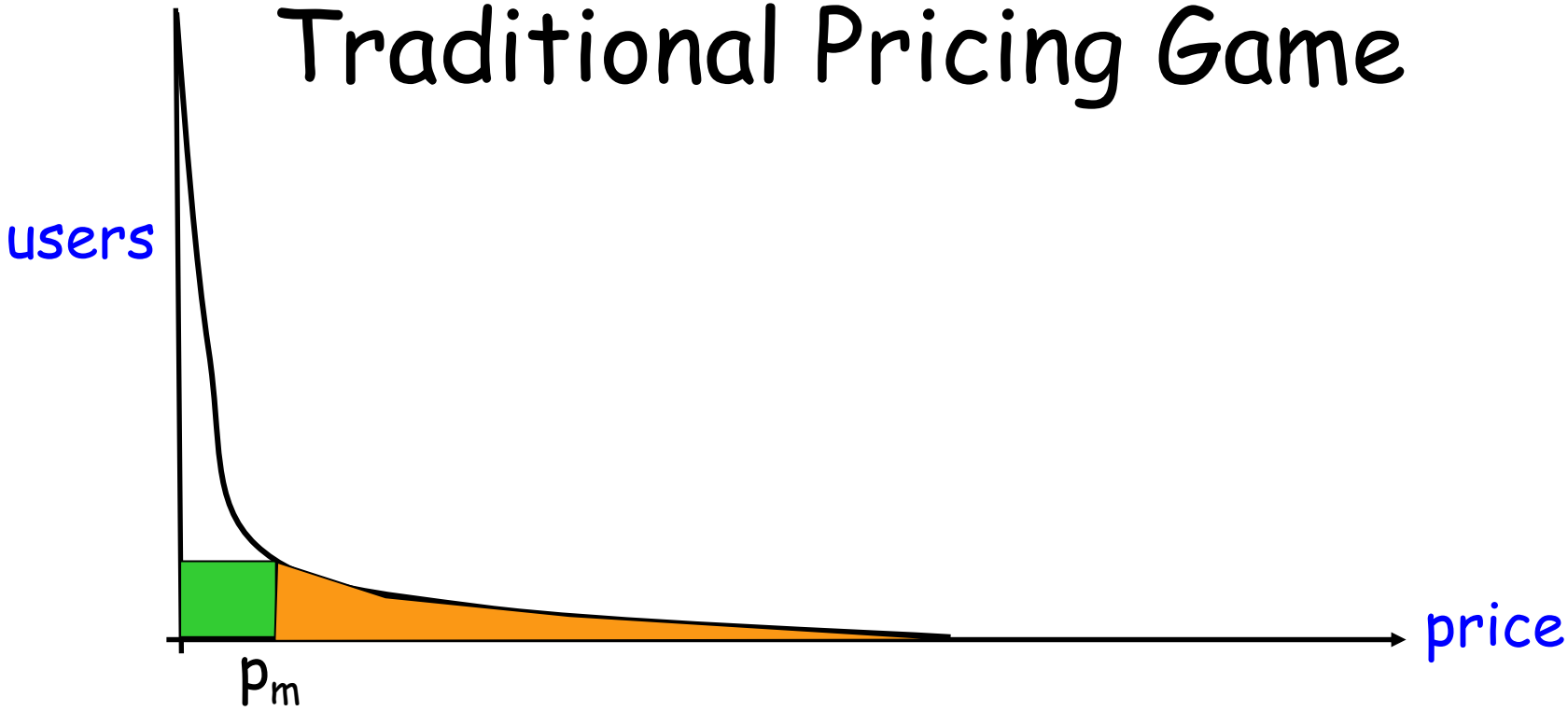
Price setting socially has pure equilibrium and is optimal ??????

Traditional Pricing Game



- Demand curve
- Price p and number of users
- The profit resulting from price p
- Monopolist profit
- **Welfare** at monopoly price

