LTL with the Freeze Quantifier and Register Automata

Stéphane Demri[1] Ranko Lazić[2]

[1]LSV, CNRS & ENS Cachan & INRIA Futurs

[2]University of Warwick

LICS, August 2006, Seattle

Overview

Data words

Finite alphabet & infinite domain:
Infinite-state computations, XML documents, Timed words, ...

$$\begin{array}{cccccc} a & a & b & b & a & b \\ URL_1 & URL_2 & URL_1 & URL_2 & URL_3 & URL_3 \end{array}$$

$$\begin{array}{cccccc} a & a & b & b & a & b \\ 3 & 2.5 & 3 & 2.5 & 4 & 4 \end{array}$$

Data words

Finite alphabet & infinite domain:
Infinite-state computations, XML documents, Timed words, ...

$$\begin{matrix} a & a & b & b & a & b \\ URL_1 & URL_2 & URL_1 & URL_2 & URL_3 & URL_3 \end{matrix}$$

$$\begin{matrix} a & a & b & b & a & b \\ 3 & 2.5 & 3 & 2.5 & 4 & 4 \end{matrix}$$

$$a \; a \; b \; b \; a \; b$$

$\mathrm{LTL}_n^{\downarrow}$

$$\begin{array}{rcl} \phi & ::= & \top \mid a \mid \uparrow_r \sim \mid \\ & & \neg\phi \mid \phi \wedge \phi \mid \\ & & \mathtt{O}(\phi, \ldots, \phi) \mid \\ & & \downarrow_r \phi \end{array}$$

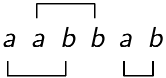
$$\begin{array}{rcl} r & \in & \{1, \ldots, n\} \\ \mathtt{O} & \in & \{\mathtt{X}, \mathtt{X}^{-1}, \mathtt{F}, \mathtt{F}^{-1}, \mathtt{U}, \mathtt{U}^{-1}, \ldots\} \end{array}$$

Freeze quantifier

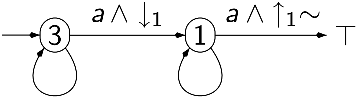
Timed logics	$x \cdot \phi(x)$	[Alur & Henzinger, JACM '94]
Hybrid logics	$\downarrow_p \phi(p)$	[Goranko, JoLLI '96]
Modal logics	$\langle \lambda x \cdot \phi(x) \rangle (c)$	[Fitting, JLC '02]

Example

$$\mathtt{F}(a \wedge \downarrow_1 \mathtt{XF}(a \wedge \uparrow_1 \sim))$$



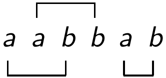
Register Automata



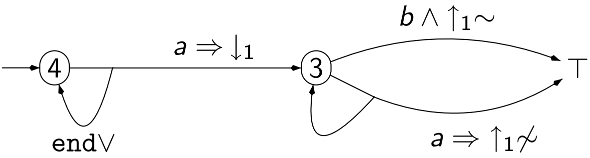
1-way or 2-way, Nondeterministic or Alternating, *n* registers

Over infinite data words: *weak parity* acceptance.
Special case of *Büchi* and *co-Büchi* acceptance.

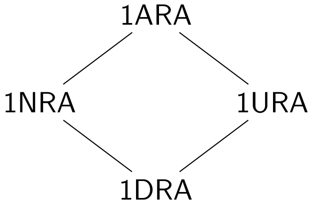
Register automata	[Kaminski & Francez, TCS '94]
Pebble automata	[Neven, Schwentick & Vianu, ToCL '04]
Timed automata	[Alur & Dill, TCS '94]
Data automata	[Bouyer, Petit & Thérien, I & C '03]



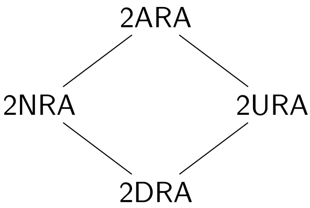
$$\mathtt{G}\,(a \Rightarrow \downarrow_1 \mathtt{X}((a \Rightarrow \neg\uparrow_1\sim)\,\mathtt{U}\,(b \wedge \uparrow_1\sim)))$$



[Kaminski & Francez, TCS '94],
[Neven, Schwentick & Vianu, ToCL '04]



[Kaminski & Francez, TCS ’94],
[Neven, Schwentick & Vianu, ToCL ’04]



Provided LogSpace $\subset$ NLogSpace $\subset$ PTime.

