Locating unstable periodic orbits: When adaptation integrates into delayed feedback control

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Finding unstable periodic orbits (UPOs) is always a challenging demand in biophysics and computational biology, which needs efficient algorithms. To meet this need, an approach to locating unstable periodic orbits in chaotic dynamical system is presented. The uniqueness of the approach lies in the introduction of adaptive rules for both feedback gain and time delay in the system without requiring any information of the targeted UPO periods *a priori*. This approach is theoretically validated under some mild conditions and successfully tested with some practical strategies in several typical chaotic systems with or without significant time delays.

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The problem of stabilizing unstable periodic orbits (UPOs) and equilibria in complex systems has attracted a great deal of attention due to its extensive applications in physics and biology. The seminal work of Ott, Grebogi, and Yorke (OGY) demonstrates that UPOs embedded in chaotic attractors can be stabilized by tiny external forces [1]. Many novel methods on chaos control were thereby proposed to achieve such stabilization in various physical, chemical, and biological systems. However, while these methods are useful for some kind of chaotic systems, they have requirements that are not always met in practice. For example, the original OGY method, although analytically well understood, does not apply to fast-dynamic systems [2]. Conventional feedback control as well as adaptive control, although theoretically sound, requires exact knowledge of the profile of the UPOs or the unstable equilibria prior to an application of such control methods [3]. To overcome these drawbacks in chaos control, delayed feedback control (DFC) was introduced by Pyragas [4]. One advantage of DFC is that it is designed to be noninvasive in the sense that the control signal vanishes when stabilization is achieved. Additionally, DFC does not require precise knowledge of either the profile of the desired goal or the form of the original system. For these advantages, DFC has been adopted in a variety of experimental systems. In particular, with the aid of bifurcation theory, DFC has been analytically shown to be feasible for systems with an odd number of real Floquet multipliers greater than unity [5].

When DFC is adopted for chaos control, the only information required *a priori* is the period of the targeted UPO. It is known that the search for a correct value of the period is difficult for a chaotic system since the values of those periods of the UPO's embedded in a chaotic attractor belong to a null set in the line of real numbers. To overcome this limitation, a scheme was proposed to improve DFC [6], where the time delay of the controlled system is adjusted according to the distance between successive maxima of the output signal. However, due to the chaotic nature of the trajectories and the influence of noise, it was found uneasy in practice to identify those "successive maxima" of the output signal. Additionally, while some other methods were proposed to iteratively adapt the delay time or the control gain to the optimum, for a given penalty function, the adaptation procedure requires many realizations and is relatively time consuming [7]. The analytical result mentioned above that DFC can stabilize some specific UPOs with an odd number of real Floquet multipliers greater than unity is only valid for those UPOs induced by a subcritical Hopf bifurcation in a very small neighborhood of an unstable equilibrium where theoretically no chaos exists. Thus, apart from the unstable DFC [8], which actually involves a complicated computation of the Floquet multiplier along the unknown UPO, it has been found to be extremely hard to directly stabilize the UPOs that are not in the vicinities of equilibria, but are truly embedded in a chaotic attractor characterized by multiple scrolls.

In this paper, we propose a method, which combines the advantages of DFC and adaptive control, to stabilize UPOs embedded in a chaotic attractor *without requiring any information* of the periods of the UPOs and the control gains *a priori*. We provide analytical analysis and numerical simulations on three representative chaotic systems to show the feasibility of this method. We also show the existence of a riddled basin corresponding to the initial values of the time-delayed variables for the controlled trajectories that can be stabilized to different UPOs.

