Communication Problems in Random Line-of-Sight Ad-hoc Radio Networks*

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Abstract

The *line-of-sight networks* is a network model introduced recently by Frieze et al. (SODA'07). It considers scenarios of wireless networks in which the underlying environment has a large number of obstacles and the communication can only take place between objects that are *close in space* and are *in the line of sight* (are visible) to one another. To capture the main properties of this model, Frieze et al. proposed a new *random networks model* in which nodes are randomly placed (with probability p) on an $n \times n$ grid and a node can see (can communicate with) all the nodes that are in at most a certain fixed distance r and which are in the same row or column.

Frieze et al. concentrated their study on basic structural properties of the random line-of-sight networks, like the connectivity, k-connectivity, etc., and in this paper we focus on the communication aspects of the random line-of-sight networks in the scenario of ad-hoc radio communication networks. We consider the classical ad-hoc radio communication model adjusted to the scenario of the line-of-sight networks and we study two fundamental communication problems: *broadcasting* and *gossiping*.

We present three algorithms for these two basic communication problems in the random line-of-sight networks. The first algorithm solves the gossiping problem assuming that each node knows its location in the grid; it performs $\mathcal{O}(r^4 p^2 \log n \log r / \log(rp) + n/r)$ steps. Our two other algorithms work even in the model of unknown networks (a node does not know its location nor knows its neighbors and the topology of the neighborhood is unknown in advance). The first one is a distributed deterministic algorithm for broadcasting that runs in $\mathcal{O}(r^4 p^2 \log n \log r / \log(rp) + n/r)$ steps, and the second is a distributed randomized algorithm for gossiping that runs in $\mathcal{O}(r^4 p^2 \log^2 n \log r / \log(rp) + n/r)$ steps. Our algorithms

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are especially efficient in the most interesting scenario: when the distance r is not too large and the product $r \cdot p$ is just a little larger than that needed to ensure the connectivity of the network. In particular, if $r = \mathcal{O}(n^{1/5}/\log^{3/5} n)$, all our algorithms achieve the asymptotically optimal number of steps, which is proportional to the diameter of the network.

1 Introduction

In this paper we study basic communication properties of *random Line-of-Sight networks*, a model of wireless networks introduced recently by Frieze et al. [12]. The model of line-of-sight networks has been motivated by wireless networking applications in complex environments with obstacles. It considers scenarios of wireless networks in which the underlying environment has obstacles and the communication can only take place between objects that are close in space and are in the line of sight (are visible) to one another. In such scenarios, the classical random graph models [3] and random geometric network models [20] seem to be not well suited, since they do not capture main properties of environments with obstacles and of line-of-sight constraints. Therefore Frieze et al. [12] proposed a new random network model that incorporates two key parameters in such scenarios: range limitations and line-of-sight restrictions. In the model of Frieze et al. [12], one places points randomly on a 2-dimensional grid and a node can see (can communicate with) all the nodes that are in at most a certain fixed distance and which are in the same row or column. One motivation is to consider urban areas, where the rows and the columns correspond to “streets” and “avenues” among a regularly spaced array of obstructions.

Frieze et al. [12] concentrated their study on basic structural properties of the line-of-sight networks like the connectivity, k-connectivity, etc. In this paper, we initiate the study of fundamental communication properties of the random line-of-sight networks in the scenario of *ad-hoc radio communication networks*. Our focus is on two classical communication problems: *broadcasting* and *gossiping*. In the broadcasting problem, a distinguished source node has a message that must be sent to all other nodes in the network. In the gossiping problem, the goal is to disseminate the messages in a network so that each node will receive messages from all other nodes.

1.1 Random line-of-sight network and communication protocols

A *line-of-sight network* is defined on a set T of grid points $\{(x, y) : x, y \in \{1, 2, \dots, n\}\}$. Let p, called a *placement probability*, be a parameter of the network, $0 \le p \le 1$. Then for each lattice point x from T we place a wireless device at x independently at random with probability p.

To measure the distance between any lattice points we use the L_1-distance: for two points $p = (i, j)$ and $q = (i', j')$, we define $\mathsf{dist}(p, q) = |i - i'| + |j - j'|$. Each node has a (common) *range* r. There is a communication link between two nodes x, y if and only if x *and* y *are on the same straight line of the grid* and $\mathsf{dist}(x, y) \le r$. Figure 1 illustrates the definition.

As observed in [12], in the scenario of wireless communication, comparing with classic geometric network model, line-of-sight networks capture some important aspects of wireless networks. For example, in urban area, two wireless devices at different streets can not communicate each other even the Euclidean distance between them is quite small.

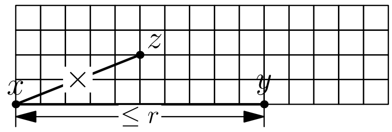

Figure 1: x and y are connected but x and z are not connected.

To study communication in the network, we consider an extension of the so-called *ad-hoc radio networks* model of communication [2, 4, 8, 10, 11, 13, 17]. We assume that all nodes have access to a global clock and work synchronously in discrete time steps called *rounds*. In radio networks the nodes communicate by sending messages through the edges of the network. In each round each node can either transmit the message to all its neighbors at once or can receive the message from one of its neighbors (be in the listening mode). A node x receives a message from its neighbor y in a given round if and only if: *(i)* x does not transmit (is in the listening mode) and *(ii)* y is the only neighbor of x that is transmitting in that round. In the case that constraint (*ii*) is violated, we say a *collision* occurs, in which case no message is received by x. In particular, we assume that x is unable to detect if a collision happened or none of its neighbors did transmit.