FO$(\sim, <, +1)$

Theorem

$$\textit{Simple } LTL^{\downarrow}_{1}(\mathtt{X}, \mathtt{X}^{-1}, \mathtt{F}, \mathtt{F}^{-1}) \quad \underset{\overleftarrow{\text{PSPACE}}}{\overset{\text{LOGSPACE}}{\longrightarrow}} \quad \mathrm{FO}^2(\sim, <, +1)$$

[Etessami, Vardi & Wilke, I & C '02]
LTL$(\mathtt{X}, \mathtt{X}^{-1}, \mathtt{F}, \mathtt{F}^{-1})$ is equivalent to FO$^2(<, +1)$.

[Bojańczyk et al., LICS '06]
FO$^2(\sim, <, +1)$ SAT is as hard as Petri net Reachability.

Simple $\mathrm{LTL}_1^{\downarrow}(\mathtt{X}, \mathtt{X}^{-1}, \mathtt{F}, \mathtt{F}^{-1})$

$$\mathtt{F}(a \wedge \downarrow_1 \mathtt{XF}(a \wedge \uparrow_1 \sim))$$

$$\exists x\,(a(x) \;\wedge\; \exists y\,(x < y \;\wedge\; a(y) \;\wedge\; x \sim y))$$

$$\mathtt{F}(a \wedge \downarrow_1 \mathtt{XF}(b \wedge \mathtt{XF}(a \wedge \uparrow_1 \sim)))$$

From LTL$^{\downarrow}$ to register automata

Theorem

$$LTL_n^{\downarrow}(\mathtt{X}, \mathtt{X}^{-1}, \mathtt{U}, \mathtt{U}^{-1}) \xrightarrow{\textsc{LogSpace}} 2ARA_n.$$

[Kaminski & Francez, TCS '94]

'... it is very likely that
[decidability of language containment of 1NRA in $1NRA_1$]
can be extended to infinite data words.'

[French, TIME '03], [Demri, Lazić & Nowak, TIME '05],
[Lisitsa & Potapov, TIME '05]:

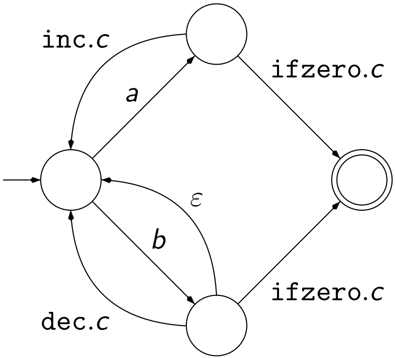
	$SAT^{<\omega}$		SAT^{ω}	
Registers	1	2	1	2
$LTL^{\downarrow}(\mathtt{X},\mathtt{F})$				
$LTL^{\downarrow}(\mathtt{X},\mathtt{U})$		Σ^0_1-comp.		Σ^1_1-comp.
$LTL^{\downarrow}(\mathtt{X},\mathtt{F},\mathtt{F}^{-1})$				Σ^1_1-comp.

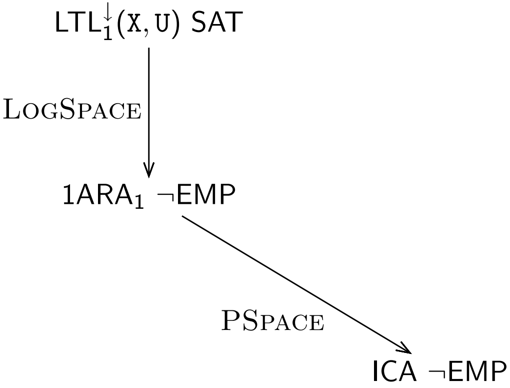
$\mathrm{LTL}_1^{\downarrow}(\mathtt{X}, \mathtt{U})$ SAT

