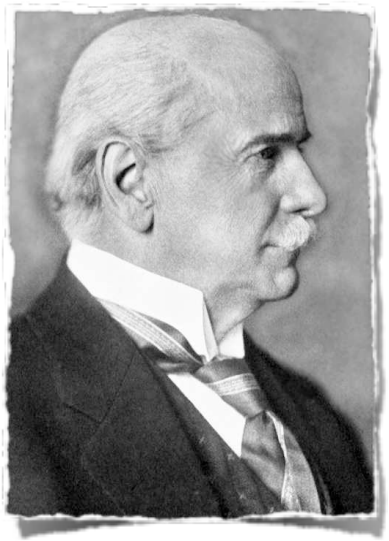
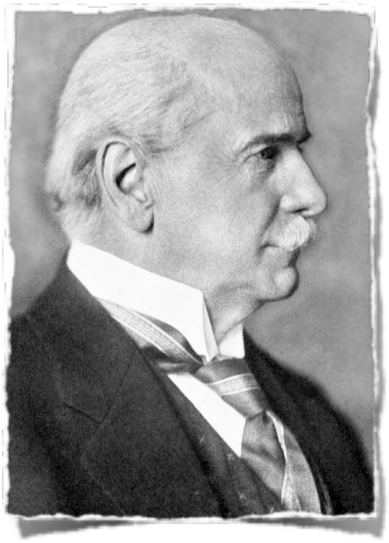
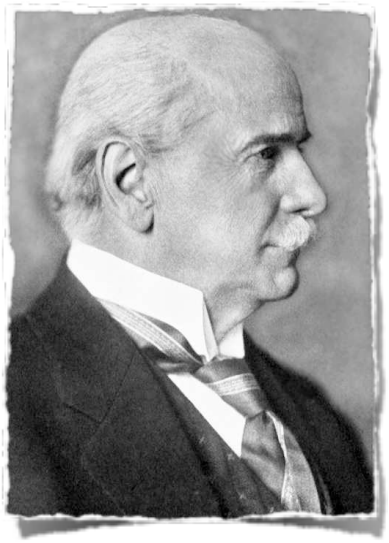
Data Streams, Dyck Languages, and *Detecting Dubious Data Structures*

Amit Chakrabarti *Dartmouth College*
Graham Cormode *AT&T Research Labs*
Ranganath Kondapally *Dartmouth College*
Andrew McGregor *University of Massachusetts, Amherst*



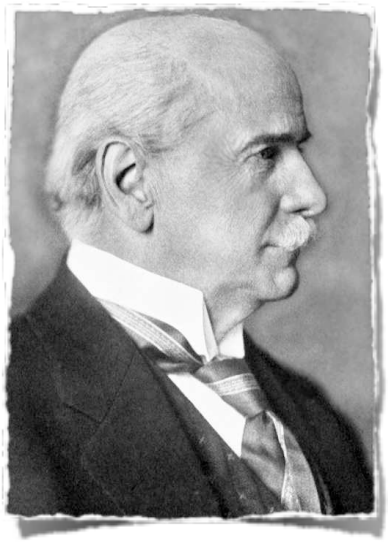


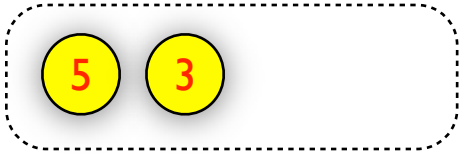
- At each step, user inserts a value into the memory or asks that the *smallest* value is extracted:



- At each step, user inserts a value into the memory or asks that the *smallest* value is extracted:

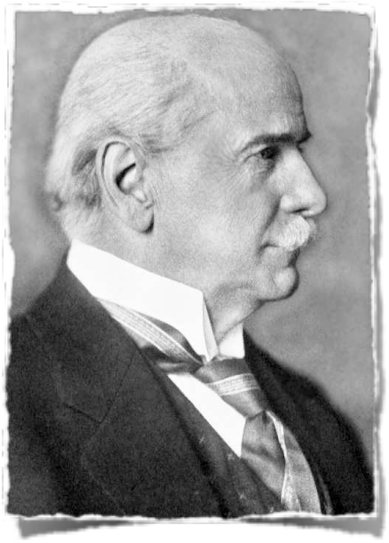
  ins(5)



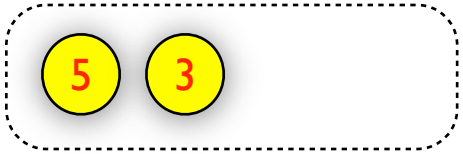


- At each step, user inserts a value into the memory or asks that the *smallest* value is extracted:

  ins(5) ins(3)



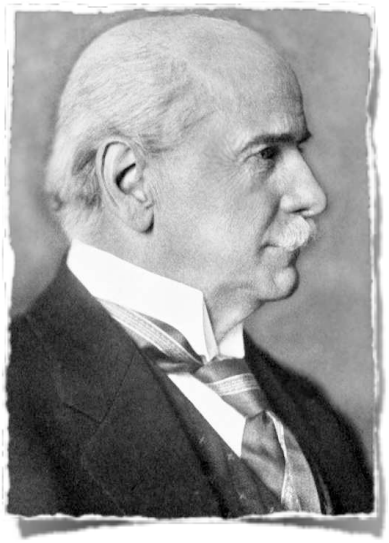
extract min!



MEMORY

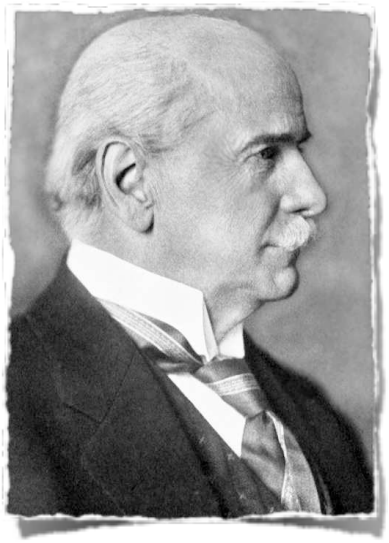
- At each step, user inserts a value into the memory or asks that the *smallest* value is extracted:

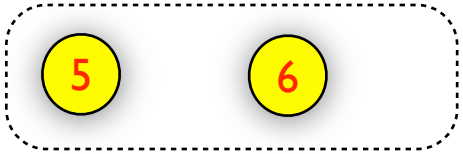
  ins(5) ins(3)



- At each step, user inserts a value into the memory or asks that the *smallest* value is extracted:

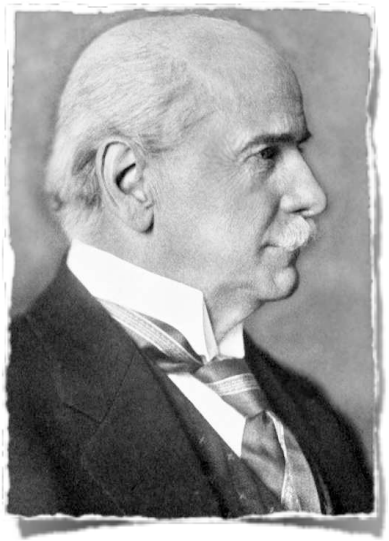
  ins(5) ins(3) ext(3)

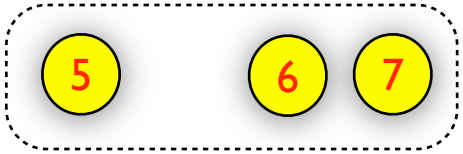




- At each step, user inserts a value into the memory or asks that the *smallest* value is extracted:

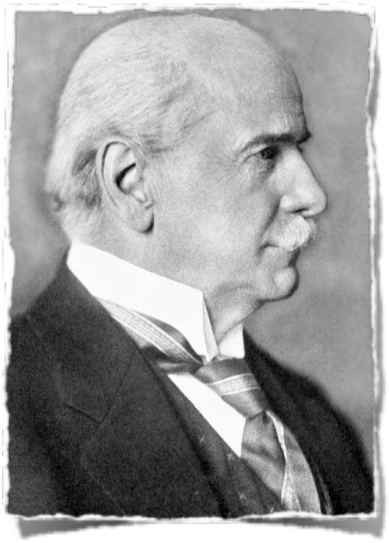
  ins(5) ins(3) ext(3) ins(6)



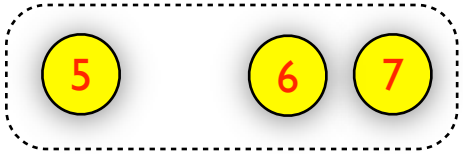


- At each step, user inserts a value into the memory or asks that the *smallest* value is extracted:

  ins(5) ins(3) ext(3) ins(6) ins(7)



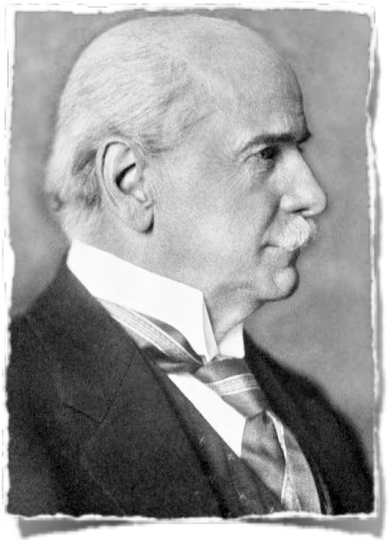
extract min!

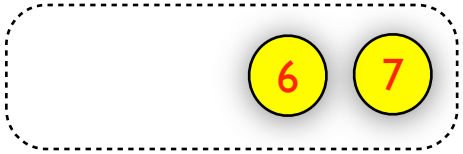


MEMORY

- At each step, user inserts a value into the memory or asks that the *smallest* value is extracted:

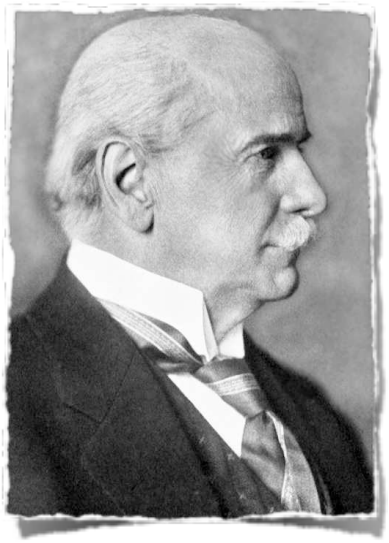
  ins(5) ins(3) ext(3) ins(6) ins(7)



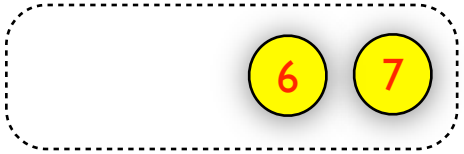


- At each step, user inserts a value into the memory or asks that the *smallest* value is extracted:

  ins(5) ins(3) ext(3) ins(6) ins(7) ext(5)



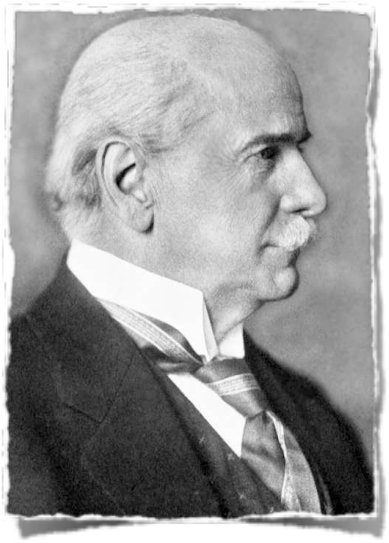
extract min!



MEMORY

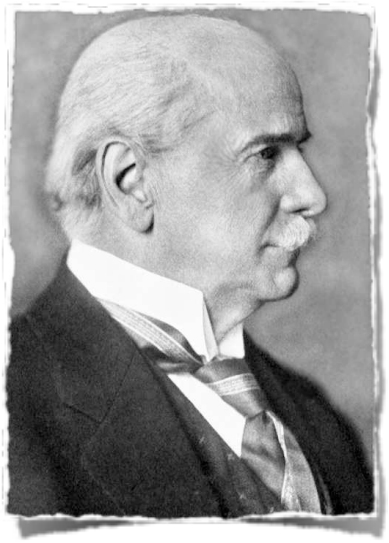
- At each step, user inserts a value into the memory or asks that the *smallest* value is extracted:

  ins(5) ins(3) ext(3) ins(6) ins(7) ext(5)



- At each step, user inserts a value into the memory or asks that the *smallest* value is extracted:

ins(5) ins(3) ext(3) ins(6) ins(7) ext(5) ext(6)

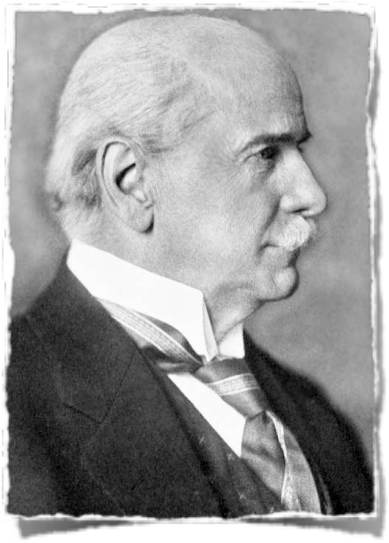


extract min!