In the classical radio network model a node cannot receive any message if more than one of its neighbors transmits because the radio signals from these nodes will interfere. Although this definition is often used to model radio ad hoc networks, we believe that this definition of the collision here is too narrow to fit the framework of ad hoc networks in the line-of-sight model. Since in a radio network messages are sent out via microwave signals, signals can interfere each other if they overlap in space (rather than just by saying that when two or more neighbors transmit). For example, in Figure 2, a sending node z can interfere the node x from receiving the message of y, even though z is not neighbor of x.

To cope with this phenomenon, we add one more constraint to ensure that node x receives the message from y: (*iii*) no neighbor of y is transmitting *and* there is no node z that is transmitting and that lies on a grid-line perpendicular to the segment $\overline{xy}$ and is at distance at most r from the segment $\overline{xy}$, see, e.g., Figure 2. If either condition (*ii*) or (*iii*) is violated, then we say a collision occurs. It is easy to see that any gossiping algorithm that works in our model will certainly work in the traditional model, but not vice versa.

All protocols designed in this paper are distributed and there is no centralized coordinator. We assume the length of the message sent in each single round is at most polynomial in n, and thus, each node can combine multiple messages into one. Throughout this paper, we assume that each node has a unique integral ID that is bounded in polynomial of the number of nodes. The node only knows its own ID, the placement probability p, and its range r; the topology of the network is unknown for any node (this is the so-called *unknown network topology* model, see, e.g., [8, 15]). In addition, we assume *in Section 3 only* that each node knows its own location in the grid. We do not use this assumption in Sections 4–5.

We say an algorithm *completes broadcasting in* T *rounds* if at the end of round T each node already received the message from the source. We say an algorithm *completes gossiping in* T *rounds* if at the end of round T each node already received messages from all other nodes.

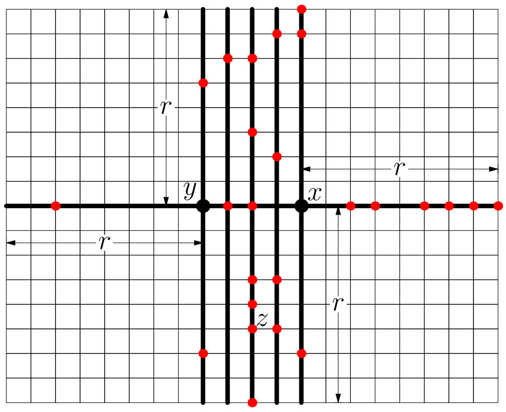

Figure 2: The nodes on bold line could cause collision when y sends a message to x.

1.2 Properties of random line-of-sight networks

Two parameters, r and p, play a critical role in the analysis of properties of the random line-of-sight networks. Frieze et al. [12] proved that when $r = o(n)$ and $r = \omega(\log n)$, if $r \cdot p = o(\log n)$, the network is disconnected with high probability, and therefore no full information exchange (including broadcasting and gossiping processes) can be performed in that case. Therefore we will assume that $r = o(n)$, $r = \omega(\log n)$, and $r \cdot p \geq c \cdot \log n$ for some sufficiently large constant c. This will ensure that the network is connected with high probability and therefore internode communication is feasible.

Assuming that $r \cdot p > c \cdot \log n$ for some sufficiently large constant c, we can also make some further assumptions about the structure of the input network. And so, it is easy to prove, that such a random line-of-sight network has minimum and maximum degree $\Theta(r\,p)$, and has *diameter* $D = \Theta(n/r)$, where these bounds hold with high probability. Besides these easy properties, we also need some other properties that are essential for our algorithms. Before we state them, let us introduce one more notation: for any $r \times r$ square in the grid T, we call the graph induced by the nodes in this sub-grid as an r-*graph*.

Lemma 1 *If* $r\,p > c\ \log n$ *for an appropriate constant* c*, then with high probability:*

(1) all r*-graphs are connected, and*

(2) the diameter of any r*-graph is* $\alpha = \Theta(\log r/\log(p\,r))$.

Proof: This has been proven in [12] (the second bound is proven in the full version of [12]). □

For simplicity of presentation, throughout the paper we will use term α as in the lemma above.

All the claims in Lemma 1 hold *with high probability*, that is, with probability at least $1-1/n^3$. Therefore, from now on, we shall implicitly condition on these events.

1.3 Related prior works

Broadcasting and gossiping have been extensively studied in the ad-hoc radio networks model of communication. Before we present prior works on these problems, we want to clarify that in all these models, broadcasting and gossiping problems are considered on the *communication graph* induced by the accessibility of nodes: if the signal of node x can reach y, then there is an edge from x to y. Let us call the set of nodes that can interfere x from getting message from y a *collision set* (of x with respect to y), denoted by $c(x, y)$. In the classical radio network model studied before, $c(x, y)$ can be implicitly expressed by the communication graph: $c(x, y) = \{z : z \text{ is the neighbor of } y\}$. But in our model, there is no such a clean structure and collision sets $c(x, y)$ cannot be defined in term of the communication graph, see Section 2 for the definition. So all algorithms, even those working in general communication graphs, do not work without any modifications in our model. To the best of our knowledge, we have not seen any work on the broadcasting and gossiping problems in ad-hoc radio networks with collisions as defined in this paper.