Traditional Pricing Game



Demand curve and **Welfare** at monopoly price p_m

No distinction between profit and user value

Optimal welfare with price 0

$\Rightarrow$ **Price of Anarchy bad**

Our Pricing Market Game

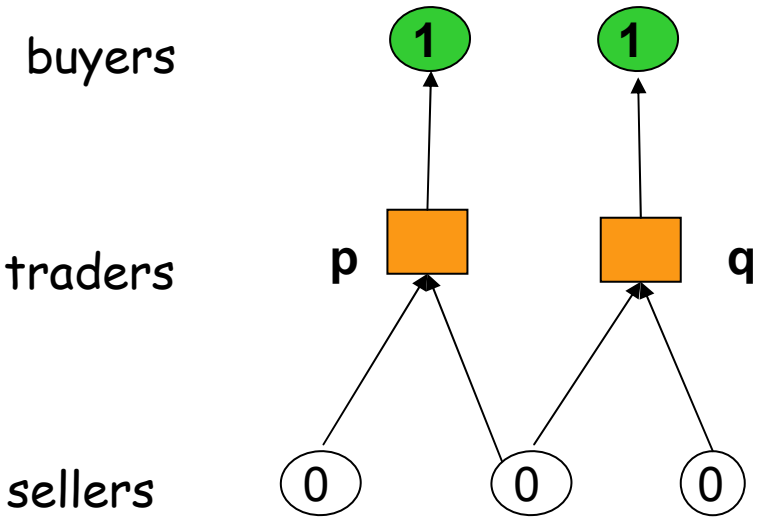
Allows individual pricing

Pure pricing with individual price:

$\Rightarrow$ No price of anarchy

But, monopolist extracts all the profit

Equilibrium exists?



Note: No price discrimination

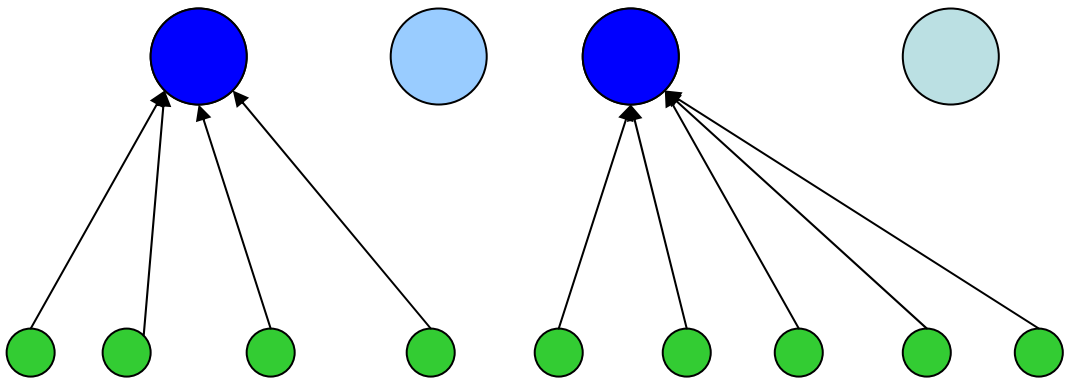
$\Rightarrow$ equilibrium may not exists

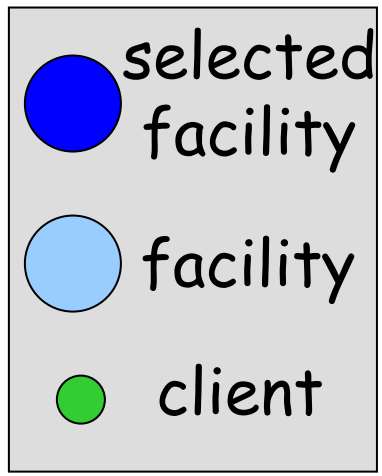
If $p \geq \frac{1}{2}$ then $\Rightarrow q=1$

If $q=1$ then $\Rightarrow p=1-\varepsilon$ then $q=1-2\varepsilon$ etc

Facility location game [Vetta'02] (revisited)

- Service providers choose locations
- and then prices (allows individual pricing)
- and users select location based on a combination of price + distance to selected location



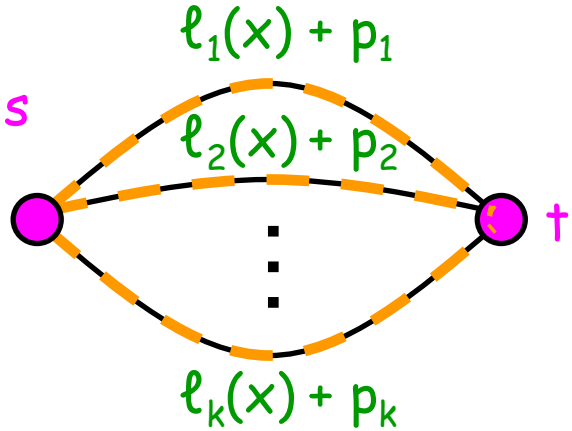


Price of Anarchy: 2

Pricing Game for Selfish Traffic (revisited)

[Acemoglu & Ozdaglar], [Hayrapetyan & T & Wexler]

- Service provides choose prices p_i (single price/link)
- users select providers minimizing price + delay (congestion based)



Price of Anarchy bound 3/2 for concave demand

Conclusion

We studied a market game where price setting is strategic behavior

[Blume, Easley, J. Kleinberg, T in EC'07]

Price setting in other context?

- Facility location
- Link pricing with delays
- Many other natural contexts to understand