MEMORY

- At each step, user inserts a value into the memory or asks that the *smallest* value is extracted:

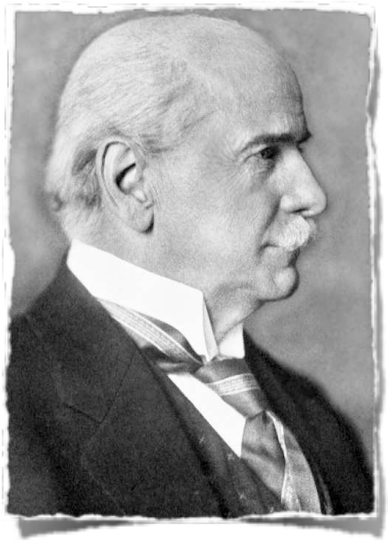
ins(5) ins(3) ext(3) ins(6) ins(7) ext(5) ext(6)



MEMORY

- At each step, user inserts a value into the memory or asks that the *smallest* value is extracted:

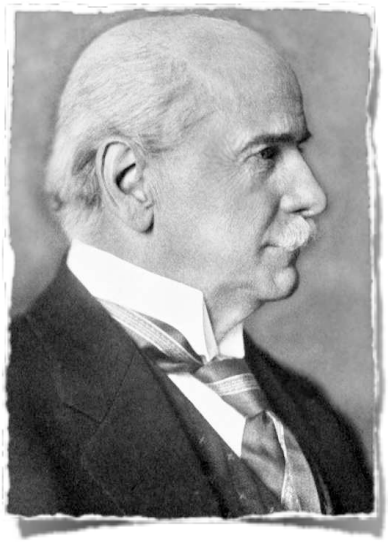
ins(5) ins(3) ext(3) ins(6) ins(7) ext(5) ext(6) ext(7)



- At each step, user inserts a value into the memory or asks that the *smallest* value is extracted:

  ins(5) ins(3) ext(3) ins(6) ins(7) ext(5) ext(6) ext(7)

? *Challenge:* Without remembering all the interaction, can you verify the priority queue performed correctly?



- At each step, user inserts a value into the memory or asks that the *smallest* value is extracted:

  ins(5) ins(3) ext(3) ins(6) ins(7) ext(5) ext(6) ext(7)

? *Challenge:* Without remembering all the interaction, can you verify the priority queue performed correctly?

- *Motivation:* Want to use cheap commodity hardware.

[Blum, Evans, Gemmell, Kannan, Naor '94]

PQ Language Problem

PQ Language Problem

- Let PQ be set of legitimate transcripts of a priority queue that starts and ends empty.

PQ Language Problem

- Let PQ be set of legitimate transcripts of a priority queue that starts and ends empty.

ins(5), ins(3), ext(3), ins(7), ext(5), ext(7) $\in$ PQ

PQ Language Problem

- Let PQ be set of legitimate transcripts of a priority queue that starts and ends empty.

  ins(5), ins(3), ext(3), ins(7), ext(5), ext(7) $\in$ PQ

  ins(5), ext(3), ins(3), ins(7), ext(7), ext(5) $\notin$ PQ

PQ Language Problem

- Let PQ be set of legitimate transcripts of a priority queue that starts and ends empty.

  ins(5), ins(3), ext(3), ins(7), ext(5), ext(7) $\in$ PQ

  ins(5), ext(3), ins(3), ins(7), ext(7), ext(5) $\notin$ PQ

- PQ *Problem:* Given *streaming* access to length N transcript, determine if it's in PQ using o(N) space.

PQ Language Problem

- Let PQ be set of legitimate transcripts of a priority queue that starts and ends empty.

  ins(5), ins(3), ext(3), ins(7), ext(5), ext(7) $\in$ PQ

  ins(5), ext(3), ins(3), ins(7), ext(7), ext(5) $\notin$ PQ

- PQ *Problem:* Given *streaming* access to length N transcript, determine if it's in PQ using o(N) space.

- *In this talk...*

  i. We'll design an algorithm that uses $O(\sqrt{N})$ space!

PQ Language Problem

- Let PQ be set of legitimate transcripts of a priority queue that starts and ends empty.

  ins(5), ins(3), ext(3), ins(7), ext(5), ext(7) $\in$ PQ

  ins(5), ext(3), ins(3), ins(7), ext(7), ext(5) $\notin$ PQ

- PQ *Problem:* Given *streaming* access to length N transcript, determine if it's in PQ using o(N) space.

- *In this talk...*

  i. We'll design an algorithm that uses $O(\sqrt{N})$ space!

  ii. Prove it's optimal via a communication lower bound.

PQ Language Problem

- Let PQ be set of legitimate transcripts of a priority queue that starts and ends empty.

  ins(5), ins(3), ext(3), ins(7), ext(5), ext(7) $\in$ PQ

  ins(5), ext(3), ins(3), ins(7), ext(7), ext(5) $\notin$ PQ

- PQ *Problem:* Given *streaming* access to length N transcript, determine if it's in PQ using o(N) space.

- *In this talk...*

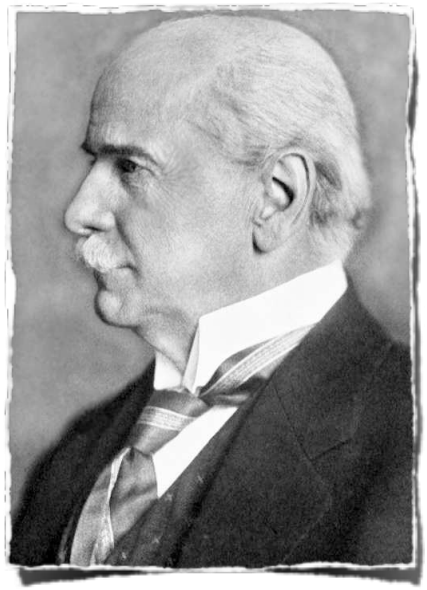
  i. We'll design an algorithm that uses $O(\sqrt{N})$ space!

  ii. Prove it's optimal via a communication lower bound.

  iii. Explore connections with other problems...

I. Memory Checking

II. Lower Bounds



III. Parenthesis and Passes

I. Memory Checking

PQ Algorithm

PQ Algorithm

- *Thm:* There exists a $O(\sqrt{N} \log N)$ space algorithm with $O(\log N)$ amortized update time for recognizing PQ.

PQ Algorithm

- *Thm:* There exists a $O(\sqrt{N} \log N)$ space algorithm with $O(\log N)$ amortized update time for recognizing PQ.

"Can verify terabytes of transcript with only megabytes!"

PQ Algorithm

- *Thm:* There exists a $O(\sqrt{N} \log N)$ space algorithm with $O(\log N)$ amortized update time for recognizing PQ.

  "Can verify terabytes of transcript with only megabytes!"

- *Prelim:* Easy to check that set of values inserted equals set of values extracted using fingerprinting.

PQ Algorithm

- *Thm:* There exists a $O(\sqrt{N} \log N)$ space algorithm with $O(\log N)$ amortized update time for recognizing PQ.

  "Can verify terabytes of transcript with only megabytes!"

- *Prelim:* Easy to check that set of values inserted equals set of values extracted using fingerprinting.

$$\prod_{u \text{ inserts}} (x - u) \stackrel{?}{=} \prod_{u \text{ extracts}} (x - u)$$

PQ Algorithm

- *Thm:* There exists a $O(\sqrt{N} \log N)$ space algorithm with $O(\log N)$ amortized update time for recognizing PQ.

  "Can verify terabytes of transcript with only megabytes!"

- *Prelim:* Easy to check that set of values inserted equals set of values extracted using fingerprinting.

$$\prod_{u \text{ inserts}} (r - u) \stackrel{?}{=} \prod_{u \text{ extracts}} (r - u)$$

PQ Algorithm

- *Thm:* There exists a $O(\sqrt{N} \log N)$ space algorithm with $O(\log N)$ amortized update time for recognizing PQ.

  "Can verify terabytes of transcript with only megabytes!"

- *Prelim:* Easy to check that set of values inserted equals set of values extracted using fingerprinting.

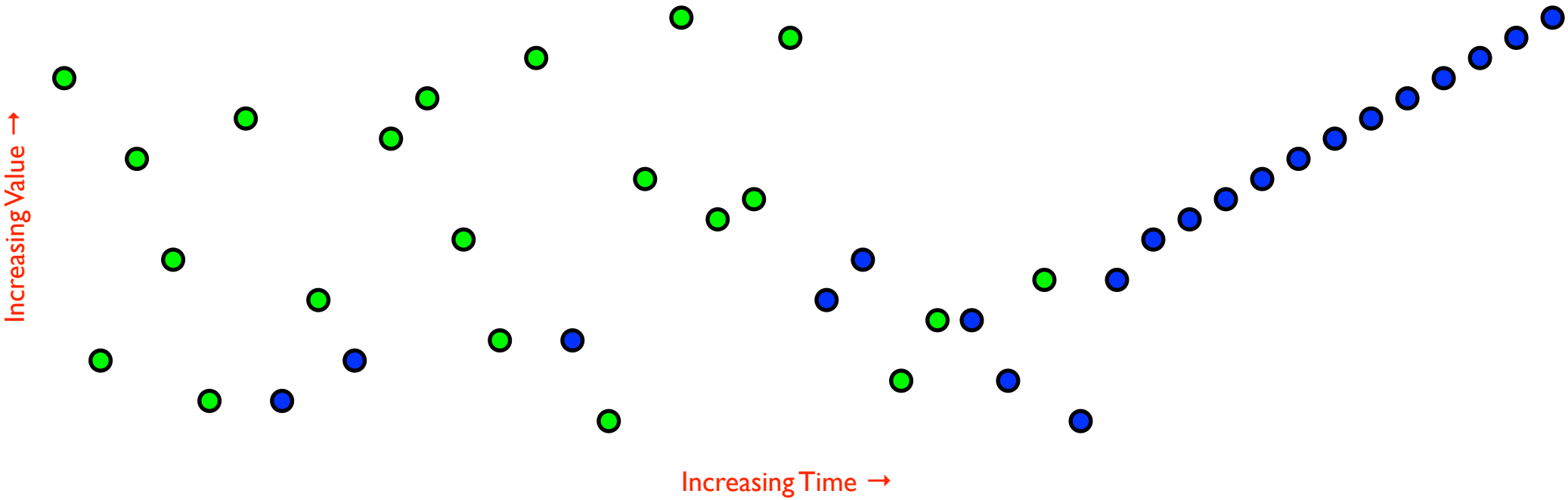
$$\prod_{u \text{ inserts}} (r - u) \stackrel{?}{=} \prod_{u \text{ extracts}} (r - u)$$

! *For this talk:* Assume inserted elements are distinct and that inserts come before their corresponding extract. I.e., we're trying to identify the following *bad pattern:*

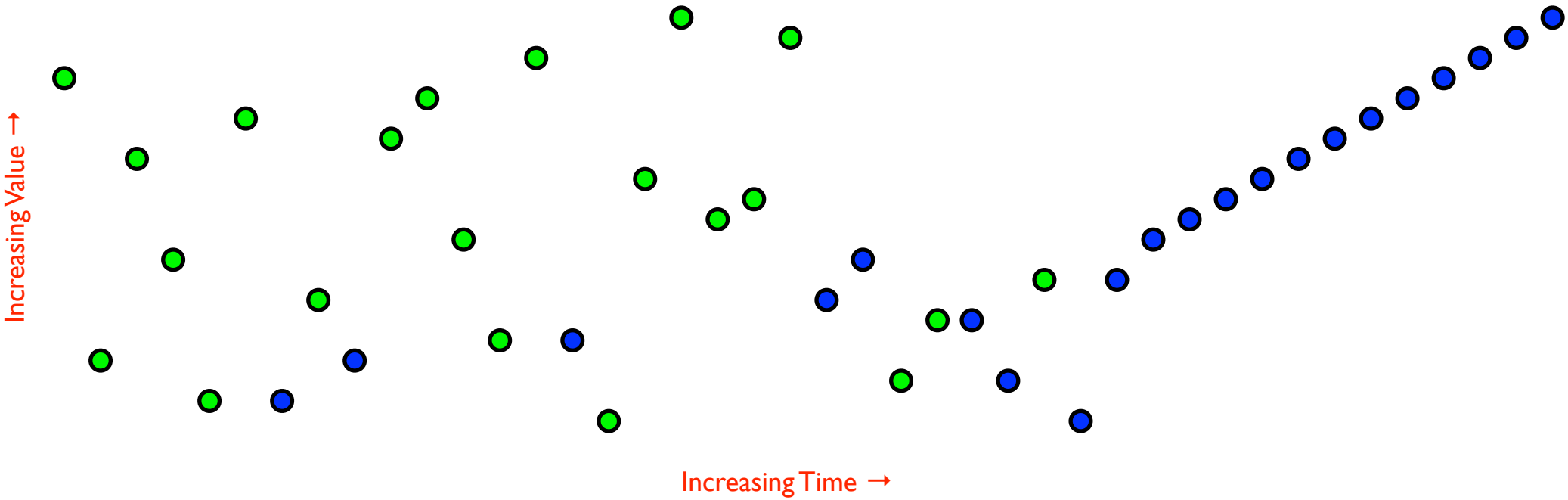
ins(u) ... ext(v) ... ext(u) for some u < v

Epochs and Local Bad Patterns...

