Methods and Problems of Wavelength-Routing in All-Optical Networks *

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Abstract

We give a survey of recent theoretical results obtained for wavelength-routing in all-optical networks. The survey is based on the previous survey in [Beauquier, B., Bermond, J-C., Gargano, L., Hell, P., Perennes, S., Vaccaro, U.: Graph problems arising from wavelength-routing in all-optical networks. In: Proc. of the 2nd Workshop on Optics and Computer Science, part of IPPS'97, 1997]. We focus our survey on the current research directions and on the used methods. We also state several open problems connected with this line of research, and give an overview of several related research directions.

1 Introduction

Motivation. Optical networking is a very quickly developing new area of research. It is a key technology in communication networks and it is expected to dominate important applications such as video conferencing, scientific visualisation, real-time medical imaging, high speed super-computing, distributed computing (covering both local and wide area). Networks which use optical transmission and maintain optical data paths through the nodes are called all-optical networks. All-optical networks exploit photonic technology for the implementation of both switching and transmission functions, so that signals in the networks can be maintained in optical form. This allows for much higher transmission rates since there is no overhead due to conversions to and from electronic form during transmission. Such networks use the technology of wavelength-division multiplexing (WDM), where the optical bandwidth is partitioned into a number of channels, and multiple laser beams are propagated concurrently along the same optical fibre on distinct light channels (wavelengths). It is evident that the number of wavelengths (the so-called *optical bandwidth*) is a limiting factor.

Optical Networks. In general, a WDM optical network consists of routing nodes interconnected by point-to-point fiber-optic links, which can support a certain number of wavelengths. Due to optical interference, two optical signals on the same wavelength coming into two input ports *must* be routed to different output ports. In this paper, we consider *switched* networks with reconfigurable wavelength-selective optical switches without wavelength converters, which

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can be based on acousto-optic filters, as done e.g. in [1]. In this kind of networks, signals for different communication requests may travel on the same communication link into a node (on different wavelengths) and then exit along (possibly) different links, keeping their original wavelength. The only constraint on all the different communication paths is that no two paths sharing the same optical link may transmit on the same wavelength.

The Wavelength-Routing Problem. A *request* consists of an ordered pair of nodes, and an *instance* of a set of requests. The *wavelength-routing problem* consists of finding settings for the switches in the network, and an assignment of *wavelengths* to the requests such that there is a directed path (dipath) between the nodes of each request, and that no arc will carry two different signals on the same wavelength.

The formal models of optical networks and of the wavelength-routing problem used in this paper are given in Section 2. It should be pointed out that we adopt the models from [11]. A thorough discussion and justification of these models can be found in [11].

Content of the Paper. In [11], a survey of the main theoretical results is given for the model of wavelength-routing in all-optical networks as introduced above. In this paper, we survey the current developments in this area. We focus on the different directions of research and on the used methods. We also state several open problems, and give an overview of several related research directions. Unfortunately, we have to omit all the proofs in this extended abstract.

Organization of the Paper. The paper is organized as follows. In Section 2, we give the fundamental definitions, and we make some simple observations. In Section 3, we present methods and results for the one-to-many and the all-to-all communication instances. In Section 4, we discuss permutation instances. Section 5 portrays other instances which are of specific interest when simulating a special network structure on another. Finally, Section 6 gives an outlook on related research directions.

2 Definitions and Simple Observations

2.1 Definitions

We adopt the models from [11] for optical networks and the wavelength-routing problem, as well as the model from [35] for the graph-embedding problem.

The Optical Network Model. An all-optical network is modelled as a symmetric directed graph $G = (V(G), A(G))$, where $V(G)$ is a set of vertices, and $A(G)$ is a set of arcs such that if $(u, v) \in A(G)$ then $(v, u) \in A(G)$. Let $P(x, y)$ denote a dipath (directed path) in G from the node x to y which consists of consecutive arcs beginning at x and ending at y. Let $dist(x, y)$ denote the *distance* from x to y in G, that is the length of a shortest dipath $P(x, y)$.

The Wavelength-Routing Problem. A *request* in G is an ordered pair of nodes (x, y) in G corresponding to a message to be sent from x to y, and an *instance* I in G is a set of requests in G. A *routing* for an instance I in G is a set of dipaths $R = \{P(x, y) | (x, y) \in I\}$.