$\downarrow$ LogSpace

$1\mathrm{ARA}_1$ $\neg$EMP

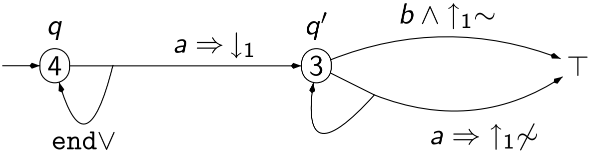
Incrementing Counter Automata



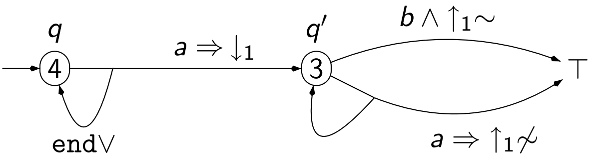

$\mathsf{LTL}_1^{\downarrow}(\mathtt{X}, \mathtt{U})$ SAT
LogSpace
$1\mathsf{ARA}_1$ $\neg$EMP
PSpace
ICA $\neg$EMP

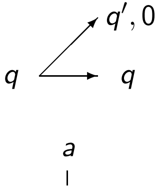
Proof.

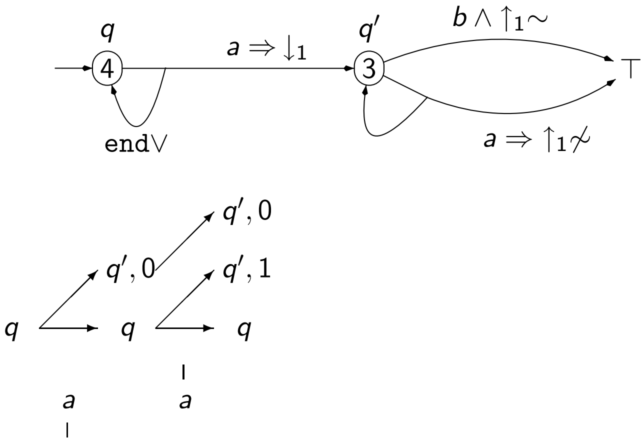
1. Quotient runs by $\sim$.
2. Represent levels and steps using counters.
3. For infinite data words, use [Miyano & Hayashi, TCS '84]: weak parity alternating $\mapsto$ Büchi nondeterministic.
4. Incrementing errors cannot cause a false acceptance. $\square$

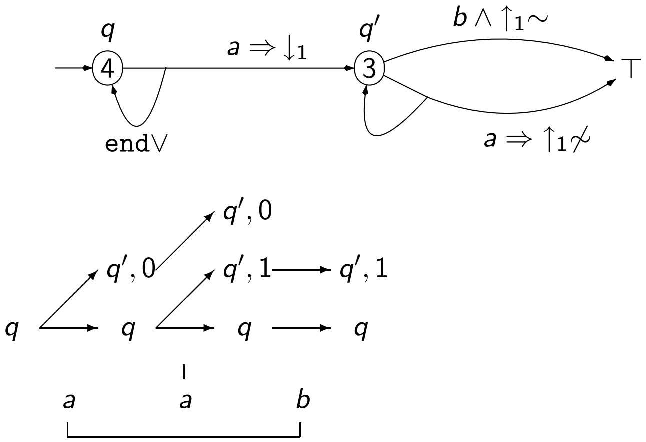

q
4
$a \Rightarrow \downarrow_1$
q′
3
$b \wedge \uparrow_1\sim$
⊤
end∨
$a \Rightarrow \uparrow_1\not\sim$