Epochs and Local Bad Patterns...

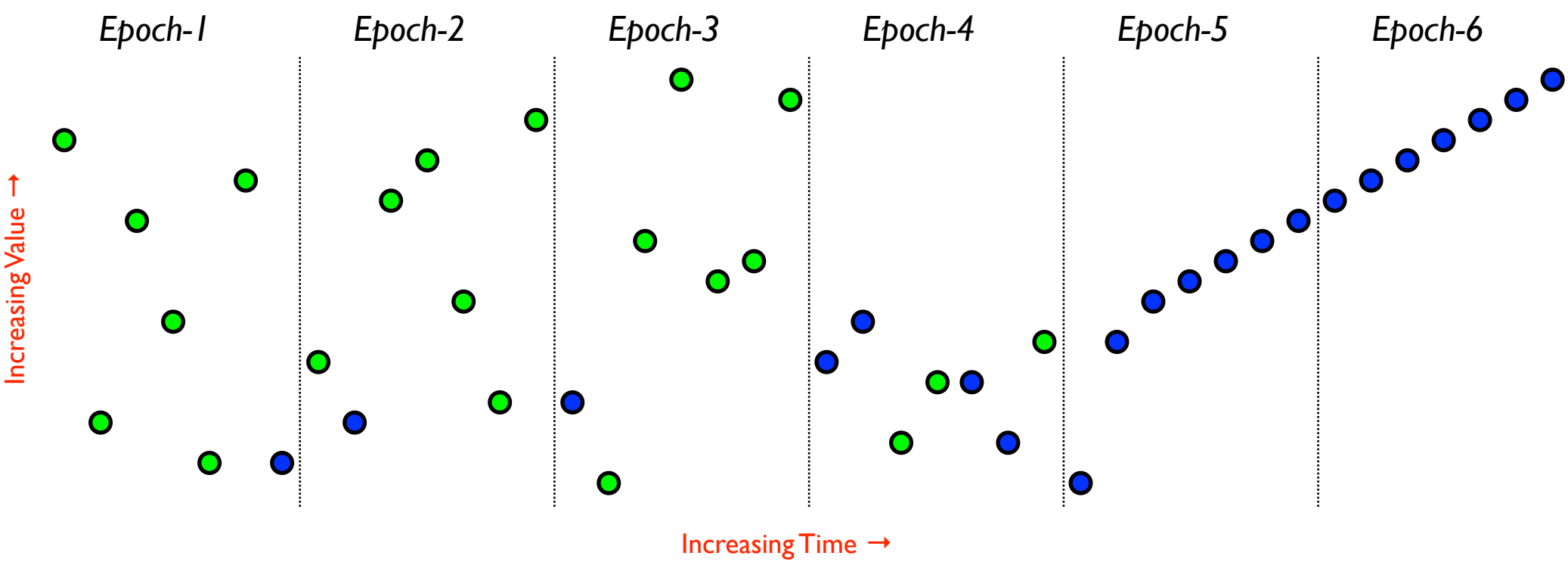


Epochs and Local Bad Patterns...



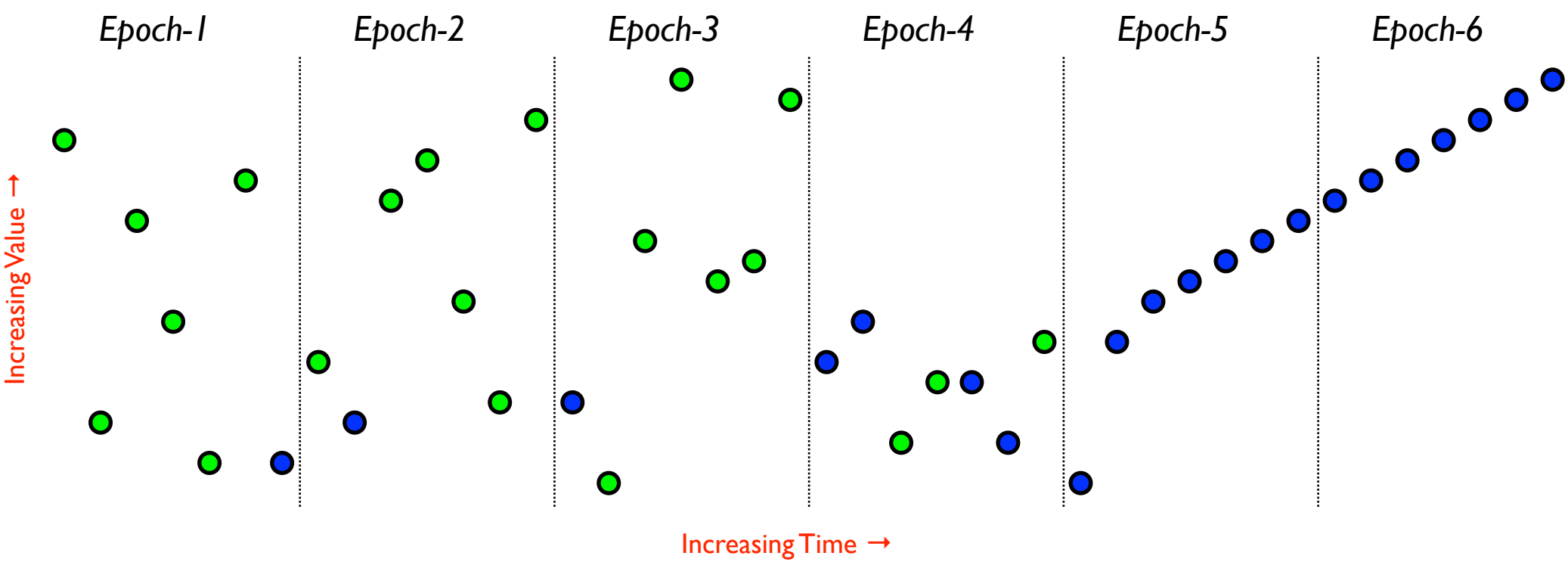
- Split length N sequence into $\sqrt{N}$ epochs of length $\sqrt{N}$

Epochs and Local Bad Patterns...



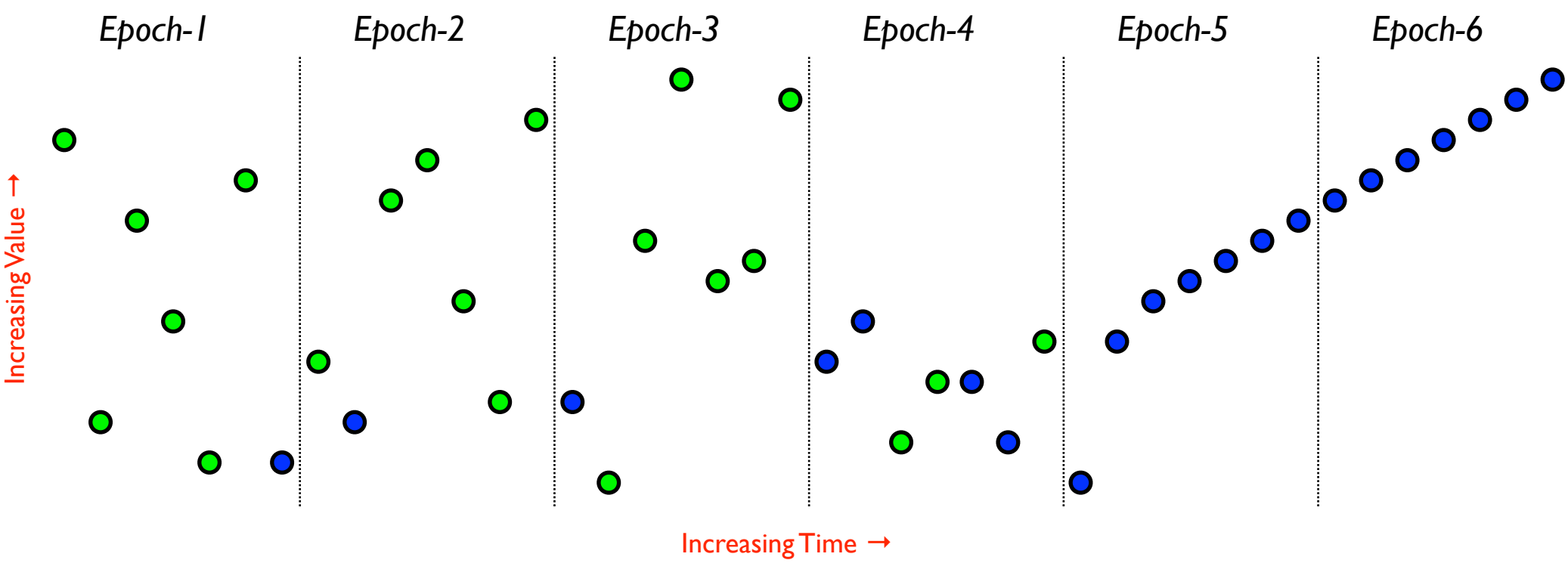
- Split length N sequence into $\sqrt{N}$ epochs of length $\sqrt{N}$

Epochs and Local Bad Patterns...



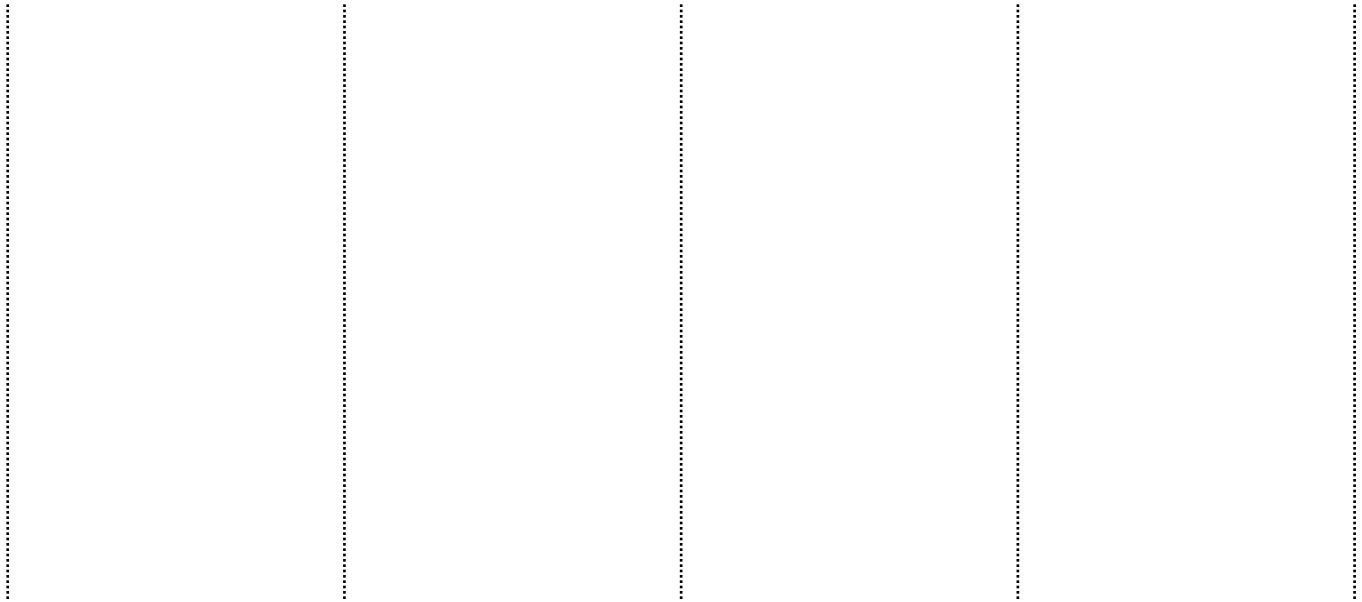
- Split length N sequence into $\sqrt{N}$ epochs of length $\sqrt{N}$
- *Defn:* Bad pattern ins(u) ... ext(v) ... ext(u) is *local* if ins(u) and ext(v) occur in same epoch and *long-range* otherwise.