1.3.1 Prior works with standard definition of collision sets.

In the *centralized scenario*, when each node knows the entire network, Kowalski and Pelc [17] gave a centralized deterministic broadcasting algorithm running in $\mathcal{O}(D + \log^2 n)$ time and Gąsieniec et al. [13] designed a deterministic $\mathcal{O}(D + \Delta \log n)$-time gossiping algorithm, where D is the diameter and Δ the maximum degree of the network.

There has been also a very extensive research in the *non-centralized (distributed) setting in ad-hoc radio networks* which we present here only very briefly. In the model of *unknown topology networks*, if we consider randomized algorithms, then we know that broadcasting can be performed in $\mathcal{O}(D \log(n/D) + \log^2 n)$ time [8, 16], and this bound is asymptotically optimal [1, 18]. The fastest randomized algorithm for gossiping in directed networks runs in $\mathcal{O}(n \log^2 n)$ time [8] and the fastest deterministic algorithm runs in $\mathcal{O}(n^{4/3} \log^4 n)$ time [14]. For undirected networks, both broadcasting and gossiping have deterministic $\mathcal{O}(n)$-time algorithms [2, 5], and it is known that these bounds are asymptotically tight [2, 15]. Clementi et al. [4] gave an algorithm that runs in $\mathcal{O}(D\,\Delta^2 \log n)$ time. In a relaxed model, in which each node knows also IDs of all its neighbors, $o(n)$-time deterministic broadcasting is possible for undirected networks of the diameter $o(\log \log n)$ [15]. For more about the complexity of deterministic distributed algorithms for broadcasting and gossiping in ad-hoc radio networks, see, e.g., [4, 8, 14, 16, 17] and the references therein.

Dessmark and Pelc [10] consider broadcasting in ad-hoc radio networks in a model of geometric networks (the model is different than that defined in our paper). They consider scenarios in which all nodes either *know their own locations on the plane*, or the labels of the nodes within some distance from them. The nodes use disks of possibly different sizes to define their neighbors. Dessmark and Pelc [10] show that then *broadcasting* can be performed in $\mathcal{O}(D)$ time.

Recently, the complexity of broadcasting in ad-hoc radio networks has been investigated in *random networks* by Elsässer and Gąsieniec [11], Chlebus et al. [6], and Czumaj and Wang [9]. In [11], the classical model of random graphs $G_{n,p}$ is considered. If the input is a random graph from $G_{n,p}$ (with $p \geq c \log n/n$ for a constant c), then Elsässer and Gąsieniec give a deterministic centralized broadcasting algorithm that runs in time $\mathcal{O}(\log(pn) + \log n/\log(pn))$, and a randomized distributed broadcasting $\mathcal{O}(\log n)$-time algorithm. Related results for gossiping are shown by Chlebus et al. [6], who consider the framework of average complexity. Czumaj and Wang [9] consider the gossiping problem in the setting of random geometric networks: n points are placed independently and uniformly at random in a unit square, and two points are connected if there at distance at most r from each other. They show that for a large number of scenarios, gossiping can be solved in random geometric networks in asymptotically optimal time $\mathcal{O}(D)$.

1.4 New contributions

In this paper we present a thorough study of basic communication primitives in random line-of-sight networks. We present three algorithms for broadcasting and gossiping in random line-of-sight networks.

We first consider in Section 3 the most powerful model in which each node knows its own geometric position in the grid (but it does not know positions of other nodes). In this model, we present a distributed deterministic algorithm that completes *gossiping* in $\mathcal{O}(r^4 p^2 \alpha \log n + n/r)$ steps, where α is defined as $\alpha = \Theta(\log r/\log(p\, r))$, as in Lemma 1.

Next, we study the model in which each node knows its own ID, the values of n, r, and p, but it is not aware of any other information about the network. We believe that this is the main model to be studied in line-of-sight networks. We present two algorithms in this model. First, in Section 4, we design a distributed *deterministic* algorithm that completes *broadcasting* in $\mathcal{O}(r^4 p^2 \alpha \log n + n/r)$ steps. Next, in Section 5, we design a distributed *randomized* algorithm that completes *gossiping* in $\mathcal{O}(r^4 p^2 \alpha \log^2 n + n/r)$ steps. These two results demonstrate that even if only a very limited information about the network is known to the nodes, still broadcasting and gossiping can be performed very fast.

The running time of $\mathcal{O}(r^4 p^2 \alpha \log^2 n + n/r)$ steps for these algorithms may seem to be unimpressive, but let us notice that this running time is especially efficient in the most interesting scenario when r is not too large and the product $r \cdot p$ is just a little larger than that needed to ensure the connectivity of the network. In particular, if $r \leq \mathcal{O}(n^{1/5}/\log^{3/5} n)$, our algorithms achieve the asymptotically *optimal number of steps* which is proportional to the diameter of the network. Thus, we demonstrate that for a large range of the input parameters (including those being the most interesting) our algorithms are achieving asymptotically optimal running time.

Finally, in all the bounds above, we have assumed the model of collisions as discussed in Section 1.1. Still, our algorithms can be also run (without any modification) in the classical model of collisions in radio networks. Furthermore, it is not difficult to see that in that case one can speed up all our algorithms to remove a factor $\Theta(r^2)$ from the first term in the running times. And so, the first two algorithms have the running time of $\mathcal{O}(r^2 p^2 \alpha \log n + n/r)$ and the third one runs in $\mathcal{O}(r^2 p^2 \alpha \log^2 n + n/r)$ time. This yields an optimal number of steps for $r = \mathcal{O}(n^{1/3}/\log n)$.