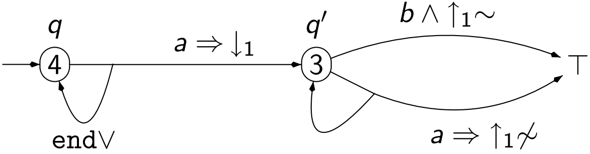
q

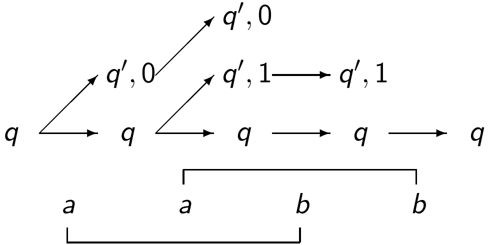

q
4
end∨
a ⇒ ↓1
q′
3
b ∧ ↑1∼
⊤
a ⇒ ↑1≁

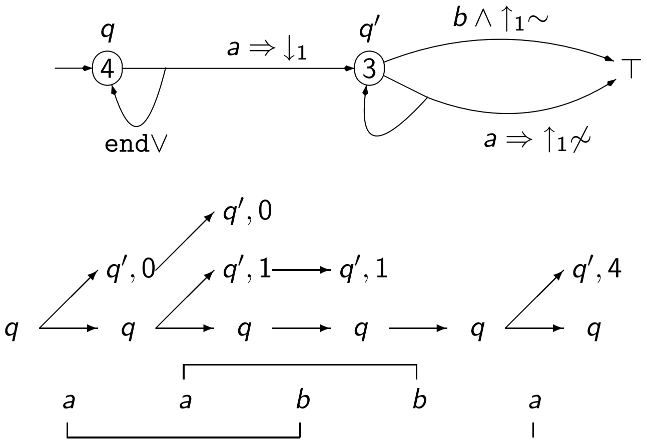

q′, 0
q
q
a

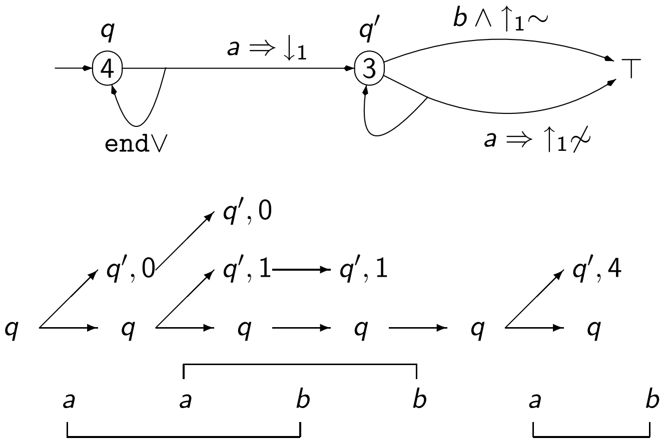

q
4
a ⇒ ↓1
q′
3
b ∧ ↑1∼
⊤
end∨
a ⇒ ↑1≁
q′, 0
q′, 0
q′, 1
q
q
q
a
a

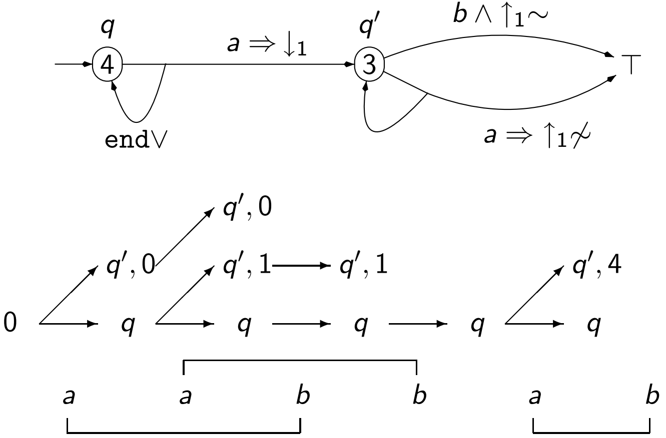

q
4
end∨
a ⇒ ↓1
q′
3
b ∧ ↑1∼
⊤
a ⇒ ↑1≁
q′, 0
q′, 0
q′, 1
q′, 1
q
q
q
q
a
a
b

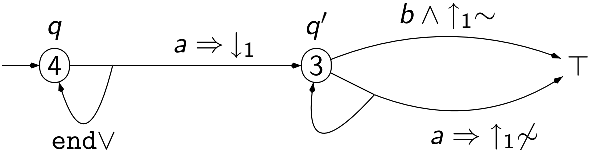

q
4
end∨
a ⇒ ↓1
q′
3
b ∧ ↑1∼