Epochs and Local Bad Patterns...

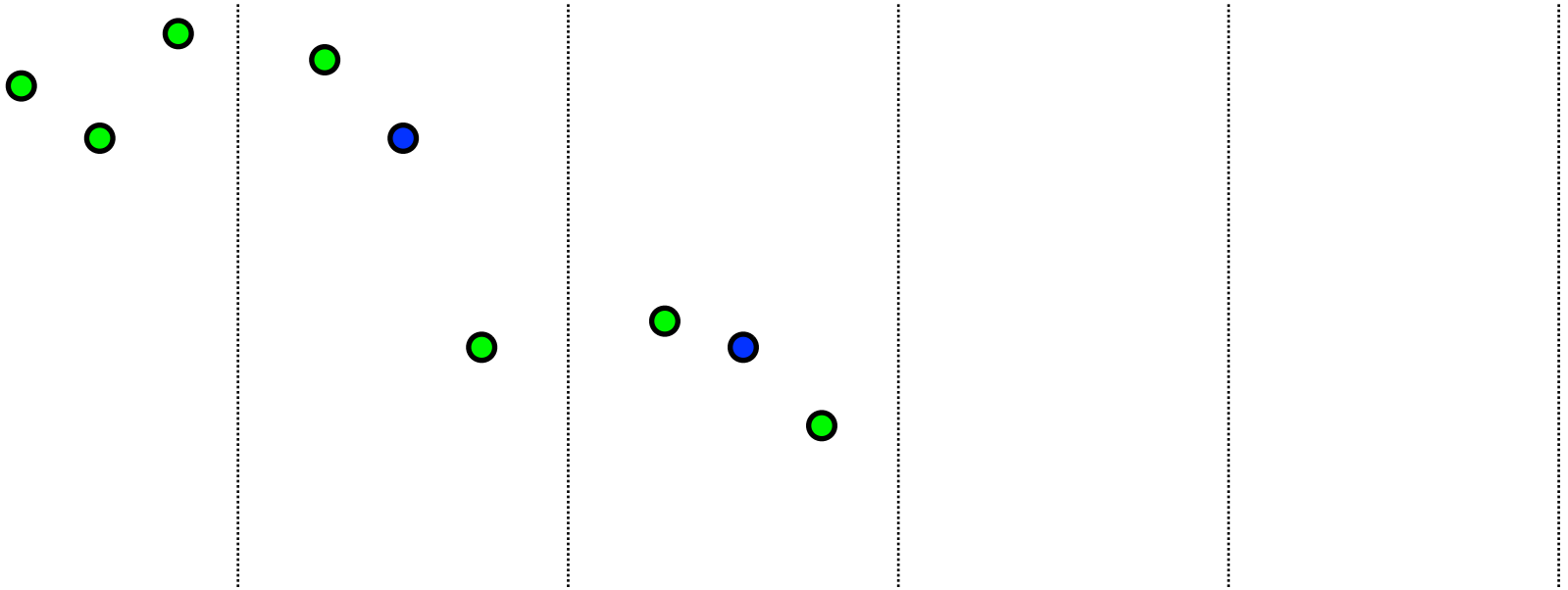


- Split length N sequence into $\sqrt{N}$ epochs of length $\sqrt{N}$
- *Defn:* Bad pattern ins(u) ... ext(v) ... ext(u) is *local* if ins(u) and ext(v) occur in same epoch and *long-range* otherwise.
- Using $O(\sqrt{N})$ space, we can buffer each epoch and check for local bad patterns.

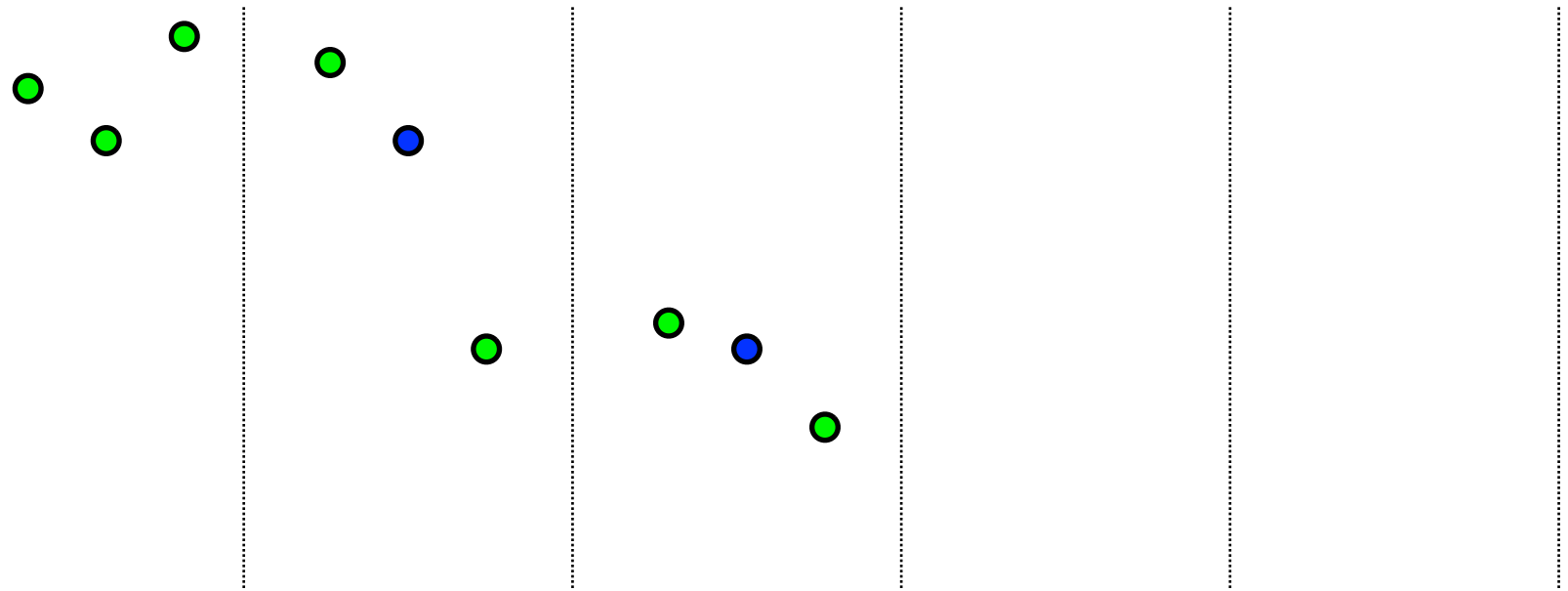
Catching Long-Range Bad Patterns... 1/2



Catching Long-Range Bad Patterns... 1/2

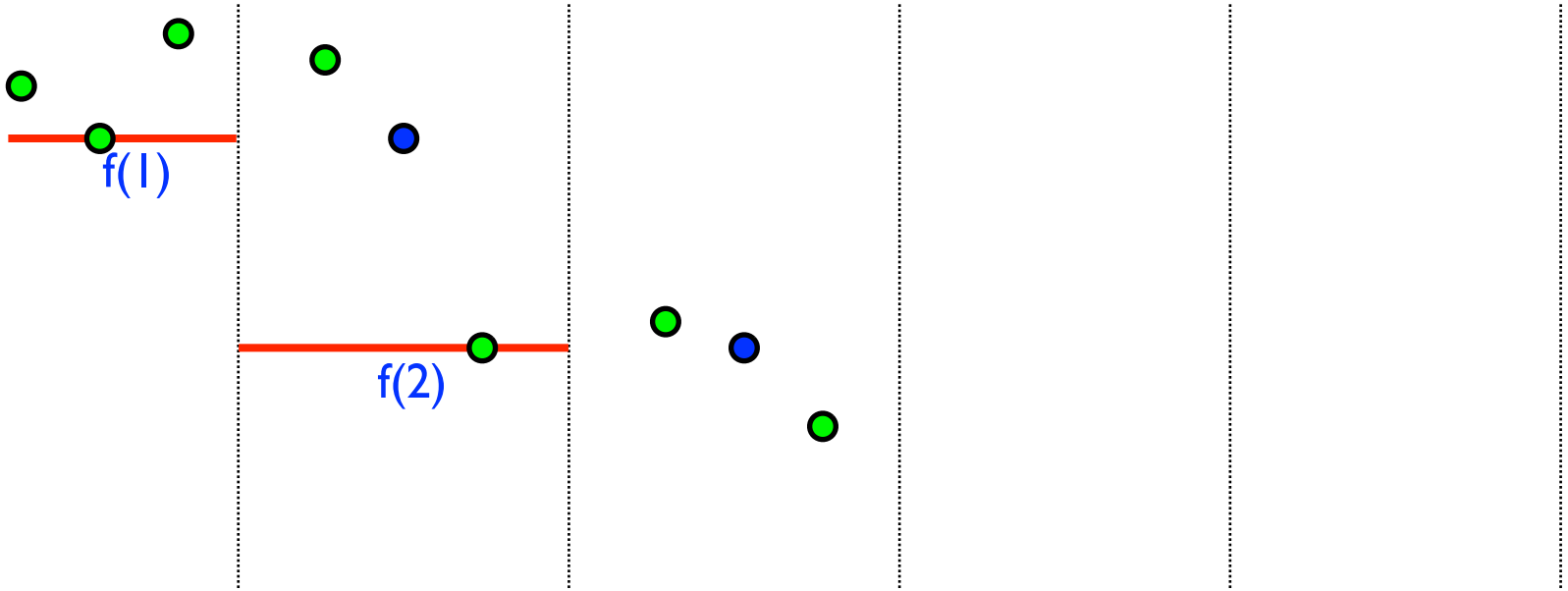


Catching Long-Range Bad Patterns... 1/2



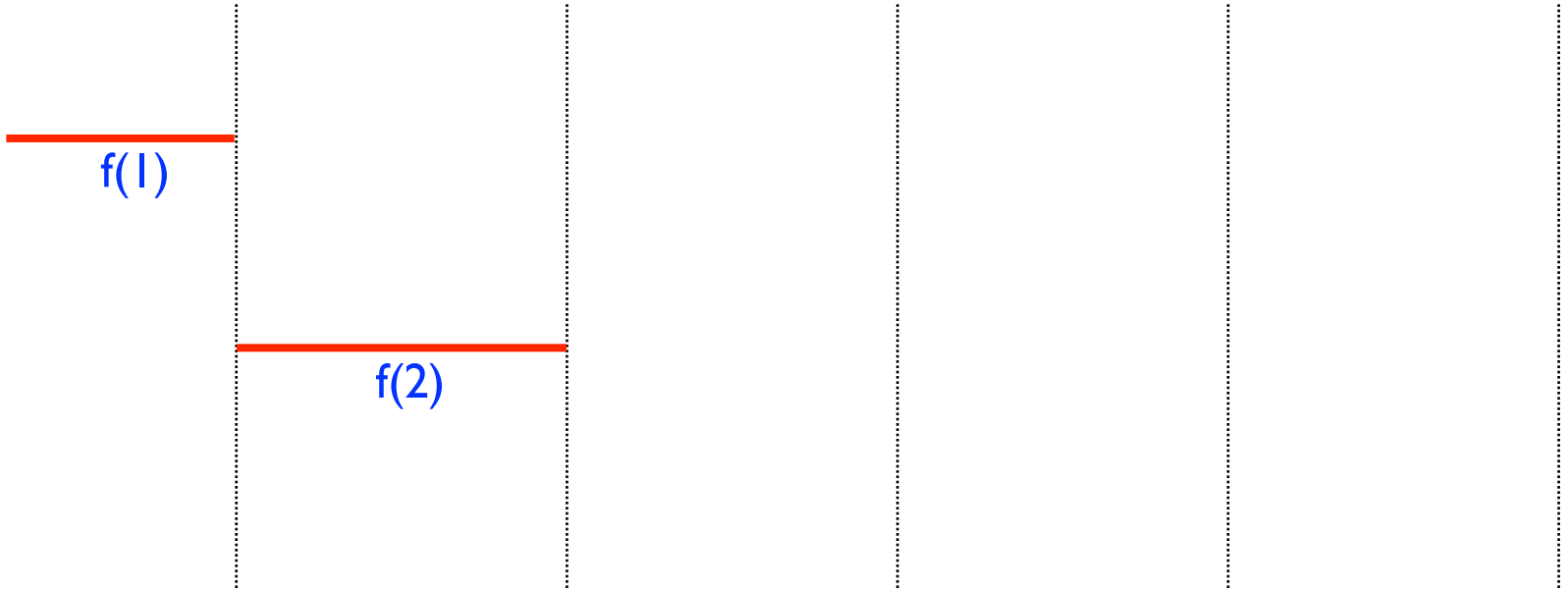
- Maintain the max value extracted between end of i-th epoch and *current time*. Call it f(i).

