TEXTURE CLASSIFICATION USING DISCRIMINANT WAVELET PACKET SUBBANDS

Nasir Rajpoot

Department of Computer Science
University of Warwick
Coventry, CV4 7AL
United Kingdom

email: nasir@dcs.warwick.ac.uk

ABSTRACT

This paper addresses the issue of selecting features from a given wavelet packet subband decomposition that are most useful for texture classification in an image. A functional measure based on Kullback-Leibler distance is proposed as a way to select most discriminant subbands. Experimental results show a superior performance in terms of classification error rates.

1. INTRODUCTION

Texture is an important regional characteristic that can be employed in order to analyse an image. There does not exist a widely accepted definition of texture. However, for the sake of simplicity, texture may be considered as a spatial area consisting of an arrangement (regular or otherwise) of primitives resembling eachother [1]. The classification of image data into different classes of texture is a challenging problem in image analysis [2]. Texture classification methods have been classified into five major categories based on the types of features they associate with a texture: statistical, structural, geometrical, model-based, and signal processing [3]. Of the signal processing methods, perhaps the most common approach is to decompose the input image into frequency subbands [4], from which, it is hoped, the features associated with textures present in the image can be extracted. Discrete wavelet transform provides a tractable way of decomposing an image into a number of frequency subbands at different scales, a feature that has been associated with human visual system [5]. A conventional dyadic wavelet transform [6], however, has a shortcoming in that it does not benefit from possibly useful features that can be obtained by further decomposing the high frequency subbands. Adaptive wavelet decomposition [7], also known as *wavelet packet* decomposition, was developed to overcome this limitation by providing better frequency localization. For the purpose of classification of image data into more than one texture classes, however, the selection of best wavelet packet decomposition has to be done in such a way that the feature images provide maximum discrimination [8]. The idea is similar to choosing a wavelet packet basis that can be most efficiently encoded by certain quantization scheme [9].

The task of a texture segmentation system is to assign class labels to each of the image pixels, and to do so efficiently and accurately. A general pattern recognition paradigm achieves this task in two stages: feature extraction and classification. In case of wavelet packet texture analysis, the extraction of features from image subbands plays a crucial role as it provides the useful information in the form of feature vectors. In this paper, following issues are addressed in the context of a two-class texture classification problem: **(1)** Can a few features associated with wavelet packet subbands be sufficient for reasonably accurate classification? **(2)** Given a wavelet packet subband decomposition consisting of n subbands, how can we select k (where $k < n$) most discriminant subbands to associate the *best* features with? It is shown that texture classification with reasonably small error rate is possible using only a few subbands. A discriminant measure based on Kullback-Leibler distance between the normalized energy of wavelet packet coefficients of two classes in a subband is proposed in order to select k most discriminant subbands.

The organization of this paper is as follows. In the next section, a brief review of both wavelet and wavelet packet transforms and texture analysis methods based on them is presented. The issue of texture classification using fewer subbands than those present in a given wavelet packet geometry is addressed in Section 3, where a discriminant measure is proposed to describe the usefulness of a subband from a classification viewpoint. Some experimental results are presented in Section 4 and the paper concludes with comments on the proposed discriminant measure and future directions.

2. WAVELET TEXTURE ANALYSIS

2.1. Introduction

The principle behind wavelets is that shifts and dilations of a prototype function $\psi(t)$ are chosen as basis functions, decomposing the signal into its components belonging to different frequencies while providing good localization in time (space) at the same time. The discrete wavelet transform can be computed with the help of filter banks that decompose the signal (image) into low and high frequency subbands. The low frequency subband is further decomposed in order to go down the transform one more level. Wavelet based texture classification methods use the wavelet subbands to extract textural features – for a review of these methods, please refer to [2, 4]. Each subband can be passed through a nonlinearity followed by a smoothing function to compute a feature image.

2.2. Wavelet Packet Decomposition

A more general form of the wavelet basis, known as the *wavelet packet basis* [7] adaptively segments the frequency axis as opposed to the adaptive segmentation of time axis in case of local cosine basis (or cosine packet basis). The frequency intervals of varying bandwidths are adaptively selected to extract specific frequency contents present in the given signal. This frequency segmentation is useful, for example, to analyze a local phenomenon occurring in the signal and belonging to a specific frequency band. The discrete wavelet packet transform of a 1-d discrete signal $\mathbf{x} = x_i$, $i = 0, 1, \ldots, N-1$ can be computed as follows. The wavelet packet coefficients are defined as

$$\begin{aligned} w_0^0(l) &= x_l & l &= 0, \ldots, N-1 \\ w_j^{2p}(l) &= \sum_k g_{k-2l}\, w_{j-1}^n(k) & l &= 0, \ldots, N2^{-j}-1 \\ w_j^{2p+1}(l) &= \sum_k h_{k-2l}\, w_{j-1}^n(k) & l &= 0, \ldots, N2^{-j}-1 \end{aligned} \quad (1)$$