- Let G be a digraph and I an instance in G. The *routing problem for* (G, I) consists of finding a routing R for the instance I in G.

- Let G be a digraph, I an instance in G, and R a routing for I in G. The *wavelength-assignment problem for* (G, I, R) consists of finding an assignment of a wavelength to each request $(x, y) \in I$, so that no two dipaths of R sharing an arc have the same wavelength.

- Let G be a digraph and I an instance in G. The *wavelength-routing problem for* (G,I) consists of finding a routing R for the instance I and an assignment of a wavelength to each request $(x,y) \in I$, so that no two dipaths of R sharing an arc have the same wavelength.

It is convenient to think of wavelengths as colours. (In this context, the wavelength-assignment problem is also denoted the *path-colouring problem.*) For a given routing R for (G,I), the smallest number of colours is denoted by $\overrightarrow{\omega}(G,I,R)$. Let $\overrightarrow{\omega}(G,I)$ denote the smallest $\overrightarrow{\omega}(G,I,R)$ over all routings R.

Special Instances.

- The *All-to-All* instance is $I_{\mathtt{A}} = \{(x,y) \mid x,y \in V(G),\ x \neq y\}$.
- A *One-to-All* instance is a set $I_{\mathtt{O}} = \{(x,y) \mid y \in V(G),\ y \neq x_0\}$, where $x_0 \in V(G)$. A *One-to-Many* instance is a subset of some instance $I_{\mathtt{O}}$.
- A *k-relation* is an instance I_k in which each node is the source and the destination of at most k requests. A *permutation* instance is a 1-relation I_1.

Routing Parameters. Let G be a digraph, I an instance in G, and R a routing for I in G. The *arc-congestion* $\overrightarrow{\pi}(G,I,R,\alpha)$ of an arc $\alpha \in A(G)$ in the routing R is the number of dipaths of R containing α. The maximum congestion of any arc of G in the routing R is called the *arc-congestion* of G in the routing R for the instance I : $\overrightarrow{\pi}(G,I,R) = \max_{\alpha \in A(G)} \overrightarrow{\pi}(G,I,R,\alpha)$. Let $\overrightarrow{\pi}(G,I)$ be the minimum congestion of G in any routing R for the instance I. (From the technological point of view, the arc-congestion problem corresponds to the case where switches in the network allow a change from one wavelength to another.) For the All-to-All instance $I_{\mathtt{A}}$, $\overrightarrow{\pi}(G,I_{\mathtt{A}})$ is called the *arc-forwarding index* of G, and $\overrightarrow{\omega}(G,I_{\mathtt{A}})$ is called the *optical index* of G.

All the definitions for the directed case have natural analogues in the undirected case. The undirected model is obtained from the directed one by replacing the two opposite arcs in G by an edge and omitting arrows in the definitions of $\overrightarrow{\omega}$ and $\overrightarrow{\pi}$. This gives rise to two new problems for an instance I on a graph G, namely $\omega(G,I)$ and $\pi(G,I)$. The analogue of the arc-congestion is the *edge-congestion* in the undirected case. The analogue of the arc-forwarding index is the *edge-forwarding index* of G in the undirected case.

The Conflict Graph. Let G be a digraph, I an instance for G, and R a routing for I in G. The *conflict graph* associated with (G,I,R), denoted by *Confl*(G,I,R), is the undirected graph (R,E) with vertex set R such that two vertices of (R,E) are adjacent if and only if they share an arc (as dipaths) in G.

Graph Embeddings. Let G_1 and G_2 be finite undirected graphs. An *embedding* of G_1 into G_2 is a one-to-one mapping f from the nodes of G_1 to the nodes of G_2. G_1 is called the *guest* graph and G_2 is called the *host* graph of the embedding f. The *dilation* of the embedding f is the maximum distance in the host between the images of adjacent guest nodes. A *routing* is a mapping r of G_1's edges to paths in G_2, $r(v,w) =$ a path from $f(v)$ to $f(w)$ in G_2. The *edge-congestion* of the embedding f is the maximum number of edges that are routed through a single edge of G_2.