Catching Long-Range Bad Patterns... 1/2



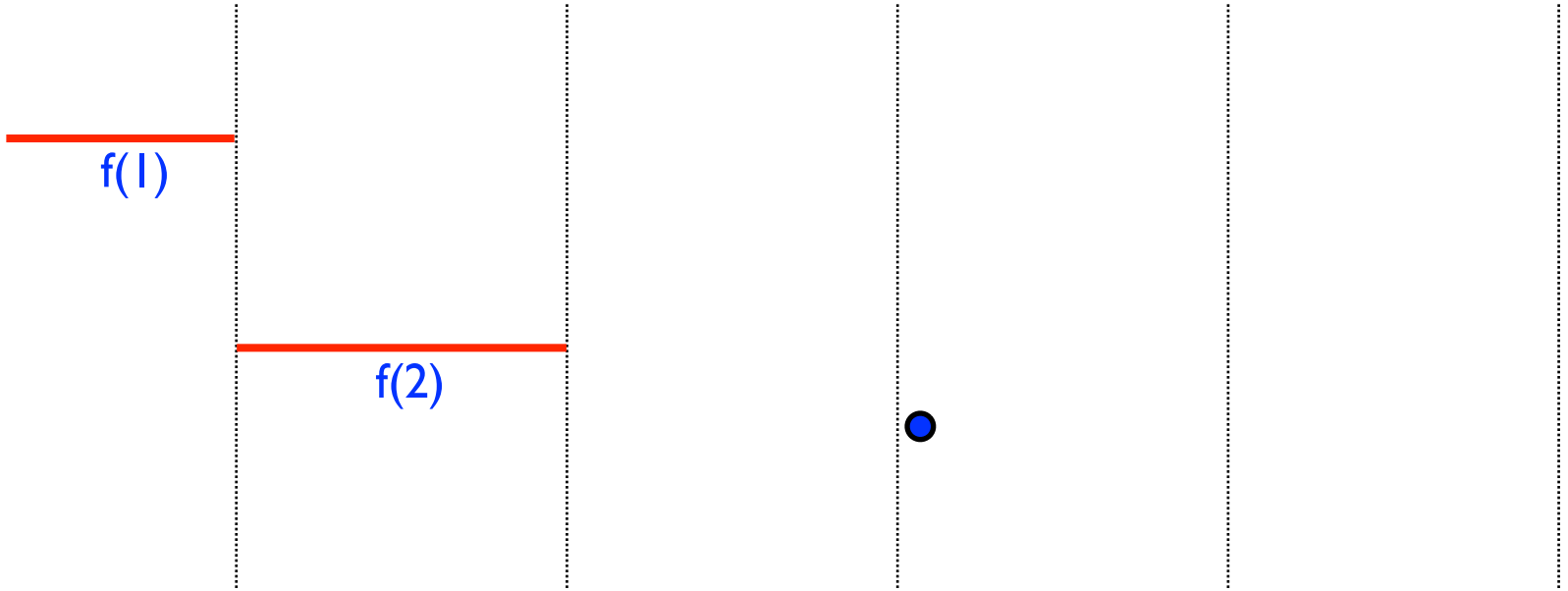
- Maintain the max value extracted between end of i-th epoch and *current time*. Call it f(i).

Catching Long-Range Bad Patterns... 1/2



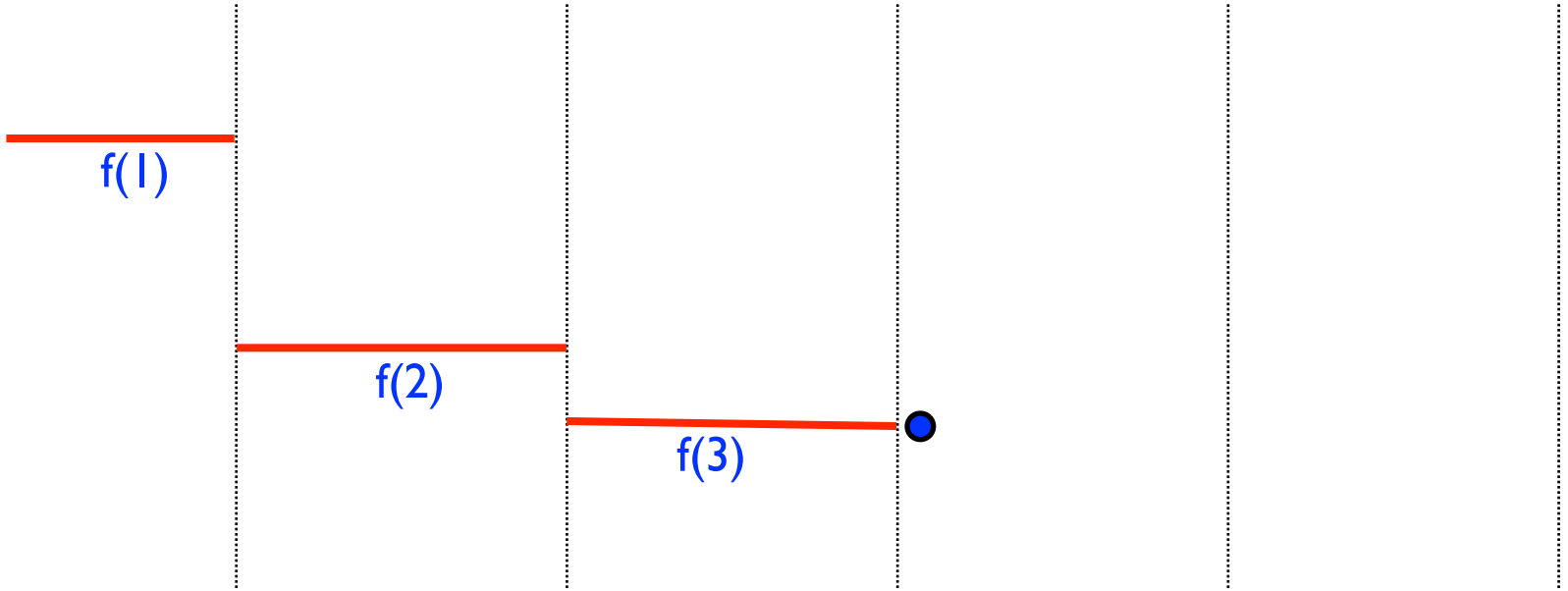
- Maintain the max value extracted between end of i-th epoch and *current time*. Call it f(i).

Catching Long-Range Bad Patterns... 1/2



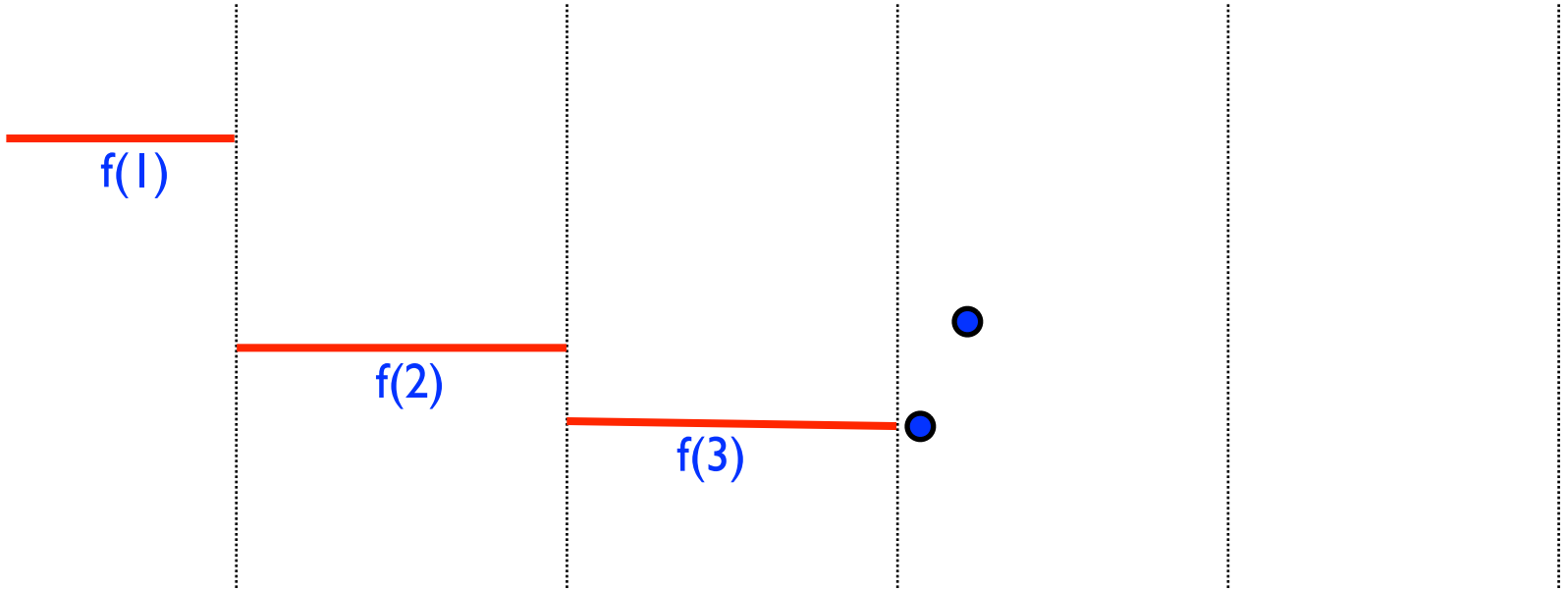
- Maintain the max value extracted between end of i-th epoch and *current time*. Call it f(i).