To begin, consider a general nonlinear system such as a chaotic system that is described by a set of ordinary differential equations:

$$\dot{y} = P(y,x), \quad \dot{x} = Q(y,x), \tag{1}$$

where the explicit form of the continuous vector field (P,Q) can be unknown in practice, $y(t)$ stands for the observed output signal, and $x(t)$ is an internal signal that is not available or not of interest. For simplicity, $y(t)$ is taken to be a scalar variable, but all results can be easily generalized to observed variables of higher dimensionality. To stabilize the UPOs embedded in a chaotic attractor generated by Eq. (1), the controlled system with an external continuous-time input is designed as

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$$\dot{y} = P(y,x) + F(t), \quad \dot{x} = Q(y,x),$$

$$F(t) = \gamma(t)\{y[t-\tau(t)] - y(t)\}, \tag{2}$$

where, differing from the original DFC in which the time delay and the control gain are constant, the time delay τ and the control gain γ here are deliberately constructed as estimating variables obeying the following adaptive rules:

$$\dot{\tau} = -r_1\{y[t-\tau(t)] - y(t)\}, \quad \dot{\gamma} = r_2\{y[t-\tau(t)] - y(t)\}^2, \tag{3}$$

with r_1 and r_2 being constant parameters. The initial values for Eqs. (2), coupled with the adaptive rules (3), are all taken as constants on the initial time interval. Adaptive rules (3) manifest that the more far the dynamics from the desired UPOs, the more rapidly both variables adjust for realization of stabilization. Particularly, the monotonicity of γ in Eq. (3) guarantees a persistent and sufficiently large control gain in the process of stabilization. In addition, to guarantee the positiveness of the time delay in practice, τ is reset to the initial value $\tau(0)>0$ if it becomes zero. In the theoretical analysis, the time-delay variable is assumed to be positive.

Next, consider the proposed method for the case with *bounded trajectories* generated by the controlled systems (2) and (3), under the assumptions [A1] that each UPO can be uniquely determined by its component projected onto the y axis and [A2] that the periods of all UPOs in the real number set are at most countable.

Note that the gain function $\gamma(t)$ obeying the adaptive rule in Eq. (3) is monotonous. Thus, if the trajectory $[y(t),x(t),\tau(t),\gamma(t)]$ is bounded, then $\gamma(+\infty)$ exists, so that an integration of the second equation in Eq. (3) yields

$$\int_0^{+\infty} r_2\{y[s-\tau(s)] - y(s)\}^2 ds = \gamma(+\infty) - \gamma(0) < +\infty.$$

This implies that the error $e(t)=y[t-\tau(t)]-y(t)$ is L^2 integrable, and therefore is bounded and satisfies $\dot{e}(t) = g(e,x) - \dot{\tau}P[y(t-\tau),x(t-\tau)] - \gamma(t)e(t) + \gamma(t-\tau)e(t-\tau) - \dot{\tau}\gamma(t-\tau)e(t-\tau)$, where $g(e,x)=P(y,x)-P[y(t-\tau),x(t-\tau)]$. Hence, $\dot{e}(t)$ is uniformly bounded due to the assumed boundedness of the trajectory. This, together with the L^2 integrability and continuity of $e(t)$, yields an error satisfying $e(t) \to 0$ as $t\to+\infty$. Moreover, since the assumed bounded trajectory is generated by the autonomous systems (2) and (3), the ω limit set of this trajectory in the corresponding functional space is nonempty, connected, and invariant [9]. If restricted to the ω-limit set, the function $\tau_t(\theta)=\tau(t+\theta)$ becomes a constant function because $e(t)\equiv 0$ and $F(t)\equiv 0$ in this limit set. This implies either that $\tau(t)$ converges to some constant which is the period of some UPO or that $\tau(t)$ either converges or fluctuates slowly in some interval as $y(t)$ converges to a constant.

The above analysis demonstrates that the proposed method is noninvasive and can stabilize bounded trajectories to periodic orbits or equilibria which are unstably embedded in the original chaotic attractor. In particular, with assumptions [A1] and [A2], the variable $\tau(t)$ restricted in the limit set is a single constant function when $y(t)$ in the limit set is not constant but periodic. This implies that $\tau(t)$ can be used to estimate the period if the stabilized orbits are nontrivially periodic.

