Assignment 4
due on Wednesday, May 24, 2017

Name:

Exercise 1 (10 points).
Let f be a polynomial computed by a multiplicatively disjoint circuit of size s. Prove that the degree of f is bounded by s.

Exercise 2 (10 + 10 points).
In this exercise, we consider algebraic branching programs with edges labeled by scalar multiples of the variables, that is, the edges have labels of the form αX_i. They can still have labels of the form α, too.

1. Let f be a homogeneous polynomial of degree d that is computed by an ABP of size s. Prove that there is an ABP of size polynomial in d and s such that at every node, a homogeneous polynomial is computed.
(Hint: Replace every node by $d + 1$ nodes.)
2. We group the nodes of equal degree together. There are now two types of edges, edges within one group and edges from degree i to degree $i + 1$. (Why?) Prove that there is an homogeneous ABP of size polynomial in d and s computing f with exactly $d + 1$ layers by removing all edges within the groups.
(Hint: Sort the nodes topologically and use induction.)

Exercise 3 (5 + 5 points).

1. Write the permanent of a generic 2×2-matrix as the projection of a determinant. (Size of your choice.)
2. The same, but now as a projection of an iterated matrix multiplication.