Catching Long-Range Bad Patterns... 1/2



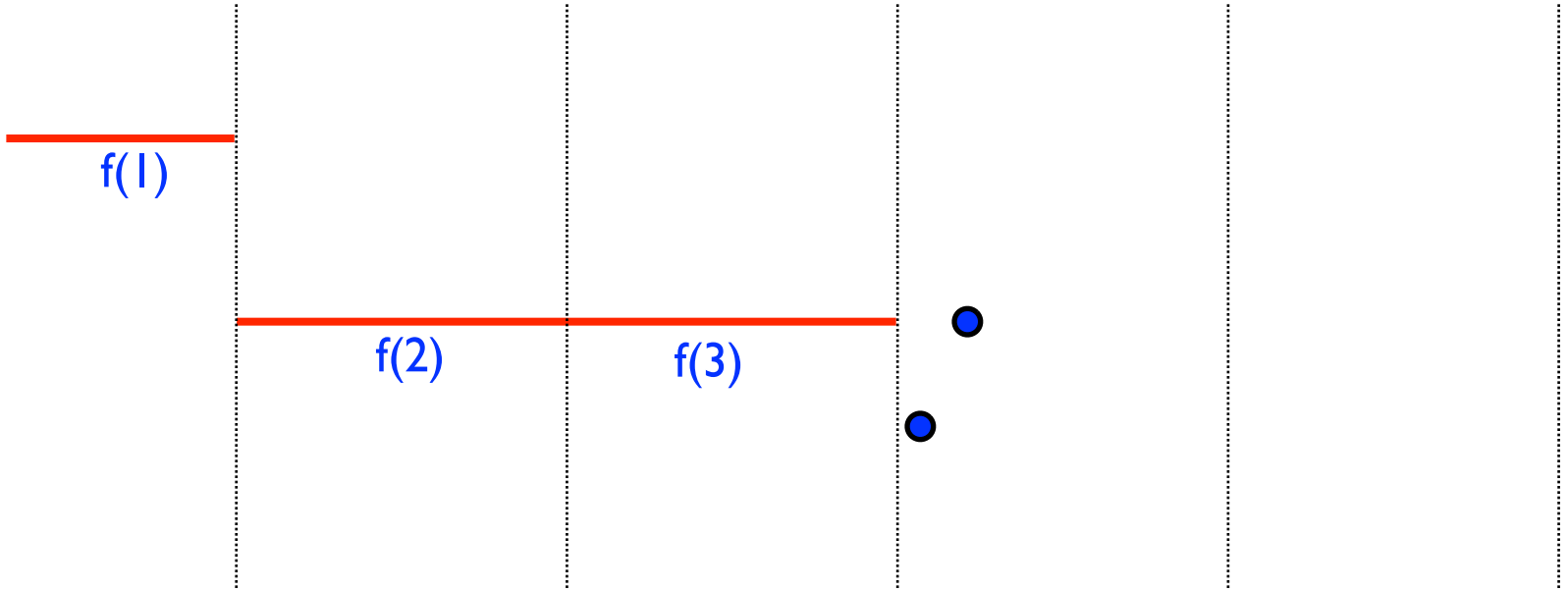
- Maintain the max value extracted between end of i-th epoch and *current time*. Call it f(i).

Catching Long-Range Bad Patterns... 1/2



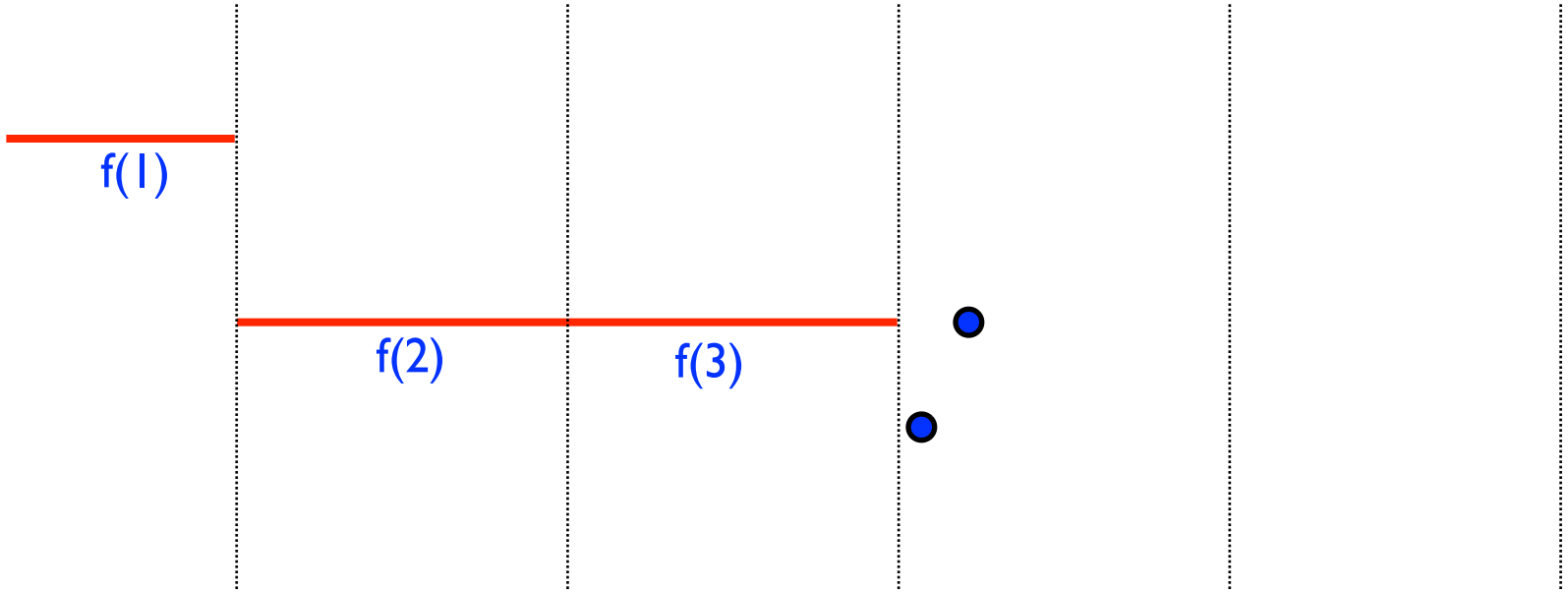
- Maintain the max value extracted between end of i-th epoch and *current time*. Call it f(i).

Catching Long-Range Bad Patterns... 1/2



- Maintain the max value extracted between end of i-th epoch and *current time*. Call it f(i).

Catching Long-Range Bad Patterns... 1/2



- Maintain the max value extracted between end of i-th epoch and *current time*. Call it f(i).
- *Defn:* Each ins(u) or ext(u) is *adopted* by earliest epoch k with $f(k) \leq u$.

Catching Long-Range Bad Patterns... 2/2

Catching Long-Range Bad Patterns... 2/2

- *Lemma:* If ins(u) ... ext(v) ... ext(u) is a long-range bad pattern then ins(u) and ext(u) are adopted by different epochs.

Catching Long-Range Bad Patterns... 2/2

- *Lemma:* If ins(u) ... ext(v) ... ext(u) is a long-range bad pattern then ins(u) and ext(u) are adopted by different epochs.
- *Proof:*
  i. Let ins(u) be adopted by k-th epoch.

Catching Long-Range Bad Patterns... 2/2

- *Lemma:* If ins(u) ... ext(v) ... ext(u) is a long-range bad pattern then ins(u) and ext(u) are adopted by different epochs.
- *Proof:*
  i. Let ins(u) be adopted by k-th epoch.
  ii. After v is extracted $f(k) \geq v > u$.

Catching Long-Range Bad Patterns... 2/2

- *Lemma:* If ins(u) ... ext(v) ... ext(u) is a long-range bad pattern then ins(u) and ext(u) are adopted by different epochs.
- *Proof:*
  i. Let ins(u) be adopted by k-th epoch.
  ii. After v is extracted $f(k) \geq v > u$.
  iii. ext(u) can no longer be adopted by k-th epoch.

Catching Long-Range Bad Patterns... 2/2

- *Lemma:* If ins(u) ... ext(v) ... ext(u) is a long-range bad pattern then ins(u) and ext(u) are adopted by different epochs.
- *Proof:*
  i. Let ins(u) be adopted by k-th epoch.
  ii. After v is extracted $f(k) \geq v > u$.
  iii. ext(u) can no longer be adopted by k-th epoch.
- *Lemma:* If there are no bad patterns, every ins(u) and ext(u) pair get adopted by the same epoch.

Catching Long-Range Bad Patterns... 2/2

- *Lemma:* If ins(u) ... ext(v) ... ext(u) is a long-range bad pattern then ins(u) and ext(u) are adopted by different epochs.
- *Proof:*
  i. Let ins(u) be adopted by k-th epoch.
  ii. After v is extracted $f(k) \geq v > u$.
  iii. ext(u) can no longer be adopted by k-th epoch.
- *Lemma:* If there are no bad patterns, every ins(u) and ext(u) pair get adopted by the same epoch.
- *Algorithm:* Using fingerprints to check: for each epoch k

  {u : ins(u) adopted by k} = {u : ext(u) adopted by k}.

Conclusions

Conclusions

- *Thm:* There exists a $O(\sqrt{N} \log N)$ space algorithm with $O(\log N)$ amortized update time for recognizing PQ.

  "Can verify terabytes of transcript with only megabytes"

Conclusions

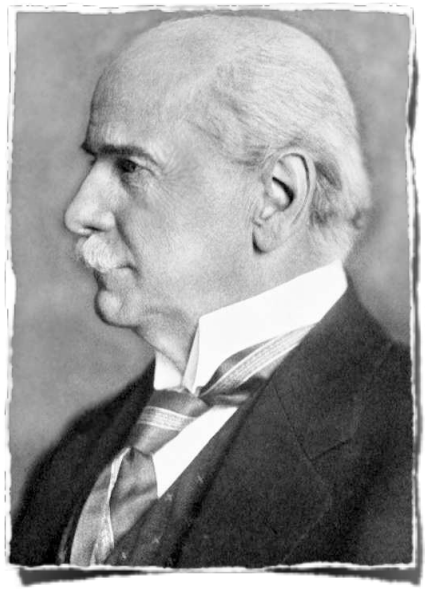
- *Thm:* There exists a $O(\sqrt{N} \log N)$ space algorithm with $O(\log N)$ amortized update time for recognizing PQ.

  "Can verify terabytes of transcript with only megabytes"

- *Extensions:* Sub-linear space streaming recognition of other data structures like stacks, double-ended queues...

I. Memory Checking

II. Lower Bounds

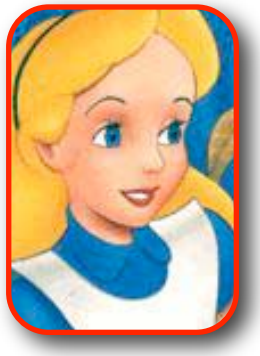


III. Parenthesis and Passes

II. Lower Bounds

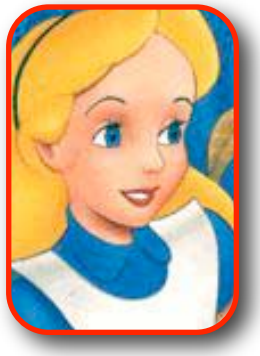
Communication Complexity

Communication Complexity



- Many space lower bounds in data stream model are based on reductions from communication complexity.

Communication Complexity

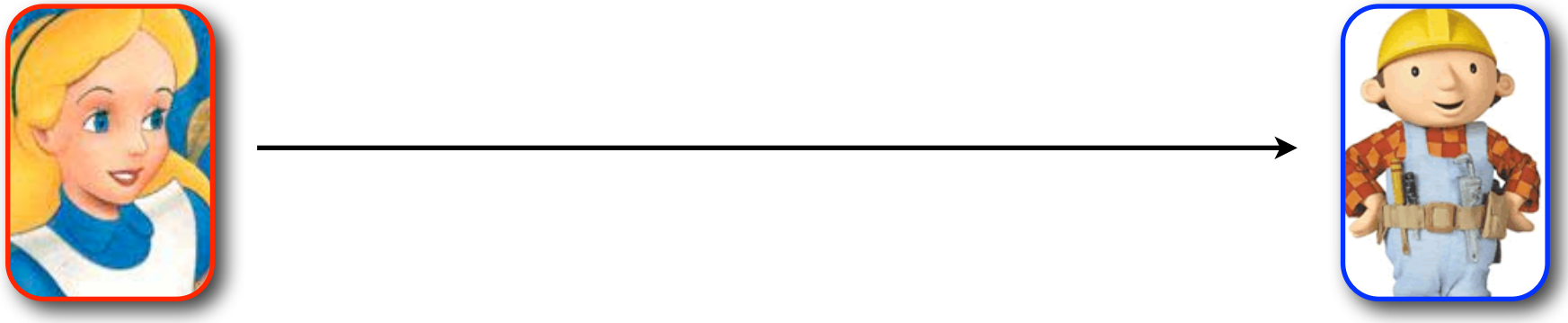


x

y, c

- Many space lower bounds in data stream model are based on reductions from communication complexity.
- *Augmented Index:* Alice has $x \in \{0,1\}^n$ and Bob has a prefix $y \in \{0,1\}^{k-1}$ of x and $c \in \{0,1\}$. Bob wants to check if $c = x_k$.