where $j = 1, 2, \ldots, J$; $J = \log_2 N$, $w_j^p(l)$ is the transform coefficient corresponding to the wavelet packet function which has relative support size 2^j, frequency $p2^j$ and is located at $l2^j$. In other words, j, p and l can be regarded as the scale, frequency and position indices of the corresponding wavelet packet function respectively. The coefficients $\{h_n\}$ and $\{g_n\}$ correspond to the lowpass and highpass filters respectively for a two-channel filter bank and the transform is invertible if appropriate dual filters $\{\tilde{h}_n\}$, $\{\tilde{g}_n\}$ are used on the synthesis side. When comparing to the wavelet decomposition, it can be regarded as a decomposition which lifts the limit of only decomposing the lowpass filtered signal so that all the highpass subbands can be further decomposed as well. This results in a combinatorial explosion of possible bases which to select a suitable basis from. Since this library of available bases provides an overcomplete representation, a fast optimization algorithm such as [10] is required to select a combination of bases from this library which is well suited to the signal under consideration.

2.3. Wavelet Packet Texture Analysis

In the case of general wavelet packet decomposition, a basis needs to be selected which has the maximum discriminating power among all possible bases in the library of wavelet packets. Apart from that, texture classification using wavelet packet subbands may proceed in the same way a system based on wavelet subbands, as in Section 2.1, works. The issue of selection of features from subband decomposition demands more scrutiny now due to a large number of possible bases that can be used to represent the image. To start with, it has to be made sure that the basis chosen to represent the image is a suitable one and indeed an optimal one for the purpose of texture classification. Chang and Kuo [11] suggested using l_1-norm as a cost function for tree prunning in a top-down manner. A subband is further decomposed only if its l_1-norm is larger than a factor of the maximum norm value at the same resolution. This approach leads to an adaptive tree-structured wavelet decomposition, a term the authors of [11] used for wavelet packet decomposition. Acharyya and Kundu [12] employ an energy based cost function and a top-down search without any decimation to compute the basis wavelet packet basis for texture segmentation. Laine and Fan [13] used energies of subbands from the full wavelet packet tree as a signature for images belonging to certain texture class.

3. TOWARDS DISCRIMINANT SUBBANDS

3.1. Motivation

As mentioned above, a wavelet packet decomposition capable of providing maximum inter-class discrimination power would be the most suitable representation for a given image in a texture analysis framework. However, it cannot always be guaranteed that using more subbands directly translates to smaller classification error. Experiments demonstrate that using only a few of the subbands instead of all of the wavelet subbands can result in smaller error rates, where error rate is the ratio of total number of misclassifications and total number of pixels with the ratio expressed as a percentage. Table 1 gives results of such experiments on six test images shown in Figure 1, while each of the images was obtained by combining two textures from the Brodatz collection. A two-level wavelet transform results in seven features out of which three were selected. The subbands were chosen by heuristical selection, whereby subbands with apparent difference in magnitudes of the transform coefficients for different texture regions are given priority over those which do not react very strongly to one texture or other.

There are ${}^nC_k = \begin{pmatrix} n \\ k \end{pmatrix}$ possible combinations of k subbands from a total of n subbands. It is not practical to employ a brute force approach which finds out the best combination by trying out each of them. This is motivation enough for finding out an efficient way of determining which of these combinations of subbands is optimal in terms of best discriminating different textures. Reduction in dimensionality of the problem may result in not only more accurate but also faster classification.

3.2. Discriminant Measure

Consider a wavelet packet subband node $\lambda_d^{p,q}$, where d is the depth and p, q represent the location at depth d of the wavelet packet tree. We use the convention that in case of an image, a subband $\lambda_d^{p,q}$ is decomposed into four subbands $\lambda_{d+1}^{2p,2q}$, $\lambda_{d+1}^{2p+1,2q}$, $\lambda_{d+1}^{2p,2q+1}$, $\lambda_{d+1}^{2p+1,2q+1}$ and $\lambda_0^{0,0}$ denotes the root node (original image). Let $\mathbf{f}_d^{p,q}$ and $\mathbf{g}_d^{p,q}$ denote the normalized energy distributions of wavelet packet coefficients corresponding to the subband node $n_d^{p,q}$ associated with classes 1 and 2 respectively given by

$$f_d^{p,q}(x,y) = \frac{\left(\mathbf{w}_d^{p,q}(x,y)^T \mathbf{c}^{(1)}\right)^2}{\|\mathbf{c}^{(1)}\|^2} \tag{2}$$

and

$$g_d^{p,q}(x,y) = \frac{\left(\mathbf{w}_d^{p,q}(x,y)^T \mathbf{c}^{(2)}\right)^2}{\|\mathbf{c}^{(2)}\|^2} \tag{3}$$

