Assignment 4
due on Wednesday, May 9, 2018

Name:

Exercise 1 (10 points)**.**
Given two metric spaces A, B and a continuous map $\varphi : A \to B$. Prove that preimages under φ of sets that are $\mathbb{C}$-closed in B are $\mathbb{C}$-closed in A.

Exercise 2 (15 points)**.**
Let $\mathbb{A} = \mathbb{C}[X_1, \ldots, X_N]_d$ with the usual action of GL_N. Prove that this action preserves Waring rank and border Waring rank, i.e.,

$$\mathrm{WR}(h) = \mathrm{WR}(gh) \qquad \text{and} \qquad \underline{\mathrm{WR}}(h) = \underline{\mathrm{WR}}(gh)$$

for all $h \in \mathbb{A}$, $g \in \mathsf{GL}_N$.

Exercise 3 (15 points)**.**
Let $0 \neq f \in \mathbb{C}[X_1, \ldots, X_n]$ and define the subset $D_f \subseteq \mathbb{C}^n$ via

$$D_f := \{x \in \mathbb{C}^n \mid f(x) \neq 0\}.$$

Prove that $\overline{D_f} = \mathbb{C}^n$.