Communication Complexity



x

y, c

- Many space lower bounds in data stream model are based on reductions from communication complexity.
- *Augmented Index:* Alice has $x \in \{0,1\}^n$ and Bob has a prefix $y \in \{0,1\}^{k-1}$ of x and $c \in \{0,1\}$. Bob wants to check if $c = x_k$.
- *Thm:* Any 1/3-error, one-way protocol from Alice to Bob for AI_n requires $\Omega(n)$ bits sent. [Miltersen et al. JCSS '98]

Multi-player Augmented Index

Multi-player Augmented Index

x^1 y^1, c^1 x^2 y^2, c^2 x^3 y^3, c^3

- We now have 2m players $A_1, \ldots, A_m, B_1, \ldots, B_m$ where each A_i and B_i have an instance (x^i, y^i, c^i) of AI_n

Multi-player Augmented Index

x^1 y^1, c^1 x^2 y^2, c^2 x^3 y^3, c^3 x^3 x^2 x^1

- We now have 2m players $A_1, \ldots, A_m, B_1, \ldots, B_m$ where each A_i and B_i have an instance (x^i, y^i, c^i) of AI_n
- Want to determine if any of the AI_n instances are false using private messages communicated in the order

$$A_1 \rightarrow B_1 \rightarrow A_2 \rightarrow B_2 \rightarrow \ldots \rightarrow A_m \rightarrow B_m \rightarrow A_m \rightarrow A_{m-1} \rightarrow \ldots \rightarrow A_1$$

Multi-player Augmented Index

x^1 y^1, c^1 x^2 y^2, c^2 x^3 y^3, c^3 x^3 x^2 x^1

- We now have 2m players $A_1, \ldots, A_m, B_1, \ldots, B_m$ where each A_i and B_i have an instance (x^i, y^i, c^i) of AI_n
- Want to determine if any of the AI_n instances are false using private messages communicated in the order

$$A_1 \rightarrow B_1 \rightarrow A_2 \rightarrow B_2 \rightarrow \ldots \rightarrow A_m \rightarrow B_m \rightarrow A_m \rightarrow A_{m-1} \rightarrow \ldots \rightarrow A_1$$

- *Thm:* Any 1/3-error protocol has a $\Omega(\min m,n)$ bit message.

Multi-player Augmented Index

x^1 y^1, c^1 x^2 y^2, c^2 x^3 y^3, c^3 x^3 x^2 x^1

- We now have 2m players $A_1, \ldots, A_m, B_1, \ldots, B_m$ where each A_i and B_i have an instance (x^i, y^i, c^i) of AI_n
- Want to determine if any of the AI_n instances are false using private messages communicated in the order

$$A_1 \rightarrow B_1 \rightarrow A_2 \rightarrow B_2 \rightarrow \ldots \rightarrow A_m \rightarrow B_m \rightarrow A_m \rightarrow A_{m-1} \rightarrow \ldots \rightarrow A_1$$

- *Thm:* Any 1/3-error protocol has a $\Omega(\min m,n)$ bit message.
- *Corollary:* Any algorithm for PQ requires $\Omega(\sqrt{N})$ space.

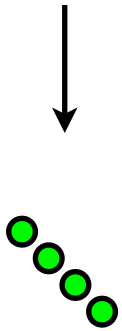
Reduction from multi-player AI to PQ...

0010 0, 0 0100 01, 1 1010 , 1 1010 0100 0010

"Ascension Problem" [Magniez, Mathieu, Nayak '10]

Reduction from multi-player AI to PQ...

0010 0, 0 0100 01, 1 1010 , 1 1010 0100 0010

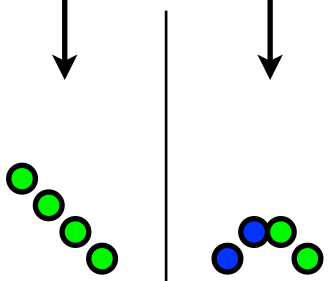


ins(12.0)
ins(11.1)
ins(10.0)
ins(9.0)

"Ascension Problem" [Magniez, Mathieu, Nayak '10]

Reduction from multi-player AI to PQ...

0010 0, 0 0100 01, 1 1010 , 1 1010 0100 0010



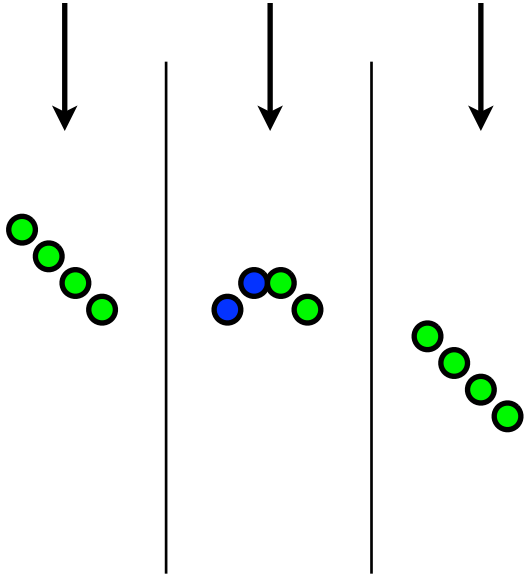
ins(12.0)
ins(11.1)
ins(10.0)
ins(9.0)

ext(9.0)
ext(10.0)
ins(10.0)
ins(9.0)

"Ascension Problem" [Magniez, Mathieu, Nayak '10]

Reduction from multi-player AI to PQ...

0010 0, 0 0100 01, 1 1010 , 1 1010 0100 0010



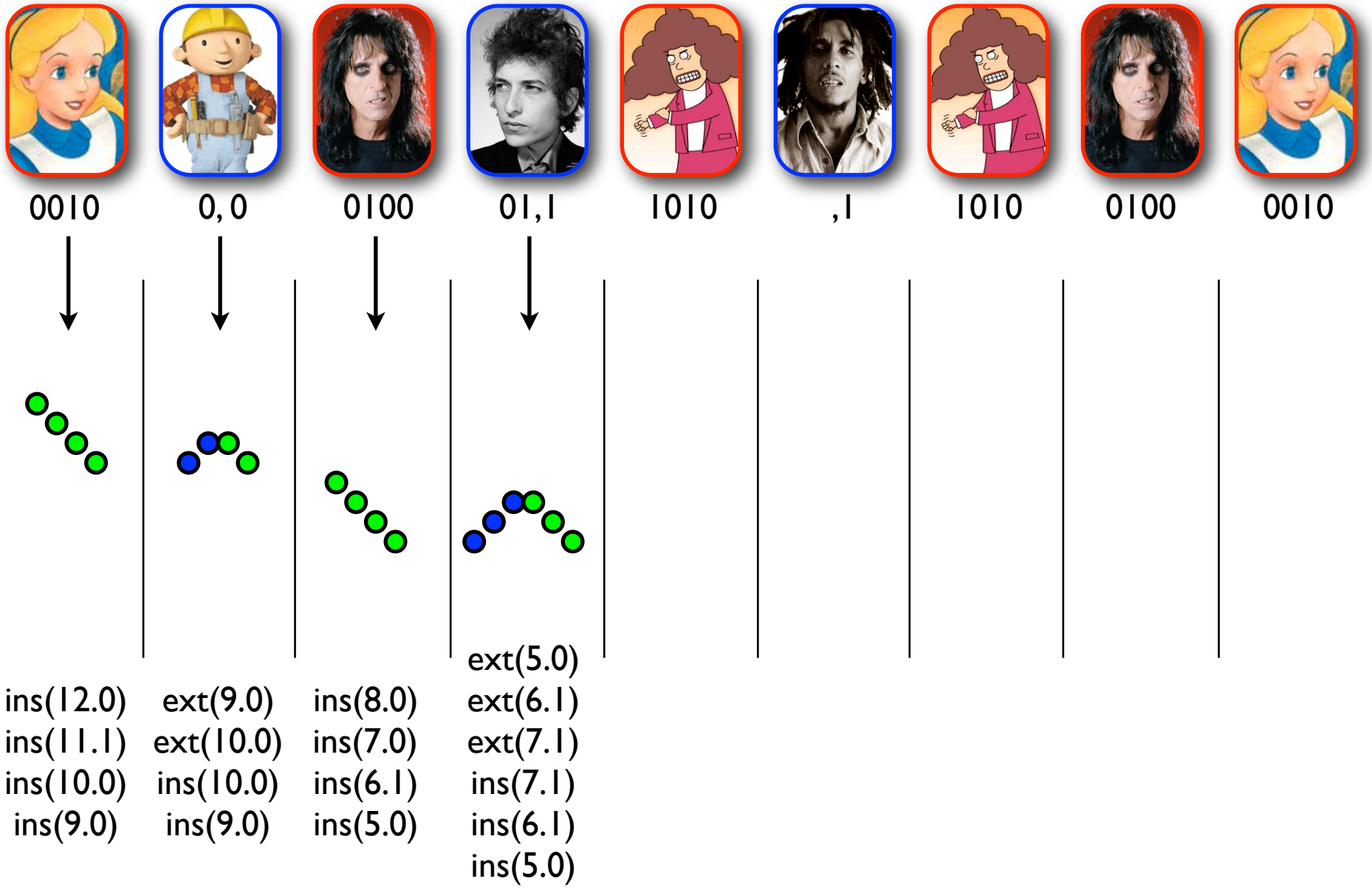
ins(12.0) ins(11.1) ins(10.0) ins(9.0)

ext(9.0) ext(10.0) ins(10.0) ins(9.0)

ins(8.0) ins(7.0) ins(6.1) ins(5.0)

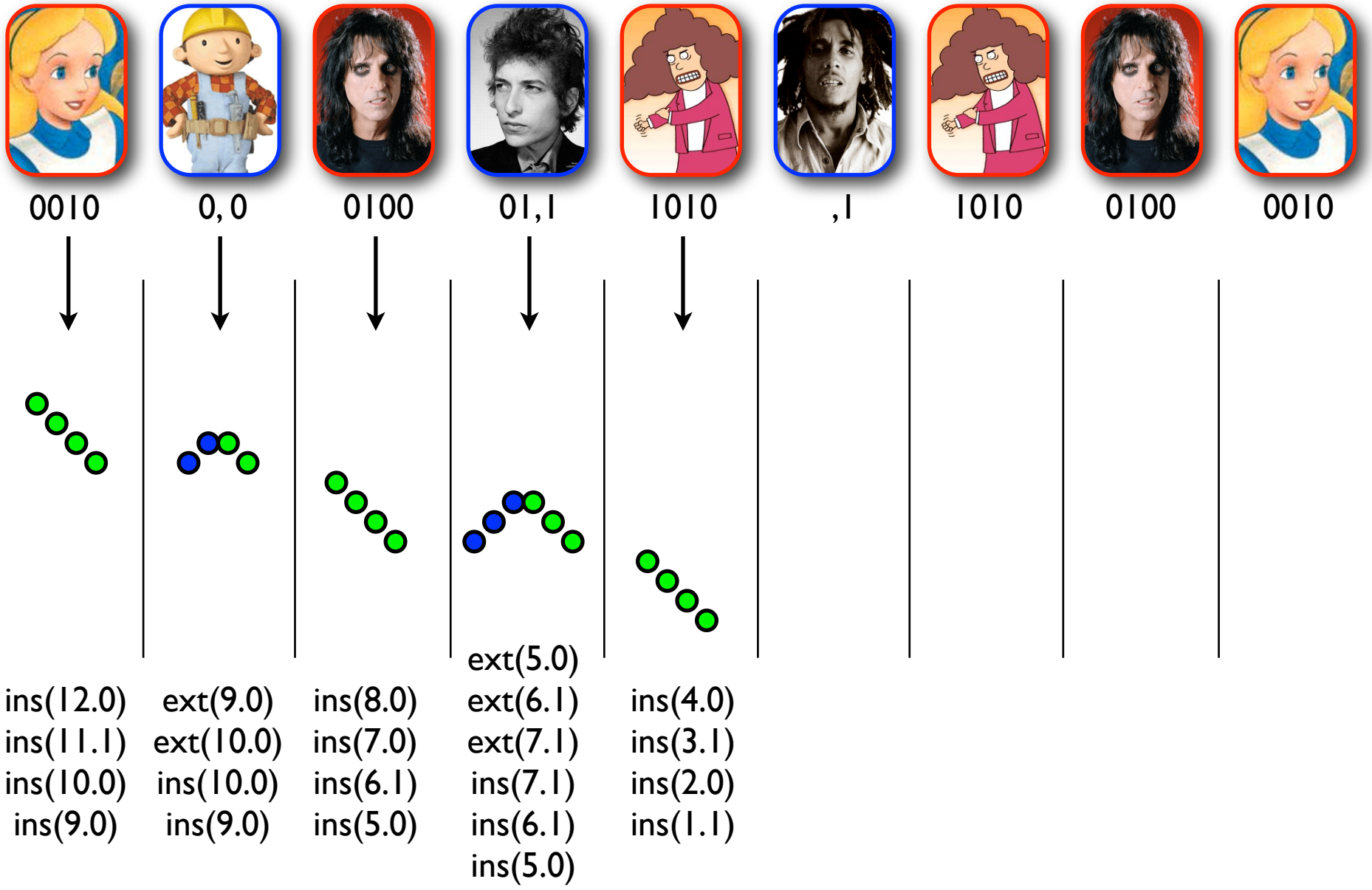
"Ascension Problem" [Magniez, Mathieu, Nayak '10]

Reduction from multi-player AI to PQ...