2 Preliminaries

Let V be the set of nodes in the grid. For any node x, define $N(x)$ to be the set of nodes are reachable from x in one hop, $N(x) = \{y \in V : \mathsf{dist}(v, u) \le r$ and x, y are on the same straight line$\}$, where $\mathsf{dist}(v, u)$ is the distance between v and u. Any node in $N(v)$ is called a *neighbor* of v, and set $N(v)$ is called the *neighborhood* of v. For any $X \subseteq V$, let $N(X) = \bigcup_{x \in X} N(x)$. Define the k*th neighborhood* of a node v, $N^k(v)$, recursively as follows: $N^0(v) = v$ and $N^k(v) = N(N^{k-1}(v))$ for $k \ge 1$. Let Δ be the *maximum degree* and D be the *diameter* of the radio network. $\Delta = \max_{v \in V}\{|N(v)|\}$ and $D = \min_{v \in V}\{k : N^k(v) = V\}$. As we mentioned earlier, in our model $\Delta = \Theta(r\,p)$ and $D = \Theta(n/r)$, with high probability.

Definition 2 (Collision sets) *Let* $x, y \in T$ *with* $x \in N(y)$. *We define the* collision set for the communication from y to x, *denoted by* $C(y, x)$, *to be the set of nodes that can interfere* x *from receiving a message from* y. *Set* $C(y, x)$ *contains all nodes* $z \in T$ *that satisfy one of the following:*

1. $z \in N(x) \cup N(y)$, *or*

2. *there is a grid point* q *such that (i)* q *lies on the segment connecting* x *and* y, *(ii) grid line* $\overline{zq}$ *is orthogonal to the grid line* $\overline{xy}$, *and (iii)* $\mathsf{dist}(z, q) \le r$.

It is easy to see that $C(y, x) = \mathcal{O}(r^2\,p)$ with high probability.

Strongly selective families. Let k and m be two arbitrary positive integers with $k \le m$. Following [4], a family $\mathcal{F}$ of subsets of $\{1, \dots, m\}$ is called (m, k)-*strongly-selective* if for every subset $X \subseteq \{1, \dots, m\}$ with $|X| \le k$, for every $x \in X$ there exists a set $F \in \mathcal{F}$ such that $X \cap F = \{x\}$. It is known (see, e.g., [4]) that for every k and m, there exists a (m, k)-strongly-selective family of size $\mathcal{O}(k^2 \log m)$.

In the last years the concept of strongly-selective families has been successfully used to design fast deterministic distributed broadcasting and gossiping algorithms. In particular, Clementi et al. [4] used this approach to obtain a deterministic distributed gossiping algorithm for general radio networks (with the standard notion of collision sets) that runs in $\mathcal{O}(D\,\Delta^2 \log n)$ time.

We can use this approach for line-of-sight networks (and for our notion of collision sets) to obtain the following result.

Lemma 3 *In random line-of-sight networks, for any integer* k, *in* ***(deterministic)*** *time* $\mathcal{O}(k\,r^4\,p^2 \log n)$ *all nodes can send their messages to all nodes in their* k*th neighborhood. The algorithm may fail with probability at most* $1/n^2$ *(where the probability is with respect to the random choice of the nodes in the line-of-sight network).*

Proof: Our arguments follow a nowadays standard approach of applying selective families to broadcasting and gossiping in radio ad-hoc networks, see, e.g., [4]. Since in our setting, for each pair of nodes x and y, we have a collision set of size $|C(y, x)| = \mathcal{O}(r^2\,p)$, with high probability, we will need to use $(n^\lambda, \Theta(r^2\,p))$-strongly-selective family, where the constant c^* hidden in the notation of $\Theta(r^2\,p)$ should be so that $\max_{x,y \in T} |C(y, x)| \le c^*\,r^2\,p$, with high probability.

Let us consider a random line-of-sight network with at most n^2 nodes. We assume (as defined in Section 1.1) that all IDs (labels of the nodes) are distinct integers bounded by a polynomial of n, that is, there are in $\{1, 2, \dots, n^\lambda\}$, for a certain positive constant λ. Let $\mathcal{F} = \{F_1, F_2, \dots\}$ be an $(n^\lambda, \Theta(r^2 p))$-strongly-selective family of size $\mathcal{O}(r^4 p^2 \lambda \log n) = \mathcal{O}(r^4 p^2 \log n)$; the existence of such family follows from our discussion above. Then, consider a protocol in which in step t only the nodes whose IDs are in the set F_t transmit. By the strong selectivity property, for every node u and for every neighbor v of u, there is at least one time step when u does not transmit and v is the only node of $C(v, u)$ that transmits in that step since $C(v, u) \le c^* r^2 p$. Therefore, with this scheme every node will receive a message from all its neighbors after $\mathcal{O}(r^4 p^2 \log n)$ steps. Hence, we can repeat this procedure to ensure that after $\mathcal{O}(k\, r^4 p^2 \log n)$ steps, every node will receive a message from its entire kth neighborhood. □

Observe that in Lemma 3, if $k = \mathcal{O}(n/(r^5 p^2 \log n))$ then the running time is $\mathcal{O}(D)$. In particular, if k is a constant and $r = \mathcal{O}\left(\frac{n^{1/5}}{p^{2/5} \log^{1/5} n}\right)$ then the running time $\mathcal{O}(D)$. Therefore, in order to use this lemma as a subroutine in an $\mathcal{O}(D)$-time algorithm, one requires that $r = \mathcal{O}\left(\frac{n^{1/5}}{p^{2/5} \log^{1/5} n}\right)$.

