Stack-Run Adaptive Wavelet Image Compression

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Abstract

We report on the development of an adaptive wavelet image coder based on stack-run representation of the quantized coefficients. The coder works by selecting an optimal wavelet packet basis for the given image and encoding the quantization indices for significant coefficients and zero runs between coefficients using a 4-ary arithmetic coder. Due to the fact that our coder exploits the redundancies present within individual subbands, its addressing complexity is much lower than that of the wavelet zerotree coding algorithms. Experimental results show coding gains of up to 1.4dB over the benchmark wavelet coding algorithm.

1 Introduction

Wavelet transform based image coding methods have gained popularity during the last decade or so [11, 10, 5, 14, 15]. This is mainly due to localization properties of wavelets in both time (space) and frequency resulting in a sparse representation with redundancies in and across subbands that can be exploited. Wavelets, however, are not well-suited [6] to represent oscillatory patterns, a special form of texture. Oscillatory variations of intensity can only be described by the small scale wavelet coefficients. Unfortunately, those small scale coefficients carry very little energy, and are often quantized to zero, even at high bit rates. Fingerprints or seismic signals are a few examples of non-wavelet-friendly signals. Wavelet packets [2] were developed in order to adapt the underlying wavelet bases to the contents of a signal. The basic idea is to allow nonoctave subband decomposition to adaptively select the best basis for a particular signal. Results from various image coding methods based on wavelet packets [6, 8] show that they are particularly good in coding images with oscillatory patterns. There has, however, been relatively much less research on wavelet packet image coding.

The zerotree quantization, proposed first by Shapiro [11], is an effective way of exploiting the self-similarities among high-frequency subbands at various resolutions.

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The success of wavelet based image coding methods has widely been perceived to be due to across-scale similarities which could enable encoding of a set of wavelet coefficients by a single codeword, called a zerotree. However, it has become increasingly clear [14, 6] that exploitation of redundancies present within subbands can also result in coding gains comparable to those of the state-of-the-art in image compression.

In this paper, we present an adaptive wavelet transform based image coder which employs a stack-run representation [14] for quantized transform coefficients in order to benefit from the intra-subband redundancies. Our compression algorithm can be divided into four parts: In the first part, an adaptive wavelet packet basis is selected for representing the given image using certain *entropy*-based cost function. We found that the Coifman-Wickerhauser (CW) entropy cost function [3] does not often result in the best wavelet packet basis yielding the highest compression performance for our coding algorithm. Next, the wavelet packet coefficients are quantized using an optimal scalar quantizer for Laplacian distribution [13]. In the third part, we represent the quantized coefficients with stack-run coding using 4-symbol set of [14] which generates a redundant symbol stream. Finally, this symbol stream is entropy coded using a high order arithmetic coder. We use a symbol set for the arithmetic coding of run/level values in which context information is used to enable multiple uses of a single symbol. Our coder offers the following advantages: (*a*) a relatively simple and therefore fast coding algorithm, (*b*) capable of progressive transmission, and (*c*) coding performance consistently comparable with and often outperforming two state-of-the-art image coders [10, 6].

This paper is organized as follows. In the next section, a brief review of wavelet packet transform is presented. The coder algorithm is described in further detail in Section 3. Experimental results of the new coder are presented in Section 4. Finally, the paper concludes with some remarks on coder's nature and performance.

2 Adaptive Wavelet Transform

Adaptive wavelet transform, also known as the wavelet packet transform, for images can be regarded as an adaptive wavelet subband decomposition that lifts the limitation of only decomposing the lowpass subband. This adaptation of representation basis to the image contents can be achieved in practical applications if a fast basis selection algorithm is employed to select the best basis from among the huge library of possible bases.

2.1 Best Basis Selection

Coifman and Wickerhauser [3] suggested to use a fast dynamic programming algorithm with a time complexity of $O(N \log N)$, where N is the number of pixels in the image, to select the best basis according to a given cost function C. A key criterion must be met in order to invoke a dynamic programming strategy [4]: the cost-function should be *separable* or *additive*; ie, $C(\mathbf{x}) = \sum_k \mu(x_k)$, where $x_k, k = 0, 1, \ldots, N - 1$, are the elements of wavelet packet decomposed image $\mathbf{x}$, and μ is a positive function

such that $\mu(0) = 0$.