Overall, the terminology for graph embeddings is exactly the same as that for wavelength-routing with the difference that G_1 and G_2 are undirected graphs. Hence, an edge $\{v_1,w_1\}$ from G_1 is viewed as the two requests $(v_1,w_1),\ (w_1,v_1) \in I$, and an edge $\{v_2,w_2\}$ from G_2 is viewed as the two arcs $(v_2,w_2),\ (w_2,v_2)$ in G.

2.2 Simple Observations

The relevance of the routing problem to the wavelength-routing problem becomes clear through the following observation:

Observation 1 *$\overrightarrow{\omega}(G,I) \geq \overrightarrow{\pi}(G,I)$ for any instance I in any digraph G.*

The relevance of the conflict graph to the wavelength-routing problem is given by the following observation:

Observation 2 *Let G be a digraph, I an instance in G, and R a routing for I in G. Then, $\overrightarrow{\omega}(G,I,R)$ is equal to the chromatic number of* $\mathrm{Confl}(G,I,R)$.

In general, minimizing the number of wavelengths (i.e., the wavelength-routing problem) is not the same as that of realizing a routing that minimizes the number of dipaths sharing an arc (i.e., the routing problem). Indeed, the wavelength-routing problem is made much harder due to the further requirement of wavelength-assignment on the paths (i.e., the wavelength-assignment problem).

3 One-to-Many and All-to-All Communication

For a general network G and an arbitrary instance I, the problem of determining $\overrightarrow{\omega}(G,I)$ has been proved to be NP-hard in [16]. In particular, it has been proved that determining $\overrightarrow{\omega}(G,I)$ is NP-hard for trees and cycles. In [21], these results have been extended to binary trees and meshes.

The problem of determining $\overrightarrow{\pi}(G,I)$ for a given digraph G and instance I has been extensively investigated, especially in the context of arc-forwarding index and edge-forwarding index (see e.g. [22, 34, 44], as well as the brief overview on forwarding indices in [11]).

From Observation 1, we know that

$$\overrightarrow{\omega}(G,I) \geq \overrightarrow{\pi}(G,I) \ . \tag{1}$$

Hence, it is natural to ask more precisely about the general relation between $\overrightarrow{\omega}(G,I)$ and $\overrightarrow{\pi}(G,I)$, especially the question in which cases equality can be achieved in (1), or in which cases one can at least come close to equality in (1).

In order to obtain equality in (1), a routing R must be found such that $\overrightarrow{\pi}(G,I,R) = \overrightarrow{\pi}(G,I)$, and the associated conflict graph $\mathit{Confl}(G,I,R)$ is $\overrightarrow{\pi}(G,I)$-vertex colourable. On the other hand, the following general relation was derived between the parameters $\overrightarrow{\omega}(G,I)$ and $\overrightarrow{\pi}(G,I)$ in [1]:

Theorem 3 **([1])**

1. *Let I be an instance in a network G. Let $L = \max_{(x,y)\in I} \mathrm{dist}(x,y)$. Then, $\overrightarrow{\omega}(G,I) = O(L \cdot \overrightarrow{\pi}(G,I))$.*

2. *For every L and $\overrightarrow{\pi}$, there exists a directed graph G and an instance I such that $L = \max_{(x,y)\in I} \mathrm{dist}(x,y)$, $\overrightarrow{\pi}(G,I) = \overrightarrow{\pi}$, and $\overrightarrow{\omega}(G,I) = \Omega(L \cdot \overrightarrow{\pi})$.*

Theorem 3 shows that the gap between $\overrightarrow{\omega}(G,I)$ and $\overrightarrow{\pi}(G,I)$ can be quite substantial in general. As a consequence, it becomes of high interest to investigate the relation between $\overrightarrow{\omega}(G,I)$ and $\overrightarrow{\pi}(G,I)$ for specific important graph classes G and/or instances I, and there has been an ongoing effort in this direction. Especially, One-to-Many instances and the All-to-All instance have been thoroughly investigated in the literature.

One-to-Many instances have been investigated in [12]. Using a network flow approach, equality was shown for (1) in this case:

Theorem 4 ([12]) *$\overrightarrow{\omega}(G, I) = \overrightarrow{\pi}(G, I)$ for any One-to-Many instance I in any digraph G.*

Results about the All-to-All instance $I_{\mathtt{A}}$ can be found in [2, 9, 11, 21, 41], and especially on rings in [13, 28, 47]. For the All-to-All instance, equality in (1) has been proved for specific networks like paths, cycles, hypercube [13], toroidal mesh of even side [9], trees [21], as well as for more general graph classes like clique-compound graphs [2], cartesian product of cycles and paths [41], cartesian product of complete graphs [9]. In [45], bounds on the number of wavelengths for inflated networks are given for the All-to-All instance. In [36, 46], bounds on the optical index of (families of) circulant 4-regular graphs are given in terms of the arc-forwarding index. There is an ongoing effort to settle the question whether the optical index is equal to the arc-forwarding index for general graphs.