It is clear that the above analysis is valid only for *bounded trajectories* of the controlled systems (2) and (3). In fact, it is not easy to analytically verify the boundedness of such trajectories for general systems. However, in practice, this boundedness can be numerically verified for many systems by designing proper piecewise or switched coupling [4]. Also to fulfill the boundedness assumption and noninvasiveness requirement, some impulsive strategy needs to be imported additionally.

Example 1. As a first example to illustrate the feasibility of the proposed method for chaos control, consider the chaotic Rössler system

$$\dot{x}_1 = -x_2 - x_3, \quad \dot{x}_2 = x_1 + 0.2x_2, \quad \dot{x}_3 = 0.2 + x_3(x_1 - 5.7). \tag{4}$$

The second variable x_2 is, as usual, taken to be the output signal. The controlled Rössler system, coupled with the proposed adaptive rules, takes the form

$$\dot{y}_1 = -y_2 - y_3, \quad \dot{y}_2 = y_1 + 0.2y_2 + F(t),$$

$$\dot{y}_3 = 0.2 + y_3(y_1 - 5.7),$$

$$\dot{\tau} = r_1\{y_2(t) - y_2[t-\tau(t)]\}, \quad \dot{\gamma} = r_2\{y_2(t) - y_2[t-\tau(t)]\}^2. \tag{5}$$

Here, to ensure the perturbation $F(t)$ be small at all times, as well as the eventual boundedness of the trajectories of the controlled system (5), take

$$F(t) = I_{\{|S(t)|<F_0\}}S(t) + F_0 I_{\{S(t)>F_0\}} - F_0 I_{\{S(t)<-F_0\}},$$

with $S(t)=\gamma(t)\{y_2[t-\tau(t)]-y_2(t)\}$, where I_A is the indication function of set A and $F_0>0$ is a small constant, i.e., $F(t)$ is the truncated function of $S(t)$. This kind of perturbation configuration was suggested in [4]. Furthermore, to ensure the noninvasiveness of the perturbation, an additional impulsive strategy is adopted [10]: $\gamma(T^*+h)$ is set to the initial value $\gamma(0)$ if (IE-1) $\max_{t\in\mathcal{I}}\{\tau(t)\} - \min_{t\in\mathcal{I}}\{\tau(t)\} < \epsilon_2$ and (IE-2) $|y_2(t)-y_2[t-\tau(t)]| < \epsilon_1$ for all $t\in\mathcal{I}=[T^*,T^*+h]$ and some $T^*>0$. Here, ϵ_1, ϵ_2, and h are parameters that can be adjusted in specific numerical experiments. This impulsive strategy means that when conditions (IE-1) and (IE-2) are satisfied for all $t\in\mathcal{I}$, the trajectories of the controlled system (5) become very close to the true UPOs. At this occasion, the large perturbation is no longer necessary. Instead, a mild perturbation gain is settled to ensure the noninvasiveness of the adaptive rules. Then, a UPO is stabilized in the practical sense provided that the requirement $|S(t)|<F_0$, together with conditions (IE-1) and (IE-2), is valid for all $t\in\mathcal{J}=[T^*+h,T^*+2h]$. This practical criterion is reasonable since the precision restriction is unavoidable in numerical simulation. Also the classical Pyragas DFC allows for the similar precision. In specific simulation, the above impulsive strategy can be adopted recursively until the UPO is stabilized with the noninvasive perturbation in the practical sense. A more detailed discussion and MATLAB program files are provided in the