The following is an immediate corollary of Lemma 3 obtained by setting $k = D$ (which corresponds to the bound from [4] in our setting):

Corollary 4 *Distributed gossiping in random line-of-sight networks can be performed in **deterministic** time* $\mathcal{O}(D\, r^4 p^2 \log n) = \mathcal{O}(n\, r^3 p^2 \log n)$. *The algorithm may fail with probability at most* $1/n^2$.

Since $r\,p = \Omega(\log n)$, the running time of this algorithm is in the best case $\Omega(n \log^4 n)$, and thus it is *superlinear*. The goal of this paper is to develop algorithms that are faster, optimally, those that achieve the running time of the order of D, the diameter of the network, which is a trivial asymptotic lower bound for broadcasting and gossiping.

3 Deterministic algorithm with position information

We consider the gossiping problem in random line-of-sight networks in the model, where each node knows its own geometric position in the grid. In such model, Dessmark and Pelc [10] give a deterministic distributed broadcasting algorithm that runs in $\mathcal{O}(D)$ time. It can be applied to solve the broadcasting problem in our model, with the same running time. We can prove a similar result for gossiping by extending the preprocessing phase from [10] and use an appropriate strongly-selective family to collect information about the neighbors of each node.

Theorem 5 *If every input node knows its location in the* $n \times n$ *grid, then the algorithm Gossiping-Known-Locations-D below will complete gossiping in a random line-of-sight network in deterministic time* $\mathcal{O}(\alpha\, r^4 p^2 \log n + n/r)$. *The algorithm may fail with probability at most* $1/n^2$.

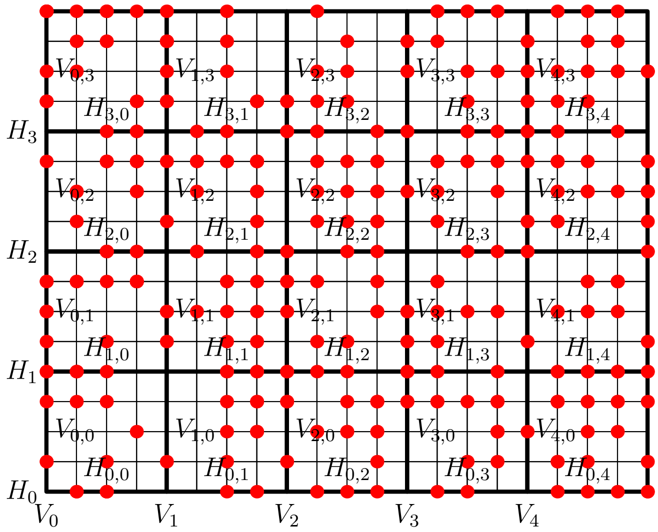

Figure 3: Horizontal and vertical segments

Observe that for $r = \mathcal{O}\left(\frac{n^{1/5}}{\alpha^{1/5}\, p^{2/5}\, \log^{1/5} n}\right)$ (and hence if $r = \mathcal{O}(n^{1/5}/\log^{2/5} n)$) the algorithm achieves the asymptotically optimal running time of $\mathcal{O}(D)$.

Let us introduce some notations first. We label the horizontal lines in the grid as $H_1, H_2, \ldots, H_n$ from bottom to top; label the vertical lines in the grid as $V_1, V_2, \ldots, V_n$ from left to right. We refer the crossing point of H_i and V_j as (H_i, V_j). For each H_i, we further divide H_i into *segments* of length of $r/2$, except for the last segment which is of length at most $r/2$, and label them as: $H_{i,1}, H_{i,2}, \ldots, H_{i,\lceil 2n/r \rceil}$ from left to right. We define $V_{j,1}, \ldots, v_{j,\lceil 2n/r \rceil}$ in a similar way, from bottom to top. See Figure 3.

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Gossiping-Known-Locations-D
Preprocessing: do local gossiping using Lemma 3 (with k = α)
        to ensure that each node knows messages and positions of its neighbors
    for j = 1 to ⌈2n/r⌉ do:
        for c = 1 to 4 do:
            for each i with i mod 4 + 1 ≡ c in parallel do:
                the node with the minimum ID in the H_{i,j} transmits
    for j = ⌈2n/r⌉ downto 1 do:
        for c = 1 to 4 do:
            for each i with i mod 4 + 1 ≡ c in parallel do:
                the node with the minimum ID in the H_{i,j} transmits
    for j = 1 to ⌈2 n/r⌉ do:
        for c = 1 to 4 do:
            for each i with i mod 4 + 1 ≡ c in parallel do:
                the node with the minimum ID in the V_{i,j} transmits
    for j = ⌈2 n/r⌉ downto 1 do:
        for c = 1 to 4 do:
            for each i with i mod 4 + 1 ≡ c in parallel do:
                the node with the minimum ID in the V_{i,j} transmits
Postprocessing: do local gossiping using Lemma 3 (with k = α)
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It is easy to see that for $r\,p \geq c \log n$ with a sufficiently large c, with high probability, there is a node in each segment. For any node x, we define $M_t(x)$ as the messages known by x at step t. For any segment S, we define $M_t(S)$ as the common messages known by all nodes in segment S at step t.