where $\mathbf{w}_d^{p,q}(x,y)^T$ denotes the basis vector corresponding to position (x, y) in the subband $n_d^{p,q}$ and $\mathbf{c}^{(1)}$ and $\mathbf{c}^{(2)}$ denote texture images corresponding to classes 1 and 2 respectively. A discriminant measure $\mathcal{D}_d^{p,q}(\mathbf{f}, \mathbf{g})$ should be able to measure how differently $\mathbf{f}$ and $\mathbf{g}$ are distributed thus relating it directly to the discrimination power of subband $n_d^{p,q}$. Such a measure can then be used to determine which of the ${}^nC_k$ combinations of subbands to use in order to reduce the classification error rate. The Kullback-Leibler distance, also known as the relative entropy, between $\mathbf{f}$ and $\mathbf{g}$ is given by

$$\mathcal{I}_d^{p,q}(\mathbf{f}, \mathbf{g}) = \sum_x \sum_y f(x,y) \log \frac{f(x,y)}{g(x,y)}. \tag{4}$$

A symmetric version of this distance measure, also known as the J-divergence, given by

$$\mathcal{D}_d^{p,q}(\mathbf{f}, \mathbf{g}) = \mathcal{I}_d^{p,q}(\mathbf{f}, \mathbf{g}) + \mathcal{I}_d^{p,q}(\mathbf{g}, \mathbf{f}) \tag{5}$$

is proposed to measure the discrimination power of a subband.

Image	Decomposition	Features	Error Rate
D9D19f	Wavelet	7	1.7
D9D19f	Selected Three	3	0.9
D9D19er	Wavelet	7	3.2
D9D19er	Selected Three	3	1.3
D15D9f	Wavelet	7	42.7
D15D9f	Selected Three	3	6.2
D15D9er	Wavelet	7	49.0
D15D9er	Selected Three	3	12.5

Table 1: Classification trials for test images

4. EXPERIMENTAL RESULTS

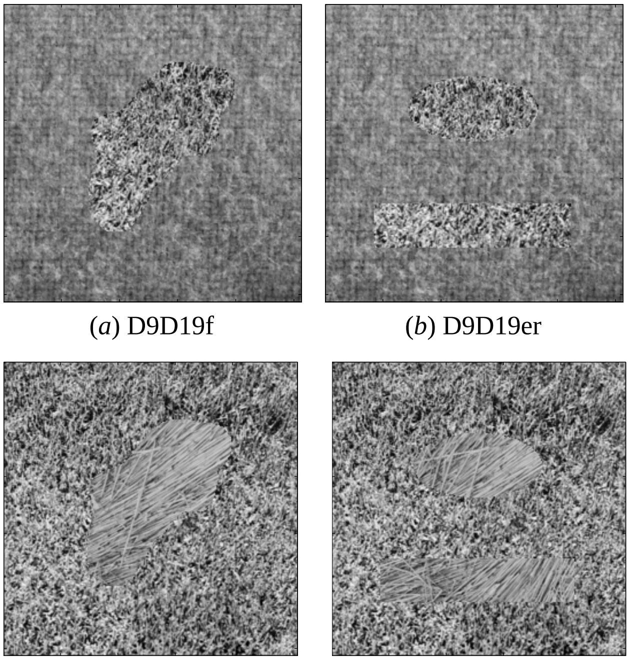
Orthonormal eight-tap filters of Daubechies [14] having four vanishing moments were used for all our experiments. While some researchers have reported that the choice of filters does not have a noticeable effect on the classification error rate [11, 15], others have disagreed [16]. The image was decomposed into a two-level full wavelet packet (full-WP) decomposition resulting in sixteen subbands. The discriminant measure of (5) was used to select four best discriminant subbands. A feature image was formed for each of the subbands by synthesising the subband and applying Gaussian smoothing on the synthesised image. Feature vectors made up of pixel values in all of the feature images were then used to classify an image pixel using an unsupervised k-means classifier. Results presented in Table 2 show a comparable performance by features based on the best discriminant subbands. The error rate is improved by using only four features instead of all sixteen.

Image	Decomposition	Features	Error Rate
D9D19f	Full-WP	16	1.9
D9D19f	Best Four	4	1.5
D9D19er	Full-WP	16	3.6
D9D19er	Best Four	4	2.1
D15D9f	Full-WP	16	42.8
D15D9f	Best Four	4	10.4
D15D9er	Full-WP	16	48.5
D15D9er	Best Four	4	16.0

Table 2: Classification results for test images

5. CONCLUSIONS

In this paper, a texture classification method using most discriminant wavelet packet subbands is presented. The symmetric version of relative entropy of normalized energy distributions of wavelet packet coefficients of textures is proposed as a solution to the problem of feature selection in



(*a*) D9D19f (*b*) D9D19er

(*c*) D15D9f (*d*) D15D9er

Figure 1: Test Images

the context of a two-class problem. Experimental results show that better performance can be achieved by selecting the most discriminant subbands. Natural extensions to this solution are the use of a classifier more sophisticated than a k-means classifier and the extension of discriminant measure to cater for more than two classes. The crucial issue of selection of an optimal wavelet packet basis for texture classification remains to be investigated.

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