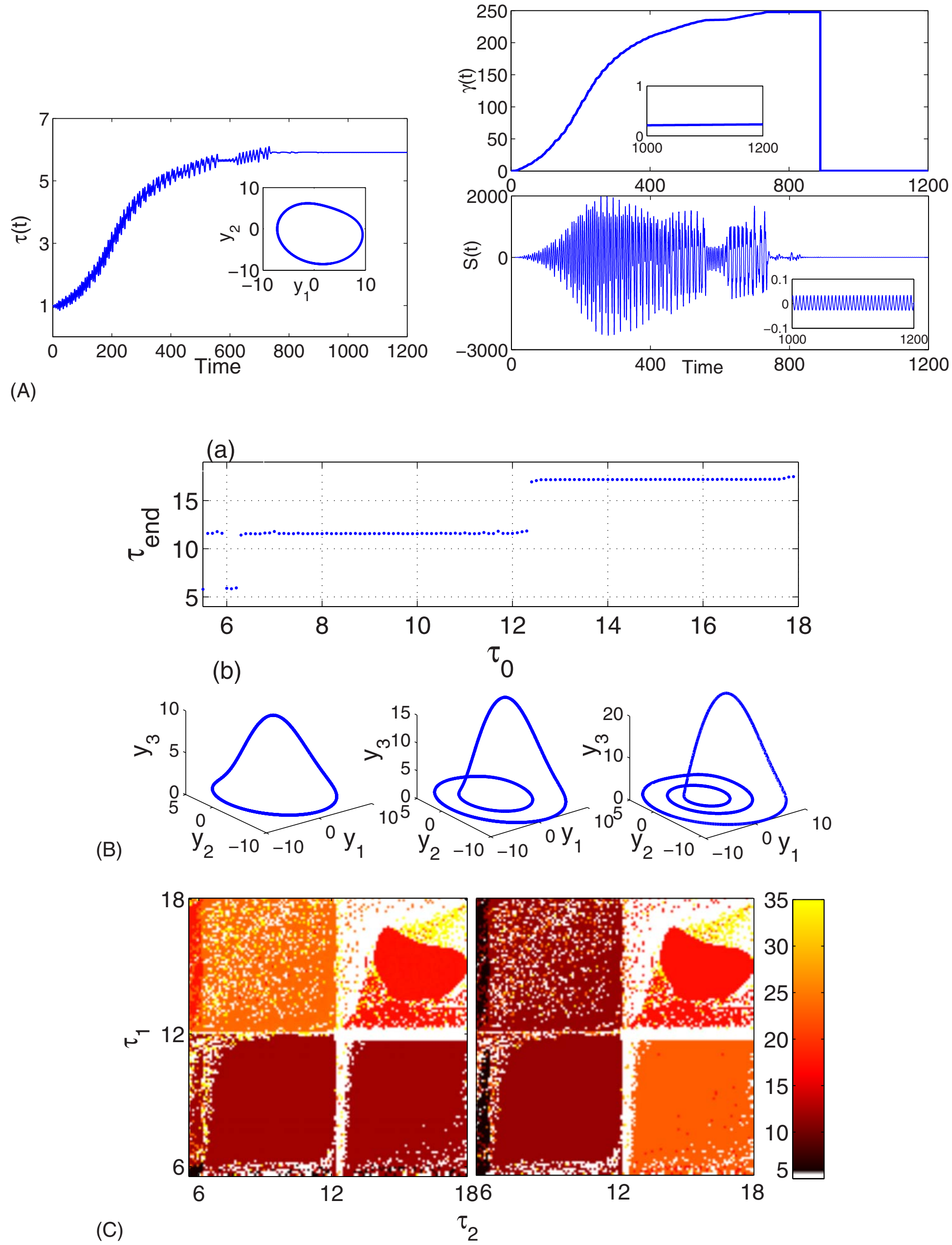

FIG. 1. (Color online) The Rössler system. (A) Equation (4). The dynamics of the time-varying delay τ, the gain γ, and the perturbation $S(t)$ show a successful stabilization of the period-1 UPO under the adaptive algorithm [Eqs. (5)] with the impulsive strategy. Here, $r_1=0.02$, $r_2=0.01$, and $F_0=0.1$. The initial values are taken as constant functions $y_1(0)=y_2(0)=y_3(0)=\tau(0)=1.0$ and $\gamma(0)=0.2$. Parameters in the impulsive strategy are taken as $\epsilon_1=0.02$, $\epsilon_2=0.1$, and $h=100$. Here and throughout, the MATLAB DDESD tool is adopted in the simulation of differential equations with delays. (B) Equation (4). (a) Identification of different periods $\tau(\text{end})$ of the UPO as the initial value $\tau(0)$ is changing in the interval [5.8,18.0]. Here, all the other parameters and initial values are the same as those in (A). (b) The stabilized period-1, period-2, and period-3 UPOs, corresponding to the three values of $\tau(\text{end})$ in (a), respectively. (C) Equations (6). Colors in the left plot and the right one, respectively, stand for the finally converged periods of τ_i with different initial values of $\tau_i(0)$, where $i=1,2$. Parameters are $r_{1i}=0.02$, $r_{2i}=0.2$, $F_0=0.1$, and other initial values $y_1(0)=y_2(0)=y_3(0)=1$, $\gamma_i(0)=0.2$. Here, different color corresponds to different converged periods: black: one time of the period of period 1; brown (light black): around two times; red (dark gray): around three times; orange (gray): around four times; yellow (light gray): around five or six times; white: higher than six times or not approaching the stabilization during the finite simulation time.

supplementary materials [10] for further justifying the non-invasiveness of the above proposed approach with the impulsive strategy, and comparing the stabilization precision of this approach with that of the classical Pyragas DFC.

As shown in Fig. 1(A), the known period-1 orbit, embedded in the chaotic attractor of the Rössler system, is stabilized by the control method proposed here. More specifically, the time-varying delay $\tau(t)$ converges to 5.909, which is regarded as the period of the period-1 UPO of the Rössler system (4), although the initial value for τ was taken as 4.0

