Polling Predicaments

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Two friends are discussing student union politics on Election Day. Each tries really hard to convince the other to switch sides. Finally, one says to the other: "Look, it's clear that we are opposed on every political issue. Our votes will surely cancel out. Why don't we save time and agree to not vote today?" The other agrees fervently and they part. Shortly after that, a friend of the first one who had over-heard the agreement says, "That was a sporting offer you made." "Not really," says the second. "This is the third time I've done this today!"

Jokes aside, humans have come a long way to develop better ways for cooperating and respecting the choice of every one via voting. Voting also puts responsibility on every citizen. As John F. Kennedy said," The margin is narrow, but the responsibility is clear." However voting can be chaotic at times. Let's check.

Suppose thirteen Warwick students are planning to have a meal together at Tocil. In order to utilise a new deal at Cost-Cutters, they choose to buy a common soft drink from among Fanta, Pepsi and Coke. Let's say the preferences of the students are as following:

Number of Students	Ranking
4	Fanta, Pepsi, Coke
2	Pepsi, Coke, Fanta
1	Coke, Fanta, Pepsi
2	Fanta, Coke, Pepsi
4	Coke, Pepsi, Fanta

The students try to decide the drink according to the usual 'show of hands' rule where every one votes for one's favourite choice. Fanta comes out as the winner: Fanta (6), Coke (5), Pepsi (2). However this result immediatly prompts a complaint by 7 people that they atleast wanted to have a cola drink! This is a typical example of how the 'single plurality' voting dilutes voter preferences, creating the possibility of choosing an option which the majority does not want. Infact the single plurality procedure which is widely used in elections around the world is one of the worst possible procedures with chances of undesirable outcomes.

In order to try a better option, the students try the 'vote for two' approach. Every student lists two choices and all the choices are summed up. In this case the new ranking is completely reversed: Pepsi (10), Coke (9), Fanta (7). Pepsi seems to be the ideal choice now.

Things seemed to have settled now when a die-hard coke fan who has imme-

diatly done some calculations suggests a better method with a glint in his eye. This time a method similar to the 'gpa grading system' is used in which 2, 1 and 0 points are assigned to voters' first, second and third alternative respectively and then points totalled. The new outcome is according to the coke fan's satisfaction: Coke (14), Fanta (13), Pepsi (12). This kind of voting system is called the Borda Count[1].

The students could have tried various other voting procedures such as pairwise comparisons in which the undefeated candidate is the winner or the 'elimination run-off' where the weakest candidate is eliminated in every round. Another option was Condorcet's[2] method which is a pairwise election system where ranked ballots are used to simulate many head-to-head elections. The winner of a Condorcet election is the candidate who wins all pairwise matchups.

Although it is unknown whether they ever came upon a consensus or not, the incident showed that instead of the preferences of the voters, the type of voting procedure can actually decide the winner! The implication of this phenomenon can be felt in economics, politics, engineering and many other fields where important choices have to be made on the basis on certain criteria

[1]Borda Count was devised by the French mathematician, Jean-Charles de Borda in 1770.

[2]Marie-Jean-Antoine-Nicolas de Caritat(1743-1794) was a French philosopher, mathematician, social scientist, economist, politician and humanitarian.

and preferences.

Things seem even more grim that when one notices a variety of other voting procedures each with its advantages and flaws. One possible procedure is dictatorship where the choice depends on the mood of only one person. This ofcourse won't go down well with every one else. In order to look for fairer procedures it is useful to identify fairness properties. There are many desirable characterisitics of a voting procedure which include:

- Pareto: If all voters share a common relative ranking between a pair of candidates then the outcome should also reflect that.
- Independence from Irrelevant Alternatives: What voters think about one candidate should not affect the relative ranking of a different pair.
- Transitivity of the societal ranking i.e if a is more popular/better than b and b is more popular/better than c, then a is more popular/better than c. Although this seems obvious it does not happen in many instances like matches between sports teams.
- No restrictions on how the voters rank the candidates.
- Each voter's binary rankings should be a complete, strict and transitive.

Nobel Prize winner, Kenneth Arrow proved the interesting paradox that if a

voting procedure with more than three candidates satisfies all the five conditions then it is a dictatorship. This famous Arrow's Impossibility Theorem[3] implies that there is no general way to aggregate preferences without running into some kind of "irrationality" or "unfairness".

Despite this negative result, things are not as hopeless as they seem. The catch in the paradox is the list of conditions which need to be satisfied. As pointed out by Professor Donald Saari [4], an expert on voting theory, relaxing the desirable properties of the procedure prevents the dreaded dictatorship.

Arrow's result kickstarted a lot of interesting research in voting theory. Famous results include Gibbard - Satterthwaite theorem which says that every general voting system is susceptible to tactical or manipulative voting. One desirable property of a voting procedure is Minimal Liberalism (decisiveness of atleast two voters over a specified pair of alternatives). Indian Noble Prize Winner, Amartya Sen proved that a general voting procedure which satisfies Pareto and Minimal Liberalism implies presence of cyclic outcomes. As a result, it attracts comments from both the politcal left and right.

We see that mathematical voting theory is important in decision making,

[3]Arrow's Impossibility Theorem was proved by Kenneth Arrow during his Ph.D. and popularized in his 1951 book Social Choice and Individual Values.

[4]Donald Saari is the author of 'Chaotic Elections! A mathematician looks at voting', American Mathematical Society, 2001 and 'Decisions and Elections; Explaining the Unexpected', Cambridge University Press, 2001.

political theory and other areas. If not anything else, some knowledge of it might prevent you from having to drink your least favourite soda!