Proof: After the preprocessing in the algorithm Gossiping-Known-Locations-D, by Lemma 3, every node knows which segment it belongs to, and every node knows all other nodes in its segment, including their messages and positions. Therefore, in every segment, all the nodes from that block can select a single representative who will be the only node transmitting (for the entire segment) in all the following time slots.

Let us call all nodes that are scheduled to send by the algorithm *representative nodes*. By our definition of the segment, in any time slot, when a segment (its representative) sends a message, the nearest sending segment is at distance of $2\,r$ from it. So after each sending, the sending segment, say $H_{i,j}$, will successfully send its message $M_t(H_{i,j})$ to segments $H_{i,j-1}$ and $H_{i,j+1}$ if there are such segments. The statement is also true for any segment $V_{i,j}$. For any two nodes v and u in the grid, the algorithm will sent the message of u to v successfully: There are two representative nodes that are within the $r \times r$ square centered at u and v respectively, with high probability. After preprocessing, the message of u will be sent to its representative node. Then after $\mathcal{O}(n/r)$ steps, the message of u will be sent to the representative node of v. After the postprocessing, the message of u will eventually reach v.

By Lemma 3, the running time of the preprocessing step is $\mathcal{O}(\alpha\, r^4\, p^2 \log n)$, and the running time of the postprocessing phase is also $\mathcal{O}(\alpha\, r^4\, p^2 \log n)$. Therefore the total running time of the algorithm Gossiping-Known-Locations-D is $\mathcal{O}(\alpha\, r^4\, p^2 \log n + n/r)$.

In the traditional model of radio networks, because the restriction of collision is weaker, the algorithm Gossiping-Known-Locations-D certainly works. We only need to consider the running time of the algorithm. Since the collision set is $\mathcal{O}(r\,p)$, we can use a smaller strongly selective family of size of $\Theta(r^2\,p^2\,\log n)$ in the preprocessing and the postprocessing. So both preprocessing and postprocessing can be done in $\mathcal{O}(\alpha\,r^2\,p^2\,\log n)$. Therefor the total running time of the algorithm Gossiping-Known-Locations-D is $\mathcal{O}(\alpha\,r^2\,p^2\,\log n + n/r)$.

□

4 Broadcasting and deterministic gossiping with a leader

We now move to a more natural model in which each node knows the values of n, r, and p, knows its own ID (which is a unique integer bounded by a polynomial of n), but it is not aware of any other information about the network. In particular, the node does not know its own location. We believe that this is the main model for the study of principles of the communication in random line-of-sight networks.

We begin our discussion with a slightly relaxed model: there is a special node (*leader*) ℓ in the network, such that ℓ knows that she is the leader, and all other nodes in the network know that they are not the leader. We will show that in this model distributed gossiping can be done *deterministically* in time $\mathcal{O}(\alpha\,r^4\,p^2\,\log n + n/r)$. This will immediately imply a deterministic distributed broadcasting algorithm with asymptotically the same running time.

We start with the same preprocessing as that used in algorithm Gossiping-Known-Locations-D from Section 3. This takes $\mathcal{O}(\alpha\,r^4\,p^2\,\log n)$ steps. After the preprocessing, each node knows its second neighborhood with high probability.

4.1 Gossiping along a grid line

We will first show that the gossiping among the nodes belonging to the same grid line can be performed in optimal $\mathcal{O}(1/r)$ time if there is a leader on the grid line.

We begin with two lemmas that estimate the size of the join neighborhood in random line-of-sight networks.

Lemma 6 *For any pair of nodes u and v belonging to the same grid line, with high probability:*

(i) if $\mathsf{dist}(u, v) \le r/2$ *then* $|N(u) \cap N(v)| \ge 1.3\,r\,p$*, and*

(ii) if $\mathsf{dist}(u, v) \ge r$ *then* $|N(u) \cap N(v)| \le 1.2\,r\,p$.

Proof: Let $B(N, P)$ be the binomial random variable with N trials and the success probability P, that is, for every integer s, $0 \le s \le N$, $\mathbf{Pr}[B(N, P) = s] = \binom{N}{s} P^s (1-P)^{N-s}$. We are using two classical Chernoff bounds: for $0 < \epsilon < 1$: $\mathbf{Pr}[B(N, P) \ge (1+\epsilon) \cdot N\,P] \le e^{-\epsilon^2 N P/4}$ and $\mathbf{Pr}[B(N, P) \le (1-\epsilon) \cdot N\,P] \le e^{-\epsilon^2 N P/2}$.

(i) If $\mathsf{dist}(u, v) \leq r/2$ then it is easy to see that $\mathbf{Pr}[|N(u) \cap N(v)| \leq 1.3\, r\, p] \leq \mathbf{Pr}[B(1.5\, r, p) \leq 1.3\, r\, p]$. Therefore, we can use Chernoff bound to obtain $\mathbf{Pr}[|N(u) \cap N(v)| \leq 1.3\, r\, p] \leq \mathbf{Pr}[B(1.5\, r, p) \leq 1.3\, r\, p] \leq e^{-r\, p/75}$, what is smaller than or equal to $1/n^3$, for $r\, p \geq 225 \ln n$.