here. Additionally, the monotonically increasing gain γ, as satisfying conditions (IE-1) and (IE-2), is set to some smaller value. After the adoption of this impulsive strategy, the non-invasive perturbation in the practical sense lasts for a longer duration, where the stabilization of the period-1 orbit is achieved numerically. It should be emphasized that no information on the period and the gain is needed throughout the entire control process.

In what follows, the structure of the basin of the initial value $\tau(0)$ achieving the period $\tau=\tau(\text{end})$ of the stabilized UPO is investigated. Specifically, consider the controlled Rössler system (5) with initial value $\tau(0)$ continuously changing in the interval [5.8,18.0]. The initial values for the other variables and parameters are the same as those specified in Fig. 1(A). As shown in Fig. 1(B), the period $\tau(\text{end})$ of the UPO can be finally identified; however, different initial values $\tau(0)$ lead to different identified periods, $\tau(\text{end})$. In fact, Fig. 1(B) shows a piecewise-interval-like basin of the initial value $\tau(0)$. With the initial value $\tau(0)$ starting from different intervals, the method leads the controlled orbit to different UPOs. It is confirmed that the method does not require any prior knowledge of the profile of the UPO. Indeed, not only it can identify the periods of the UPOs, but also it can locate the UPOs embedded in the chaotic attractor for the initial value $\tau(0)$ selected from a wide range. Numerical simulation also shows that this basin is quite robust when the initial values for the other variables in the Rössler system are slightly perturbed.

Next, the structure of the basin is investigated for the periods of UPOs with multiple observed outputs and time-delayed variables. Again, consider the Rössler system with a chaotic strange attractor. However, differing from Eqs. (5), the controlled system here is designed as

$$\dot{y}_1 = -y_2 - y_3 + F_1(t), \quad \dot{y}_2 = y_1 + 0.2y_2 + F_2(t),$$
$$\dot{y}_3 = 0.2 + y_3(y_1 - 5.7), \tag{6}$$

where each control coupling $F_i(t)$ is the truncated function of $S_i(t)=\gamma_i(t)\{y_i[t-\tau_i(t)]-y_i(t)\}$ (with $i=1,2$), as defined in Example 1, and the adaptive rules satisfy

$$\dot{\tau}_i = r_{1i}\{y_i(t) - y_i[t-\tau_i(t)]\}, \quad \dot{\gamma}_i = r_{2i}\{y_i(t) - y_i[t-\tau_i(t)]\}^2. \tag{7}$$