2.2 Role of a Cost Function

The notion of the *best* wavelet packet basis is limited to the cost function based on which the full wavelet packet tree is pruned to obtain an optimal tree. It is, therefore, crucial that an appropriate cost function is chosen taking into consideration the quantization strategy employed by the coder. Initially, Coifman and Wickerhauser [3] used

$$h(\mathbf{x}) = -\sum_k \left(\frac{|x_k|^2}{||\mathbf{x}||^2} \log \frac{|x_k|^2}{||\mathbf{x}||^2} \right) \tag{1}$$

as a cost function. Ramchandran and Vetterli [9] used the optimal bit allocation algorithm of Shoham and Gersho [12] to distribute the budget across the nodes of the wavelet packet tree, and they select the best basis according to the following rate-distortion criterion

$$C(\mathbf{x}, Q, \lambda) = D(\mathbf{x}, Q) + \lambda R(\mathbf{x}, Q). \tag{2}$$

Given a set of quantizers Q, the rate $R(\mathbf{x}, Q)$ is estimated with the first-order approximation of the entropy $H(Q(\mathbf{x}))$, and the distortion $D(\mathbf{x}, Q)$ is defined as the mean square error (MSE). The selection of the best basis involves three embedded nonlinear optimization problems. The overall complexity of the approach in [9] is, therefore, quite high.

Since a scalar quantizer is used to quantize the transform coefficients, we chose to select the best wavelet packet basis by simply using entropy as a criterion for the split/merge decisions. This also enables to reduce substantially the computational complexity of the algorithm. We experimented with three additive entropy-based cost functions: the CW-entropy $h(\mathbf{x})$, the l_p-norm (where $p = 1$) given by

$$C_1(x) = \sum_k |x_k| \tag{3}$$

and the energy entropy given by

$$C_2(x) = \sum_k \log(x_k^2). \tag{4}$$

3 The Coder Algorithm

3.1 Quantization

Within each subband, distribution of the wavelet packet coefficients can be approximated by a Laplacian distribution [1]. We used the optimal entropy constrained scalar quantizer for Laplacian distribution presented in [13]. It is a uniform scalar quantizer with a symmetric dead-zone $[-\Delta+\delta, \Delta-\delta]$ and a step size Δ. The optimal reconstruction offset in the sense of MSE is: $\delta = 1 - \Delta(e^{-\Delta}/(1 - e^{-\Delta}))$. A dichotomic search is employed to find the optimal value of Δ that uses up the bit budget.

3.2   Stack-Run Representation

The stack-run representation of [14] partitions the quantized transform coefficients into two groups containing zero valued and nonzero valued, referred to as *significant*, coefficients. Each significant coefficient is represented in binary notation as a stack or a column of bits with the MSB at the top and the LSB at the bottom. A subband is encoded by starting in one corner of the subband and performing a raster scan described in terms of stack and run representations, where 'stack' is the magnitude and sign of the significant coefficient and 'run' is the number of zerovalued coefficients encountered before the next significant coefficient.

The symbol set S consists of four symbols; $S = \{0, 1, +, -\}$, with the symbols having following meanings: '0' is used to signify a binary bit value of 0 in encoding of significant coefficients, '1' is used for binary 1 in significant coefficients, but it is not used for the MSB, '+' is used to represent the MSB of a significant positive coefficient; in addition '+' is used for binary 1 in representing run lengths, and '−' is used for the MSB of negative significant coefficients, and for binary 0 in representing run lengths. Since all binary run lengths start with 1, one can omit the final (eg, MSB) '+' from most runlength representations without loss of information. A potential problem occurs in representation of a run of length of one which would not be representable if all MSB '+' symbols were eliminated. For tackling this, it is necessary to retain the MSB '+' symbol for all runs of length $2^j - 1$, where j is an integer. Because the symbols '+' and '−' are used to code both runs and levels, a level of magnitude one, which would be represented only by the '+' or '−' that is its MSB, would be indistinguishable from a run. This is handled by simply incrementing by one the absolute value of all levels prior to performing the symbol mapping.

3.3   Entropy Coding

Both the geometry of the best wavelet packet basis selected and the quantized coefficients are converted into a stream of symbols from S using stack-run representation described above. This symbol stream is entropy coded using an order-3 arithmetic coder [7] to give actual bitstream.

