A Rigorous Analysis Of Linsker's Hebbian Learning Network

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Abstract — **We propose a novel approach for a rigorous analysis of the nonlinear asymmetric dynamics of Linsker's unsupervised Hebbian learning network. The results provide for the first time comprehensive explanations of the origin of the various structured connection patterns and of the roles of the different system parameters of the model. Our theoretical predictions are corroborated by numerical simulations.**

I. Introduction

For the purpose of understanding the self-organization mechanism of primary visual system, Linsker has proposed a multilayered unsupervised Hebbian learning network with random uncorrelated inputs and localized arborization of synapses between adjacent layers. In Linsker's network, the dynamical equation for the development of the synaptic strength $\omega_\tau(i)$ between a cell in the present layer $\mathcal{M}$ and i-th cell in the preceding layer $\mathcal{L}$ at time τ is given as:

$$\omega_{\tau+1}(i) = f\{\omega_\tau(i) + k_1 + \sum_{j=1}^{N_\mathcal{L}}[q(i,j) + k_2 r(j)]\omega_\tau(j)\} \quad (1)$$

where k_1, k_2 are system parameters which are particular combinations of the constants of the Hebb rule, $f(\cdot)$ is a nonlinear clipping function defined by $f(x) = \omega_{max}$, if $x > \omega_{max}$; $= x$, if $|x| \le \omega_{max}$; and $= -\omega_{max}$, if $x < -\omega_{max}$. Here the asymmetric generalized covariance matrix $q = \{q(i,j)\}_{i,j=1}^{N_\mathcal{L}} = \{Q_{ij}r(j)\}_{i,j=1}^{N_\mathcal{L}}$ is obtained from the covariance matrix $\{Q_{ij}\}$ of activities of $\mathcal{L}$-cells' inputs to the $\mathcal{M}$-cell and multiplying it by a non-negative synaptic density function (SDF) $r(\cdot)$. Based on a rigorous analysis of the parameter space of the above nonlinear asymmetric dynamics, we propose an alternative approach for analyzing Linsker's model.

II. The Whole Set Of Fixed Point Attractors And The Criterion For The Division Of Parameter Regimes

Because of the special form of the nonlinear function $f(\cdot)$, *the whole set of fixed point attractors* of the dynamics in equation (1) is given by

$$\Omega_{FP} = \{\omega | \omega(i)\{k_1 + \sum_{j=1}^{N_\mathcal{L}}[q(i,j) + k_2 r(j)]\omega(j)\} > 0, \forall i\}.$$

Then we define $\Omega^+(\omega) = \{i|\omega(i) = 1\}$ as the set of cells at preceding layer $\mathcal{L}$ with excitatory weight for a connection pattern ω, and $\Omega^-(\omega) = \{i|\omega(i) = -1\}$ as the set of $\mathcal{L}$-cells with inhibitory weight for ω. We introduce the *slope function*

$$c(\omega) = \sum_{j\in\Omega^+(\omega)} r(j) - \sum_{j\in\Omega^-(\omega)} r(j)$$

which is the difference of sums of the SDF $r(\cdot)$ over $\Omega^+(\omega)$ and $\Omega^-(\omega)$, and two k_1-*intercept functions*

$$d_1(\omega) = \max_{i\in\Omega^+(\omega)}\Big(\sum_{j\in\Omega^-(\omega)} q(i,j) - \sum_{j\in\Omega^+(\omega)} q(i,j)\Big)$$

and

$$d_2(\omega) = \min_{i\in\Omega^-(\omega)}\Big(\sum_{j\in\Omega^-(\omega)} q(i,j) - \sum_{j\in\Omega^+(\omega)} q(i,j)\Big).$$

Now *the new rigorous criterion for the division of stable parameter regimes to ensure the development of various structured connection patterns* is

$$d_2(\omega) > k_1 + c(\omega)k_2 > d_1(\omega).$$

That is, for a given SDF $r(\cdot)$, the parameter regime of (k_1, k_2) to ensure that ω is a stable attractor of dynamics (1) is a band between two parallel lines $k_1 + c(\omega)k_2 > d_1(\omega)$ and $k_1 + c(\omega)k_2 < d_2(\omega)$. Thus the *existence* of such a structured receptive field ω as an attractor of dynamics (1) is determined by k_1-intercept functions $d_1(\cdot)$ and $d_2(\cdot)$, and therefore by the covariance matrix q or SDFs $r(\cdot)$ of all preceding layers. On the other hand, the *shape* of a receptive field ω is governed by k_1, k_2 and $c(\cdot)$, and thus only by the SDF $r(\cdot)$ between the present and the preceding layers.

III. Concluding Remarks

Circularly symmetric but localized arbor functions $r(\cdot)$ introduce the *asymmetry* of the generalized covariance matrix q. The resulting *asymmetric* q causes the *diversity* of systematic behaviors in Linsker's network. The *nonlinear* function $f(\cdot)$ in equation (1) which establishes bounds on the synaptic strengths gives rise to the *coexistence* of many attractors for the same family of parameters, and therefore supports the diversity of pattern formation.

Without *localized* synaptic arbor density function, there would be no structured covariance matrix. And without structured covariance matrix which includes localized correlation in afferent activities, no structured receptive fields would exist.

Our rigorous approach unifies the treatment of many diverse problems about dynamical mechanisms and is also applicable to analyzing other unsupervised self-organization systems of connection evolution.

References

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