With these new configurations, numerical simulations confirm successful locations of the UPOs in the Rössler system from most of the initial values. However, those converged values of the time-delayed variables τ_i ($i=1,2$) are no longer equal to the exact primary periods of the UPO, but to multiplies of the primary periods. With different initial values of $\tau_1(0)$ and $\tau_2(0)$, the colors in Fig. 1(C), panels (a) and (b), represent the converged values of the variables τ_i ($i=1,2$). Interestingly, as shown in Fig. 1(C), the basins for the identified periods of the time-delayed variables show some fractal structures when the initial values of $\tau_i(0)$ ($i=1,2$) become larger. This phenomenon may be attributed to the existence of the infinitely many highly unstable periodic orbits with larger periods embedded in the chaotic attractor. These findings not only reinforce the feasibility of the proposed adaptively time-varying delayed feedback control method, but also lead to a better understanding of the complexity of those stabilized UPOs. Also note that some experiments do not approach the stabilization during the numerical simulation. This requires longer simulation duration for the recursive use of the above algorithm.

Example 2. To further demonstrate the feasibility of the method, consider a possible stabilization of the UPOs embedded in a chaotic attractor with torsion or a system with an odd number of real Floquet multipliers greater than unity. As reported in [3], the original DFC [4] suffers from failing to stabilize UPOs with an odd number of real Floquet multipliers that are greater than unity. Although this viewpoint has been reconsidered recently in [5], by virtue of bifurcation theory, the corresponding analytical result and argument are valid only for those UPOs in the vicinity of an equilibrium. For those UPOs with torsion, Pyragas proposed a modification known as the unstable delayed feedback controller, claiming that it meets the odd number gap by artificially enlarging a set of multipliers (greater than unity) to an even number [8]. However, this modification involves a complicated computation of the Floquet multipliers, requiring the exact profile of the periodic orbit that is generally unknown *a priori*, and therefore is inconvenient to use.

Here, the Lorenz system is taken as an example for illustration, which is commonly regarded as a benchmark for testing methods of stabilizing UPOs with torsion. Using our method, the controlled Lorenz system is

$$\dot{y}_1 = \sigma(y_2 - y_1),$$

$$\dot{y}_2 = ry_1 - y_2 - y_1y_3 + F(t),$$

$$\dot{y}_3 = y_1y_2 - by_3,$$

$$\dot{\tau} = r_1[y_2(t) - y_2(t-\tau)], \quad \dot{\gamma} = r_2[y_2(t) - y_2(t-\tau)]^2,$$

where the control coupling $F(t)$ as well as those strategies is the same as that used in Example 1, and the parameters are $\sigma=10$, $r=28$, and $b=8/3$. Without $F(t)$, the system has a chaotic attractor with double scrolls. In particular, a period-1 UPO with period $\tau_1 \approx 1.5586$ and a period-2 UPO with period $\tau_2 \approx 2\tau_1$ are unstably embedded in the chaotic attractor. With the proposed control coupling, these two UPOs with torsion are stabilized, as shown in Fig. 2. Interestingly, a small perturbation on the initial value, $y_2(0)$, can lead the controlled orbits to different periodic orbits. This means that the period estimation is sensitive to the choice of the initial value of the coupling variable y_2 in the Lorenz system. It is different from the numerical results obtained for the Rössler system, presented in Figs. 1(A) and 1(B).

Example 3. Finally, the proposed method is applied to time-delayed systems with noise perturbation. Consider the noise-perturbed Mackey-Glass system, a model for blood cell regeneration [11], described by

