Assignment 11
due on Wednesday, July 12, 2017

Name:

Exercise 1 (5+5 points)**.**
Prove the following:

1. $\Lambda^2 V = \langle v \otimes w - w \otimes v \mid v, w \in V \rangle$.
2. For all $t \in V^{\otimes 2}$, $(1,2)t = -t$.

Exercise 2 (5 + 5 points)**.**
Prove that there are no nontrivial linear subspaces of S^2V and $\Lambda^2 V$, respectively, that are invariant under the $\mathsf{GL}(V)$-action.

Exercise 3 (5 + 10 points)**.**
Let V be an n-dimensional vector space. Define the projection operators $p_{\substack{1\\2}}$ by

$$v_1 \otimes v_2 \otimes v_3 \to \frac{1}{2}(v_1 \otimes v_2 \otimes v_3 - v_2 \otimes v_1 \otimes v_3)$$

and $p_{1\,3}$ by

$$v_1 \otimes v_2 \otimes v_3 \to \frac{1}{2}(v_1 \otimes v_2 \otimes v_3 + v_3 \otimes v_2 \otimes v_1)$$

Let $U := p_{13}(p_{\substack{1\\2}}(V))$. In the same way, let $U' = p_{12}(p_{\substack{1\\3}}(V))$.

1. Prove that U and U' are $\mathsf{GL}(V)$-invariant.
2. Prove that $V^{\otimes 3} = S^3V \oplus U \oplus U' \oplus \Lambda^3 V$.