⊤
a ⇒ ↑1≁

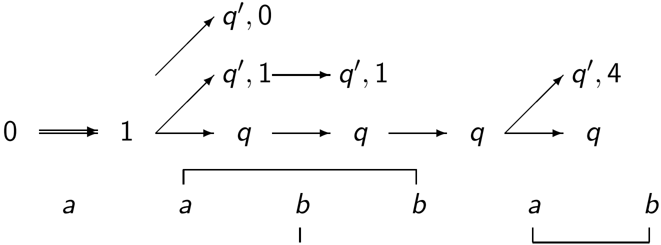

q′, 0
q′, 0
q′, 1
q′, 1
q
q
q
q
q
a
a
b
b

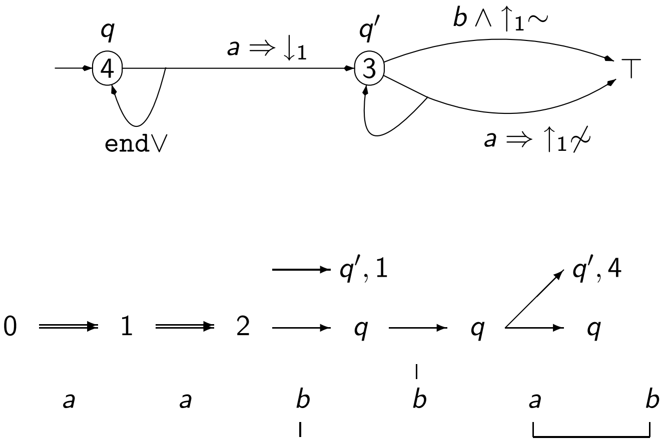

q
q′
4
3
a ⇒ ↓1
b ∧ ↑1∼
⊤
end∨
a ⇒ ↑1≁
q′, 0
q′, 0
q′, 1
q′, 1
q′, 4
q
q
q
q
q
q
a
a
b
b
a

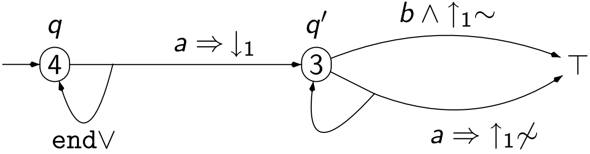

q
4
a ⇒ ↓1
q′
3
b ∧ ↑1∼
⊤
end∨
a ⇒ ↑1≁
q′, 0
q′, 0
q′, 1
q′, 1
q′, 4
q
a
b

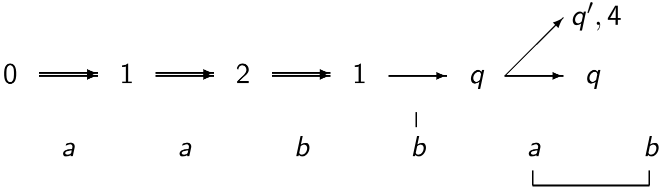

q
4
a ⇒ ↓1
q′
3
b ∧ ↑1∼
⊤
end∨
a ⇒ ↑1≁
q′, 0
q′, 0
q′, 1
q′, 1
q′, 4
0
q
q
q
q
q
a
a
b
b
a
b