(ii) If $\mathsf{dist}(u, v) \geq r$ then, similarly, $\mathbf{Pr}[|N(u) \cap N(v)| \geq 1.2\, r\, p] \leq \mathbf{Pr}[B(r, p) \geq 1.2\, r\, p] \leq e^{-r\, p/100} \leq 1/n^3$, for $r\, p \geq 300 \ln n$. □

Lemma 7 *For any node u, if the distance between u and the nearest boundary is greater than or equal to r, then, in each of four directions, with high probability, there is a neighboring node v of u such that* $1.2\, r\, p \leq |N(u) \cap N(v)| \leq 1.3\, r\, p$.

Proof: Fix a node u and one direction. It is easy to see that with high probability, there is a node v such that $0.74\, r \leq \mathsf{dist}(u, v) \leq 0.76\, r$. $1.24\, r\, p \leq \mathbf{E}[|N(u) \cap N(v)|] \leq 1.26\, r\, p$. Similarly as in the proof of Lemma 6, one can show that the following holds with high probability: $1.2\, r\, p \leq |N(u) \cap N(v)| \leq 1.3\, r\, p$. □

The process of the gossiping among the nodes on a grid-line is initialized by one specific node (call it a *launching node*). The launching node u checks its second neighborhood $N^2(u)$, and selects one *representative node*, say v, such that $1.2\, r\, p \leq |N(u) \cap N(v)| \leq 1.3\, r\, p$. Then, u sends a message to v with the aims: *(i)* u transmits its message to v and *(ii)* u informs v that it is picked as representative node. Because of Lemmas 6 and 7, we know $r/2 \leq \mathsf{dist}(u, v) \leq r$.

The process of gossiping along a grid line is working in steps. At the beginning of each step, a node ϖ_t receives a message from node ϖ_{t-1}, and ϖ_t is informed that it is the representative node. Then ϖ_t will pick a representative node ϖ_{t+1} for the next step, and then send a message to ϖ_{t+1} to inform about it. ϖ_t picks ϖ_{t+1} by checking $N^2(\varpi_t)$, and selecting as ϖ_{t+1} any node fulfilling:

1. $1.2\, r\, p \leq |N(\varpi_t) \cap N(\varpi_{t+1})| \leq 1.3\, r\, p$,
2. $|N(\varpi_{t-1}) \cap N(\varpi_{t+1})| \leq 1.3\, r\, p$.

Because of Lemmas 6 and 7, it is easy to see that $r/2 \leq \mathsf{dist}(\varpi_{t+1}, \varpi_t) \leq r$. Moreover, ϖ_{t+1} and ϖ_{t-1} are at the different sides of ϖ_t, for otherwise $\mathsf{dist}(\varpi_{t+1}, \varpi_{t-1}) \leq r/2$ and then the second constraint would be violated with high probability. If in one step ϖ_t is unable to find the ϖ_{t+1} as defined above, then the distance between ϖ_t and its nearest boundary is less than r. In that case ϖ_t simply stops the process and makes itself as the *last representative node*.

We run this process for $2\, n/r$ steps and call these steps *Phase 1*. Then the last representative node, say ϖ_{t+1}, picks ϖ_t as next representative node and initialize the process again, for another $2\, n/r$ steps. These steps define *Phase 2*. Then, the representative nodes in Phase 2 send their messages in reverse order and with that the gossiping along straight-line will be done. These steps form *Phase 3*. The total running time is $\mathcal{O}(n/r)$.

4.2 Broadcasting and gossiping with the leader in the whole grid

Now, we are ready to present our gossiping algorithm in the model with a distinguished leader ℓ. First, the leader will pick an arbitrary direction and do gossiping along the corresponding grid line. As a by product of this algorithm, a set of representative nodes will be chosen and the minimum distance between any pair of them is greater than or equal to $r/2$. Next, each of the representative nodes treat itself as the pseudo-leader, and do gossiping along a grid line in parallel, in an orthogonal direction to that first chosen by the leader. There are two issues to be solved. First, since the distance between these pseudo leaders could be as small as $r/2$, we need to interleave the transmissions in adjacent pseudo-leaders, which yields a constant-factor slow-down. Second, by checking its second neighborhood, a pseudo-leader can indeed find a node in orthogonal direction that first chosen by the leader. (ϖ_i can pick a node y from its neighbors such that $1.2\,r\,p \leq |N(\varpi_t) \cap N(y)| \leq 1.3\,r\,p$ and $|N(\varpi_{t-1}) \cap N(y)| \equiv 1$.)

Next, we repeat the whole process (starting from the first gossiping along a grid line initialized by the leader) once again. It is easy to see that gossiping is done among all representative nodes. For any pair of nodes u and v, there are two representative nodes that are within the $r \times r$ squares centered at u and v respectively, with high probability. After preprocessing, u will send its message to its representative nodes. The message of u then will be sent to the representative node of v in the following steps. Let us run the postprocessing defined in the algorithm of the Section 3, the message of u will be sent to v. The running time is $\mathcal{O}(\alpha\, r^4\, p^2\, \log n + n/r)$. Because of the same reason as discussed in section3, for the traditional model of radio networks, the running time can be reduced to $\mathcal{O}(\alpha\, r^2\, p^2\, \log n + n/r)$.

Theorem 8 *If there is a leader in the random line-of-sight network, then gossiping can be completed in **deterministic** time* $\mathcal{O}(\alpha\, r^4\, p^2\, \log n + n/r)$. *The algorithm may fail with probability at most* $1/n^2$.