Question 5 *Does the equality $\overrightarrow{\omega}(G, I_{\mathtt{A}}) = \overrightarrow{\pi}(G, I_{\mathtt{A}})$ hold for the All-to-All instance $I_{\mathtt{A}}$ in any symmetric digraph G?*

Instances different from One-to-Many instances and from the All-to-All instance are of importance in the context of graph embeddings and will be discussed later on in Section 5.

4 Permutations

Some lower bounds on the numbers of wavelengths required for permutation instances in any network were given in [37]. From the algorithmic point of view, determining $\overrightarrow{\omega}(C_n, I_1)$ and $\omega(C_n, I_1)$ is NP-hard, which can be shown by a modification of the NP-hardness proof of the wavelength problem in a ring for general instances, given in [16]. However polynomial 2-approximation algorithms exist for both problems [16, 39, 43]. The problems $\overrightarrow{\pi}(C_n, I)$ and $\pi(C_n, I_1)$ are solvable in polynomial time, which follows from the more general results of [48] and [18], respectively. This motivates study of the best possible upper bounds on the above four measures. In [38], the following is shown:

Theorem 6 ([38]) *The following inequalities hold, for $\overrightarrow{\omega}$ and w and for $\overrightarrow{\pi}$ and π, for an I_1 permutation request on the n-vertex ring C_n:*

$$\overrightarrow{\omega}(C_n, I_1) \leq \left\lceil \frac{n}{3} \right\rceil, \quad \overrightarrow{\pi}(C_n, I_1) \leq \left\lceil \frac{n}{4} \right\rceil, \quad \omega(C_n, I_1) \leq \left\lceil \frac{n}{2} \right\rceil, \quad \pi(C_n, I_1) \leq \left\lfloor \frac{n}{2} \right\rfloor.$$

All bounds are the best possible for worst-case instances.

5 Network Simulation

In this section, we consider the problem of simulating a guest network G_1 on a host network G_2.

Customarily, the *simulation* problem is formalized as the *emdedding* problem of one graph in another (for a formal definition of the *embedding* problem, see Section 2). The "quality" of an embedding is measured by the parameters *dilation* and *edge-congestion*. The importance of the different parameters becomes apparent through the following result.

Proposition 7 ([29]) *If there is an embedding of G_1 into G_2 with dilation d and edge-congestion c, then there is a simulation of G_1 by G_2 with slowdown $O(d+c)$.*

Most of the early work was focused on dilation (for an overview, see e.g. [35]), where, due to the result of Leighton et al. above, more recent work also considers edge-congestion (see e.g. [14, 15, 23, 24, 33, 40, 42].

When simulating a guest network G_1 on an all-optical host network G_2, the guest network G_1 corresponds to the instance I and the host network G_2 corresponds to the all-optical network G, and we have to solve the wavelength-routing problem for (G, I). Due to the inequality $\vec{\omega}(G,I) \geq \vec{\pi}(G,I)$, the edge-congestion of the considered embeddings is of increased interest at this point, especially lower bounds. Recently, there has been some work in this direction [14, 33, 42]. Unfortunately, at present, the author is not aware of any specific work in the area of graph embeddings which takes the path-colouring problem into account.

6 Conclusion and Outlook

In this paper, we have given a survey of the recent developments in wavelength-routing in all-optical networks. Unfortunately, we had to leave out several other important lines of research in this area. One of these is the area of on-line routing in all-optical networks. In this scenario, requests can dynamically change and are given at different times. We refer the reader to [5, 8, 30] and references therein for an account of this area. Another area is the area of probabilistic algorithms for on-line routing in all-optical networks (see e.g. [7, 17, 31, 32]). Recent progress has been made in the routing model where wavelength converters are allowed [3, 4, 20]. Finally, specific communication problems were investigated in the so-called *linear cost* model [10].

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