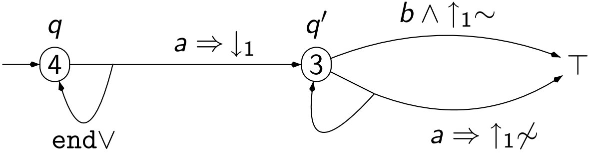

q
4
end∨
a ⇒ ↓1
q′
3
b ∧ ↑1∼
⊤
a ⇒ ↑1≁

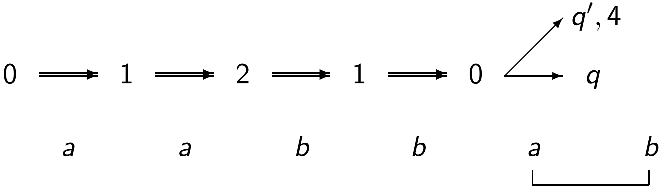

q′, 0
q′, 1
q′, 1
q′, 4
0
1
q
q
q
q
a
a
b
b
a
b

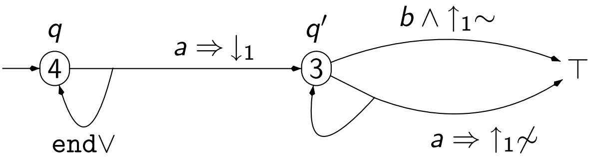

q
4
a ⇒ ↓1
q′
3
b ∧ ↑1∼
⊤
end∨
a ⇒ ↑1≁
q′, 1
q′, 4
0
1
2
q
q
q
a
a
b
b
a
b

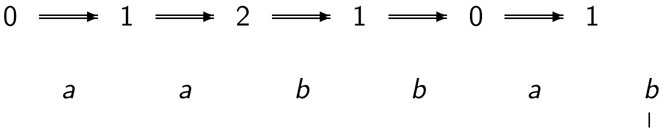

q
4
end∨
a ⇒ ↓1
q′
3
b ∧ ↑1∼
⊤
a ⇒ ↑1≁

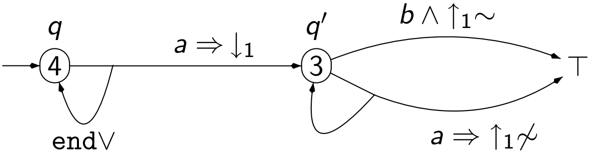

0
1
2
1
q
q′, 4
q
a
a
b
b
a
b


q
4
end∨
a ⇒ ↓1
q′
3
b ∧ ↑1∼
⊤
a ⇒ ↑1≁


0
1
2
1
0
q′, 4
q
a
a
b
b
a
b

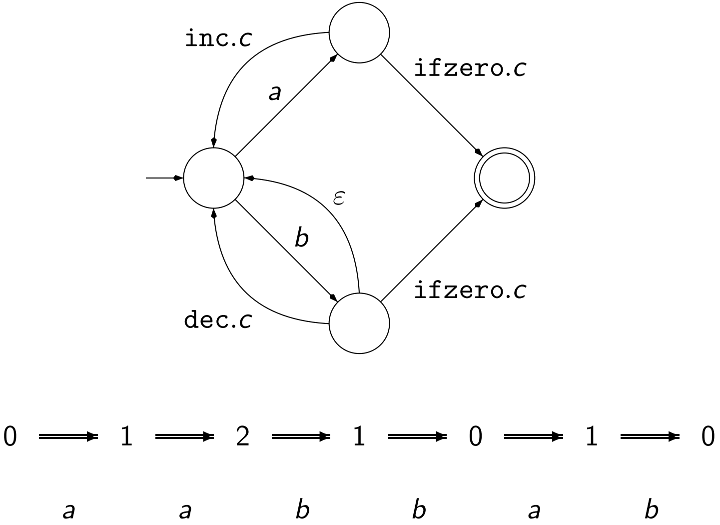

q
q'
4
3
$a \Rightarrow \downarrow_1$
$b \wedge \uparrow_1 \sim$
$\top$
end$\vee$
$a \Rightarrow \uparrow_1 \not\sim$


0
1
2
1
0
1
a
a
b
b
a
b



$$0 \Longrightarrow 1 \Longrightarrow 2 \Longrightarrow 1 \Longrightarrow 0 \Longrightarrow 1 \Longrightarrow 0$$