4   Experimental Results

We implemented the stack-run wavelet packet (SRWP) image coder, generating an actual bitstream, and decoder. Nine different variations of the SRWP coder were tested on six standard 256-level greyscale images each having a resolution of 512×512: *Barbara*, *Houses*, *Lighthouse*, *Fingerprints*, *Goldhill*, and *Lena*. The coder variations were based on three different filters: symmlet-8, biorthogonal 17/11, and biorthogonal 9/7 filters. Each of these variations employed three cost functions given in equations (1), (3), and (4) – denoted respectively by CW, log, and l_p – for basis selection purposes. Compression results in terms of the peak-signal-to-noise-ratio (PSNR) in decibels (dB) are given in Tables 1–6. Comparisons are provided for two state-of-the-art image coders: the wavelet based SPIHT coder [10] and the fast adaptive

wavelet packet (FAWP) coder [6]; results for these algorithms were not available for last two images used for evaluating SRWP. The experimental results show that the compression performance of our SRWP coder is comparable to and often better than that of SPIHT and FAWP. Coding gains achieved by the SRWP coder over both other coders are noticeable, especially for *Barbara* (1.2dB over SPIHT and 0.4dB over FAWP) and *Fingerprints* (1.1dB over SPIHT and nearly 0.2dB over FAWP). Empirical rate-distortion curves for the best SRWP coder variation, SPIHT, and FAWP are given in Figure 1. Visual results for four test images compressed at 0.25 bits per pixel (bpp.) are provided in Figure 2.

5 Conclusions

An adaptive wavelet packet image coding algorithm based on stack-run representation of the quantized transform coefficients is presented. Our coder is relatively simple and does not need to maintain any list of coefficients, as is the case with EZW [11] and SPIHT. Decent coding gains over SPIHT and FAWP coders demonstrate the successful exploitation of intra-subband redundancies. Experimental results also show that the l_1-norm and energy cost functions typically result in the best wavelet packet basis whose coefficients are more efficiently encoded by our algorithm. Another interesting result is the consistently better performance of the symmlet-8 filters over filters in popular use. Future work on this coder may focus on the development of a cost function that takes into account the Laplacian distribution of wavelet packet coefficients.

6 Acknowledgements

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Bit rate	SRWP (*symm-8*)			SRWP (*17/11*)			SRWP (*9/7*)			SPIHT	FAWP
(bpp.)	CW	log	l_p	CW	log	l_p	CW	log	l_p		
1.0	37.13	**37.65**	37.53	37.00	37.39	37.28	36.04	36.41	36.26	36.41	37.24
0.5	32.43	**32.87**	32.81	32.28	32.59	32.51	31.34	31.53	31.49	31.39	32.82
0.25	28.80	28.97	**28.98**	28.60	28.73	28.72	27.84	27.78	27.87	27.58	29.12

Table 1: Compression results (PSNR values in dB) for 512×512 *Barbara* image

Bit rate	SRWP (*symm-8*)			SRWP (*17/11*)			SRWP (*9/7*)			SPIHT	FAWP
(bpp.)	CW	log	l_p	CW	log	l_p	CW	log	l_p		
1.0	30.61	30.88	**30.89**	30.39	30.61	30.62	30.14	30.23	30.22	30.84	30.64
0.5	26.41	26.56	**26.58**	26.22	26.19	26.26	25.84	25.87	25.85	26.15	26.48
0.25	**23.37**	23.31	23.31	23.16	22.94	23.03	22.71	22.71	22.70	23.17	23.41

Table 2: Compression results (PSNR values in dB) for 512×512 *Houses* image

Bit rate	SRWP (*symm-8*)			SRWP (*17/11*)			SRWP (*9/7*)			SPIHT	FAWP
(bpp.)	CW	log	l_p	CW	log	l_p	CW	log	l_p		
1.0	33.98	**34.16**	34.07	33.60	33.93	33.74	33.34	33.51	33.42	34.03	34.06
0.5	30.27	**30.40**	30.35	30.01	30.15	30.05	29.68	29.80	29.77	30.25	30.44
0.25	27.70	27.75	**27.78**	27.50	27.55	27.54	27.17	27.20	27.27	27.43	27.98

Table 3: Compression results (PSNR values in dB) for 512×512 *Lighthouse* image

Bit rate	SRWP (*symm-8*)			SRWP (*17/11*)			SRWP (*9/7*)			SPIHT	FAWP
(bpp.)	CW	log	l_p	CW	log	l_p	CW	log	l_p		
1.0	37.11	36.48	**37.12**	36.57	36.01	36.64	34.82	33.57	35.02	36.01	36.85
0.50	31.92	31.70	**31.95**	31.51	31.36	31.58	30.25	29.14	30.26	31.27	31.79
0.25	27.84	**27.90**	27.86	27.54	27.60	27.49	26.51	25.48	26.52	27.12	27.59