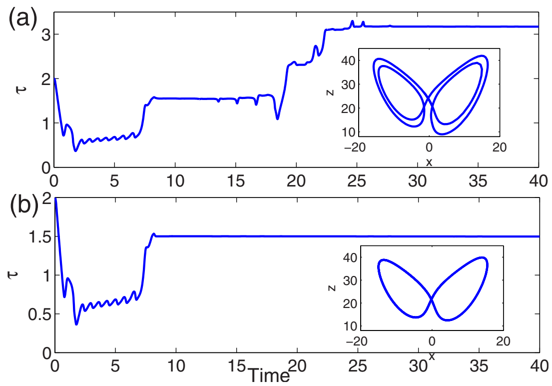

FIG. 2. (Color online) Stabilization of the UPOs in the Lorenz system. (a) The identified period and the period-2 UPO with the initial values $y_1(0)=14.88$, $y_2(0)=18.71$, $y_3(0)=31.50$, $\tau(0)=2$, and $\gamma(0)=1$. Here, $r_1=0.1$ and $r_2=1$. (b) The identified period and the period-1 UPO with the same initial values and parameters, except for $y_1(0)=14.88-0.01$, slightly different from that in (a).

$$\dot{x} = \frac{ax(t-\tau_s)}{1+x^b(t-\tau_s)} - cx(t) + \xi(t). \tag{8}$$

When the parameters are $a=2$, $b=10$, $c=1$, $\tau_s=3$, and white noise $\xi(t)$ with a strength of 0.5, system (8) exhibits chaotic dynamics. By using the method described in (2) and (3), the controlled orbit is almost surely stabilized to the equilibrium $x^*=1$, which is unstable in the uncontrolled and noise-perturbed system (8). Figure 3(a) shows an almost successful stabilization, where the time-delayed variable converges almost surely to a constant, the trivial period of the equilibrium.

Theoretically, the choice of parameters r_i does not influence the convergence of the method. However, they should be chosen as small as possible in practice since accuracy limit in discrete algorithms for simulating continuous systems is unavoidable in real applications. This is illustrated in Fig. 3(b), where the Rössler system (4) with Eqs. (5) is considered.

In conclusion, we have proposed, simulated, and analyzed an adaptively time-varying delayed feedback controller, which can be used to effectively stabilize unstable periodic orbits or unstable steady states that are embedded in chaotic strange attractors. This method not only possesses all of the merits of the conventional delayed feedback control technique, but also has several advantages of practical importance including (i) no requirements of prior knowledge of the period of the UPO and the control gain; (ii) online and automatic search of the period with little computational complexity $O(T)$ [12], where T denotes the duration of one realization of solving the differential equations; (iii) practically noninvasive and successful location of UPOs' bifurcation from an unstable equilibrium or having finite torsion; and (iv) robustness against the noise perturbation. More importantly, this method is theoretically proved with some reasonable assumptions and numerically validated with some practical impulsive strategies. A similar idea can be used to identify various parameters such as the periods of oscillations and the time delays in modeling biological systems, based on, for example, high throughput data available in systems biology studies [13].

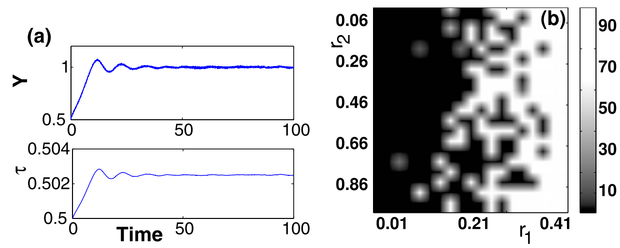

FIG. 3. (Color online) (a) An almost successful stabilization of the unstable equilibrium $x^*=1$ embedded in the chaotic Mackey-Glass system with noise perturbation. Parameters are $r_1=r_2=0.01$ and initial values $y(0)=0.7$, $\tau(0)=0.5$, and $\gamma(0)=15$. (b) The amplitude of $\tau(t)$ for $t\in[7000,8000]$ when different pairs of parameters r_i are used in the Rössler system (4) with Eqs. (5). The more black the color, the more successful the estimation of the period.

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[10] See supplementary material at http://link.aps.org/supplemental/10.1103/PhysRevE.82.046214 for a detailed discussion on the noninvasiveness of the adaptive approach with the impulsive strategy (IE-1) and (IE-2), a comparison between the proposed approach and the classical Pyragas' method, and several MATLAB program files displaying the stabilization procedure of the UPOs through the proposed approach.

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