Assignment 6
due on Wednesday, May 23, 2018

Name:

Exercise 1 (10 points)**.**
Given an arithmetic circuit of size s that computes a polynomial h in variables $x_1, \ldots, x_n$. Prove that for each $i \in \{1, \ldots, n\}$ there exists an arithmetic circuit of size $\leq 4s$ that computes the partial derivative $\frac{\partial h}{\partial x_i}$.

Exercise 2 (10 points)**.**
Given an arithmetic circuit of size s that computes a polynomial h of degree d. Prove that for each $i \in \{0, \ldots, d\}$ there exists an arithmetic circuit of size $O(d^2 s)$ that computes the homogeneous degree i part of h.

Exercise 3 (10 points)**.**
Let $(h_n) \in \mathrm{VP}$ and let $p(n)$ be minimal such that that $h_n \in \mathbb{C}[x_1, \ldots, x_{p(n)}]$. For each n let $g_n \in \mathrm{End}_{p(n)}$. Prove that $(g_n h_n) \in \mathrm{VP}$.

Exercise 4 (10 points)**.**
Define the polynomials $\mathrm{gensum}_{i,0} = \mathrm{genprod}_{i,0} = x_i$. For $d > 0$ define

$$\begin{aligned} \mathrm{gensum}_{i,d} &= \mathrm{genprod}_{i,d-1} + \mathrm{genprod}_{i+2^{d-1},d-1}, \\ \mathrm{genprod}_{i,d} &= \mathrm{gensum}_{i,d-1} \cdot \mathrm{gensum}_{i+2^{d-1},d-1}\,. \end{aligned}$$

(1) Prove that $\mathrm{gensum}_{i,d}$ is a polynomial in the variables $x_i, x_{i+1}, \ldots, x_{i+2^d-1}$.

Define $\mathrm{gen}_n = \mathrm{gensum}_{1,\lceil \log n \rceil}$, where $\lceil \log n \rceil$ is the binary logarithm rounded up.

(2) Prove that (gen_n) is $\mathrm{VP_e}$-complete.