$$a \quad a \quad b \quad b \quad a \quad b$$


inc.c
a
ifzero.c
ε
b
ifzero.c
dec.c
0 1 2 1 0 1 0
a a b b a b

Infinite words

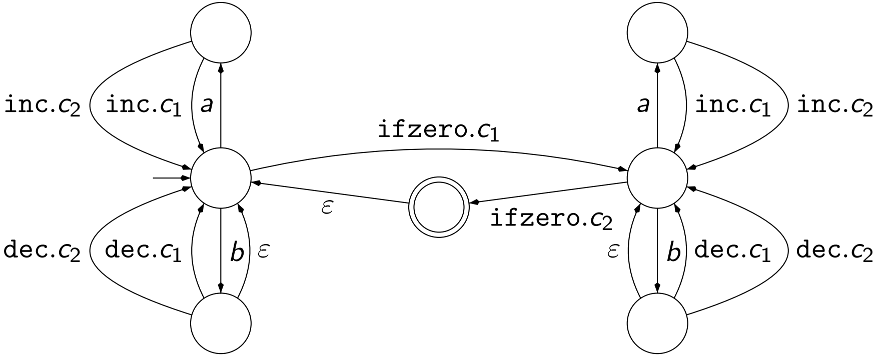
$$\mathtt{G}\,(a \Rightarrow \downarrow_1 \mathtt{X}((a \Rightarrow \neg\uparrow_1\sim)\,\mathtt{U}\,(b \wedge \uparrow_1\sim)))$$

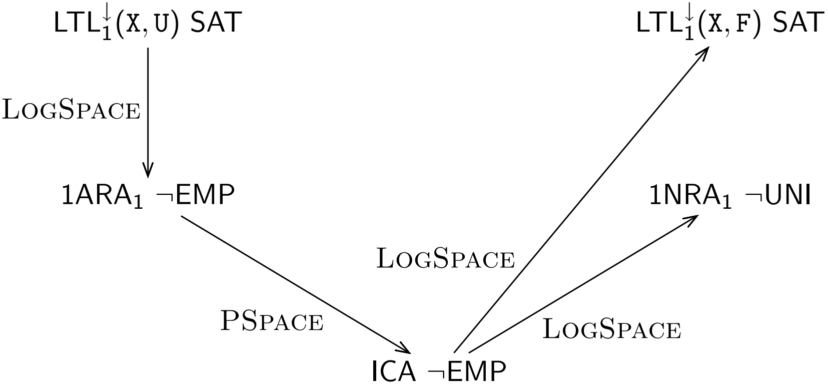
We should accept:

$$a \quad a \quad b \quad a \quad b \quad a \quad b \quad a \quad b \quad \cdots$$

but reject:

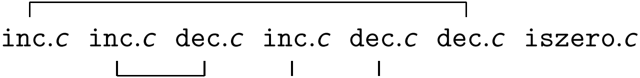
$$a \quad a \quad b \quad a \quad b \quad a \quad a \quad a \quad a \quad \cdots$$


inc.c_2
inc.c_1
a
ifzero.c_1
a
inc.c_1
inc.c_2
ε
ifzero.c_2
dec.c_2
dec.c_1
b
ε
ε
b
dec.c_1
dec.c_2


$LTL_1^{\downarrow}(X, U)$ SAT
$LTL_1^{\downarrow}(X, F)$ SAT
LogSpace
$1ARA_1$ ¬EMP
$1NRA_1$ ¬UNI
LogSpace
PSpace
LogSpace
ICA ¬EMP

Proof.

Encode computations of ICA as data words:



□

Proof.

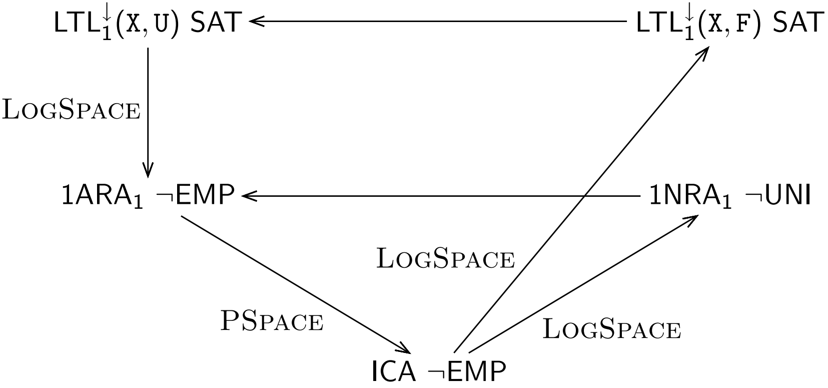
Encode computations of ICA as data words:

Proof.

