

Assignment 13  
due on Wednesday, July 18, 2018

Name:

**Exercise 1** (10 points).

Let  $V := \mathbb{C}^n$  with its canonical action of  $\mathrm{GL}_n$ . Prove that  $V$  is irreducible and determine its isomorphism type  $\lambda$ .

**Exercise 2** (10 points).

Let  $V := \mathbb{C}^{n \times n}$  denote the vector space of  $n \times n$  matrices. The group  $\mathrm{GL}_n$  acts on  $V$  by left multiplication. Decompose  $V$  into a direct sum of irreducibles and determine their isomorphism types  $\lambda$ .

**Exercise 3** (10 points).

Let  $\mathbb{A} = \mathbb{C}[x, y]_2$ . Prove that  $\mathbb{C}[\mathbb{A}]_3$  is not an irreducible  $\mathrm{GL}_2$ -representation.

**Exercise 4** (10 points).

Prove that  $\mathbb{C}[x_1, x_2, \dots, x_n]_d$  is an irreducible  $\mathrm{GL}_n$ -representation and determine its isomorphism type  $\lambda$ .