Since in the broadcasting problem, there always is a given source node which can be used as a leader, Theorem 8 immediately implies that broadcasting can also be done in the same time.

Theorem 9 (Deterministic broadcasting in random line-of-sight networks) Distributed broadcasting in random line-of-sight networks can be performed in deterministic time $\mathcal{O}(\alpha\, r^4\, p^2\, \log n + n/r)$.

5 Fast distributed randomized gossiping

We continue our study of the model of random line-of-sight networks in which each node knows the values of n, r, and p, knows its own ID, but it is not aware of any other information about the network. As we have seen in the previous section, if the nodes in the network can elect a leader then gossiping could be done within the time bounds stated in Theorem 8. The problem of leader election is difficult in our setting, but as we will show in this section, randomized leader election can be solved efficiently.

At the beginning, each node independently and uniformly at random selects itself as the *leader* with probability $\frac{\log n}{n^2 p}$. By simple probabilistic arguments, one can prove that exactly $\Theta(\log n)$ leaders are chosen with high probability. Then we want to execute a distributed *minimum finding* algorithm to eliminate all of them but one, and the one chosen will have the lowest ID.

Each of these leaders will pick four representative nodes along four directions, respectively, and execute the process of *Phase 1* as described in Section 4.1. Let us call it *fast transmission*. We interleave fast transmission with the preprocessing of the algorithm in Section 3. Let us call it *slow transmission*. In the odd steps, every node follows the schedule of fast transmission, and in the even steps, every node follow the schedule of slow transmission. Since we have more than one leader, it is possible that transmission *collisions* can occur. We are able to detect these collisions, because after ϖ_t picks ϖ_{t+1} as the next representative node and informs it, in the next step, ϖ_t is expected to receive an acknowledgement of the successful transmission from ϖ_{t+1}. If the acknowledgement is not received, ϖ_t knows that a collision happened.

When a node, say u, detects a collision, in the following $\mathcal{O}(\alpha r^4 p^2 \log n)$ steps u will send nothing (stay in the listening mode) in odd steps, and run slow transmission as before in even steps. By the property of the strongly selective family, u will eventually receive the messages of other representative nodes that are transmitting for their own leaders during this period. Then u compares the ID of its own leader with all other leader's ID that it just received. If (at least) one of those ID is smaller than the ID of its leader, u will send an "eliminating" message back to its leader, reversely along the path through which the leader sent the message to it. The leader eliminates itself after getting this message. If the ID of u's leader is the smallest one, u resumes the fast transmission. Altogether, a node will encounter at most $\mathcal{O}(\log n)$ collisions when it transmits toward any boundary. [1] Therefore the slow down caused by collisions is small and the total running time is $\mathcal{O}(r^4 p^2 \alpha \log^2 n + n/r)$.

Now we can see that the leader is either eliminated or successfully transmits its ID along four directions. Thus, for any pair of surviving leaders, there is a pair of representative nodes in one $r \times r$ square. If we run the postprocessing from Section 3, the two representative nodes will exchange information about their leaders. Again, the representative node that holds the larger ID will transmit an "eliminating" message to its leader. The running time is $\mathcal{O}(r^4 p^2 \alpha \log^2 n + n/r)$. After this procedure, all but one leader with the smallest ID will survive. Once again, because of the same reason as disscussed in section 3, the running time can be reduced to $\mathcal{O}(r^2 p^2 \alpha \log^2 n + n/r)$ for the model of traditional radio networks.

Now, we can summary our result above.

Theorem 10 *In the random line-of-sight network, there is a distributed randomized algorithm that finds a single node which is approved by all the nodes in the network as a leader, and which completes the task in time $\mathcal{O}(r^4 p^2 \alpha \log^2 n + n/r)$, with high probability.*

Finally, we can combine Theorem 10 with the result from the previous section, Theorem 8, to obtain the following result.

[1] It seems that if two nodes with distance less than or equal to r send along the same direction, there are a lot of collisions. But in this case, both nodes will notice the collision at once, so after waiting for $\mathcal{O}(\alpha r^4 p^2 \log n)$ steps, the one with small ID will stop transmitting.

Theorem 11 (Randomized gossiping) In the random line-of-sight network, distributed gossiping can be completed in randomized time $\mathcal{O}(r^4 p^2 \alpha \log^2 n + n/r)$, with high probability.

6 Conclusions

We have presented three efficient algorithms for broadcasting and gossiping in the model of random sight-of-line networks. If $r = \mathcal{O}(n^{1/5}/\log^{3/5} n)$, then all our algorithms perform $\mathcal{O}(D)$ steps to complete the task, what is clearly asymptotically optimal. While it is certainly very interesting to extend the optimality of these bounds to larger values of r and this is an interesting open problems of extending our results to larger values of r or providing lower bounds for the running times for larger values of r, we believe that the case $r = \mathcal{O}(n^{1/5}/\log^{3/5} n)$ covers the most interesting cases, when the graph is relatively sparse and each node is able to communicate only with the nodes that are not a large distance apart. Therefore, in our opinion the most interesting specific open problem left in this paper is to extend the result from Section 5 to obtain a distributed *deterministic* algorithm for gossiping. Even more interesting is a more general question: what are important aspects of random sight-of-line networks to perform fast communication in these networks. Our work is only the very first step in that direction.

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