Encode computations of ICA as data words:

$\texttt{inc}.c$ $\texttt{inc}.c$ $\texttt{dec}.c$ $\texttt{inc}.c$ $\texttt{dec}.c$ $\texttt{dec}.c$ $\texttt{iszero}.c$

□


LTL↓1(X, U) SAT
LTL↓1(X, F) SAT
LogSpace
1ARA1 ¬EMP
1NRA1 ¬UNI
LogSpace
PSpace
LogSpace
ICA ¬EMP

Theorem

	Minsky CA	*Incrementing CA*
$\neg EMP^{<\omega}$	Σ^0_1-*complete*	R *(a), not* PR *(b)*
$\neg EMP^{\omega}$	Σ^1_1-*complete*	Π^0_1-*complete (c)*

Proof.

(a) Reverse computations, and obtain Reset Petri net Coverability [Dufourd, Finkel & Schnoebelen, ICALP '98].

(b) Reverse computations, and adapt [Schnoebelen, IPL '02]: Reachability is not PR for Lossy Channel Systems.

(c) Adapt [Ouaknine & Worrell, FoSSaCS '06]: Recur. State for Insertion Chan. Mach. with Empt. Test. □

[Kaminski & Francez, TCS '94]

'... it is very likely that
[decidability of language containment of 1NRA in $1NRA_1$]
can be extended to infinite data words.'

[French, TIME '03], [Demri, Lazić & Nowak, TIME '05],
[Lisitsa & Potapov, TIME '05]:

	$SAT^{<\omega}$		SAT^{ω}	
Registers	1	2	1	2
$LTL^{\downarrow}(X, F)$				
$LTL^{\downarrow}(X, U)$		Σ_1^0-comp.		Σ_1^1-comp.
$LTL^{\downarrow}(X, F, F^{-1})$				Σ_1^1-comp.

[Demri & Lazić, LICS '06]

$1NRA_1$ $\neg UNI^{\omega}$ is Π^0_1-complete.

[French, TIME '03], [Demri, Lazić & Nowak, TIME '05], [Lisitsa & Potapov, TIME '05]:

	$SAT^{<\omega}$		SAT^{ω}	
Registers	1	2	1	2
$LTL^{\downarrow}(\mathtt{X}, \mathtt{F})$				
$LTL^{\downarrow}(\mathtt{X}, \mathtt{U})$		Σ^0_1-comp.		Σ^1_1-comp.
$LTL^{\downarrow}(\mathtt{X}, \mathtt{F}, \mathtt{F}^{-1})$				Σ^1_1-comp.

[Demri & Lazić, LICS '06]

$1NRA_1$ $\neg UNI^\omega$ is Π^0_1-complete.

[French, TIME '03], [Demri, Lazić & Nowak, TIME '05], [Lisitsa & Potapov, TIME '05], [Demri & Lazić, LICS '06]:

	$SAT^{<\omega}$		SAT^ω	
Registers	1	2	1	2
$LTL^{\downarrow}(\mathtt{X}, \mathtt{F})$	$\mathrm{R} \setminus \mathrm{PR}$	Σ^0_1-comp.	Π^0_1-comp.	Σ^1_1-comp.
$LTL^{\downarrow}(\mathtt{X}, \mathtt{U})$	$\mathrm{R} \setminus \mathrm{PR}$	Σ^0_1-comp.	Π^0_1-comp.	Σ^1_1-comp.
$LTL^{\downarrow}(\mathtt{X}, \mathtt{F}, \mathtt{F}^{-1})$	Σ^0_1-comp.	Σ^0_1-comp.	Σ^1_1-comp.	Σ^1_1-comp.

Characterisation of languages

Corollary

$$\begin{aligned} & \{\mathrm{L}(\mathcal{C}) \ : \ \mathcal{C} \textit{ an Incrementing CA}\} \\ = \ & \{f(\mathrm{str}(\mathrm{L}(\phi))) \ : \ f \textit{ a homomorphism}, \ \phi \textit{ in } LTL_1^{\downarrow}(\mathtt{X}, \mathtt{F})\} \\ = \ & \{f(\mathrm{str}(\mathrm{L}(\phi))) \ : \ f \textit{ a homomorphism}, \ \phi \textit{ in } LTL_1^{\downarrow}(\mathtt{X}, \mathtt{U})\} \end{aligned}$$