Table 4: Compression results (PSNR values in dB) for 512×512 *Fingerprints* image

Bit rate	SRWP (*symm-8*)			SRWP (*17/11*)			SRWP (*9/7*)		
(bpp.)	CW	log	l_p	CW	log	l_p	CW	log	l_p
1.0	36.29	**36.35**	36.34	35.98	36.12	36.10	35.72	35.77	35.77
0.50	32.83	**32.86**	32.87	32.57	32.65	32.64	32.28	32.33	32.33
0.25	30.16	30.16	**30.21**	29.95	29.94	29.96	29.61	29.64	29.65

Table 5: Compression results (PSNR values in dB) for 512×512 *Goldhill* image

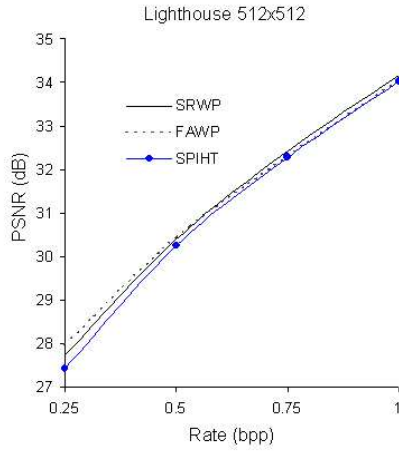| Bit rate	SRWP (*symm-8*)			SRWP (*17/11*)			SRWP (*9/7*)		
(bpp.)	CW	log	l_p	CW	log	l_p	CW	log	l_p
1.0	39.83	**39.85**	39.84	39.52	39.66	39.66	39.10	39.13	39.12
0.50	36.42	36.42	**36.43**	36.06	36.18	36.19	35.41	35.43	35.45
0.25	32.98	**33.01**	32.98	32.75	32.71	32.70	31.94	31.96	31.98

Table 6: Compression results (PSNR values in dB) for 512×512 *Lena* image
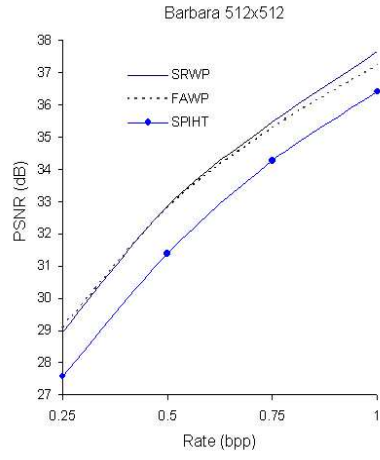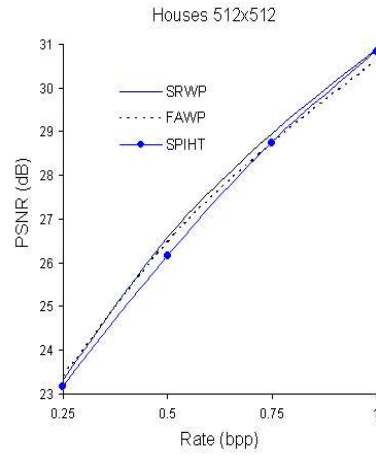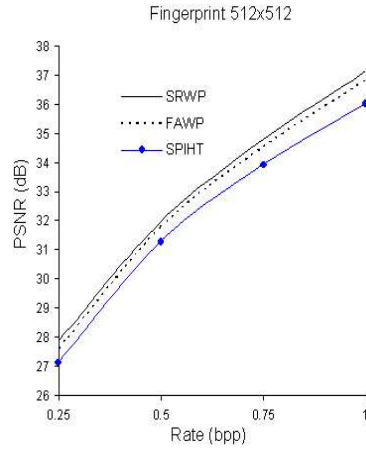
(a) (b)

(c) (d)

Figure 1: Comparative rate-distortion curves
(a) *Barbara*, (b) *Houses*, (c) *Lighthouse*, and (d) *Fingerprints*

(a) (b)

(c) (d)

Figure 2: Decoded images compressed by SRWP at 0.25 bpp.
(a) *Barbara*, (b) *Houses*, (c) *Lighthouse*, and (d) *Fingerprints*

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