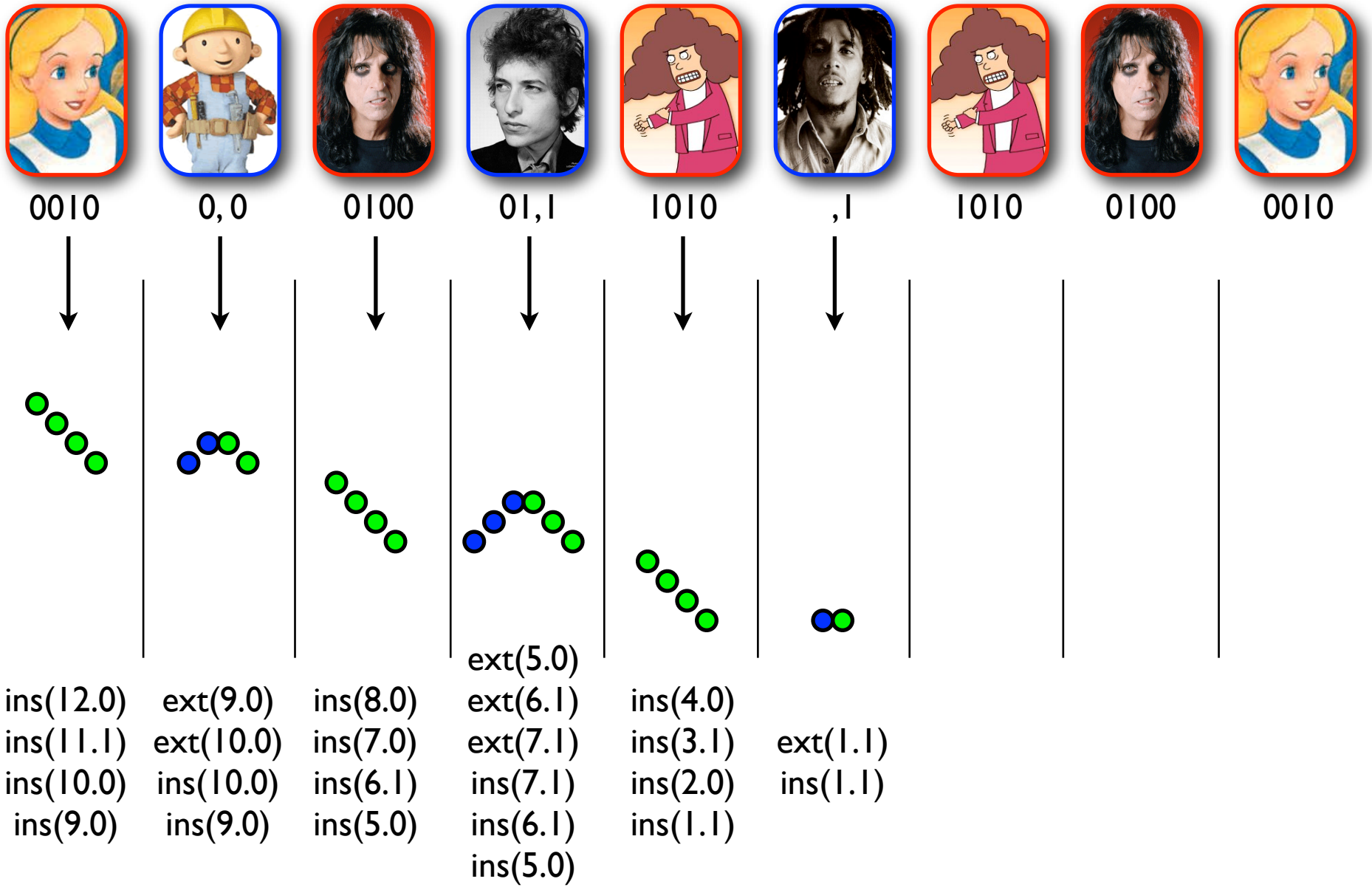
"Ascension Problem" [Magniez, Mathieu, Nayak '10]

Reduction from multi-player AI to PQ...



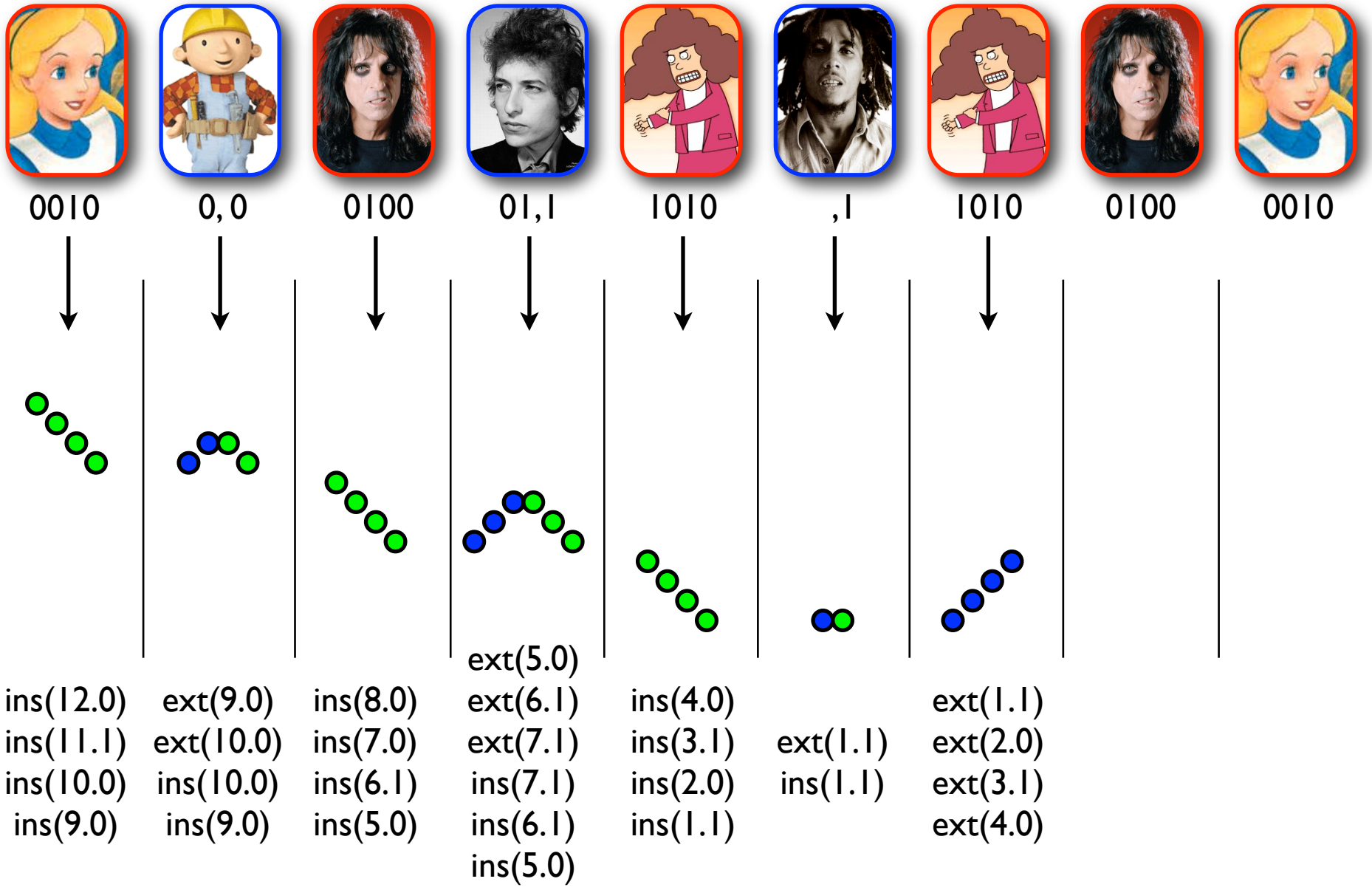
"Ascension Problem" [Magniez, Mathieu, Nayak '10]

Reduction from multi-player AI to PQ...



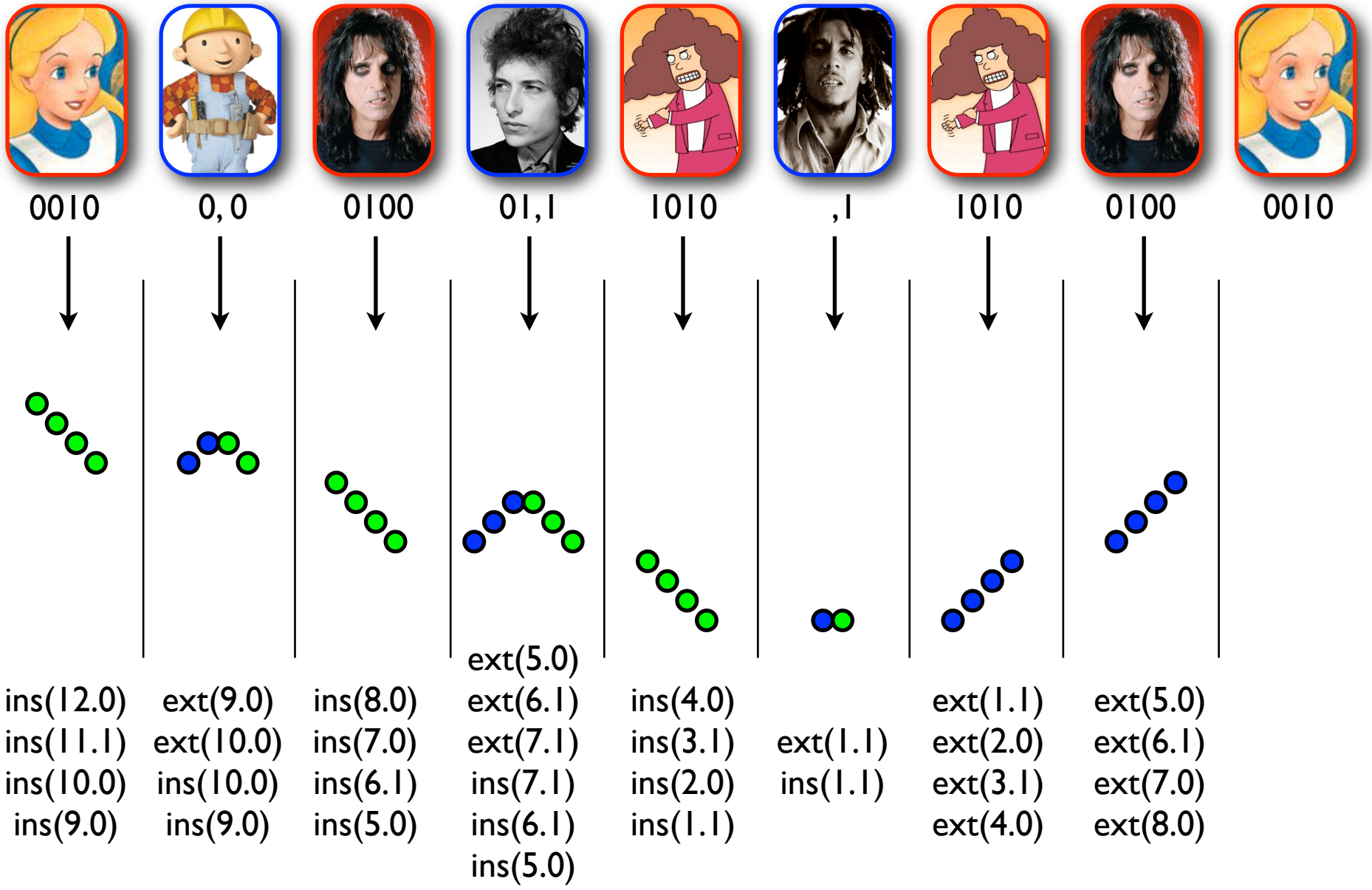
"Ascension Problem" [Magniez, Mathieu, Nayak '10]

Reduction from multi-player AI to PQ...



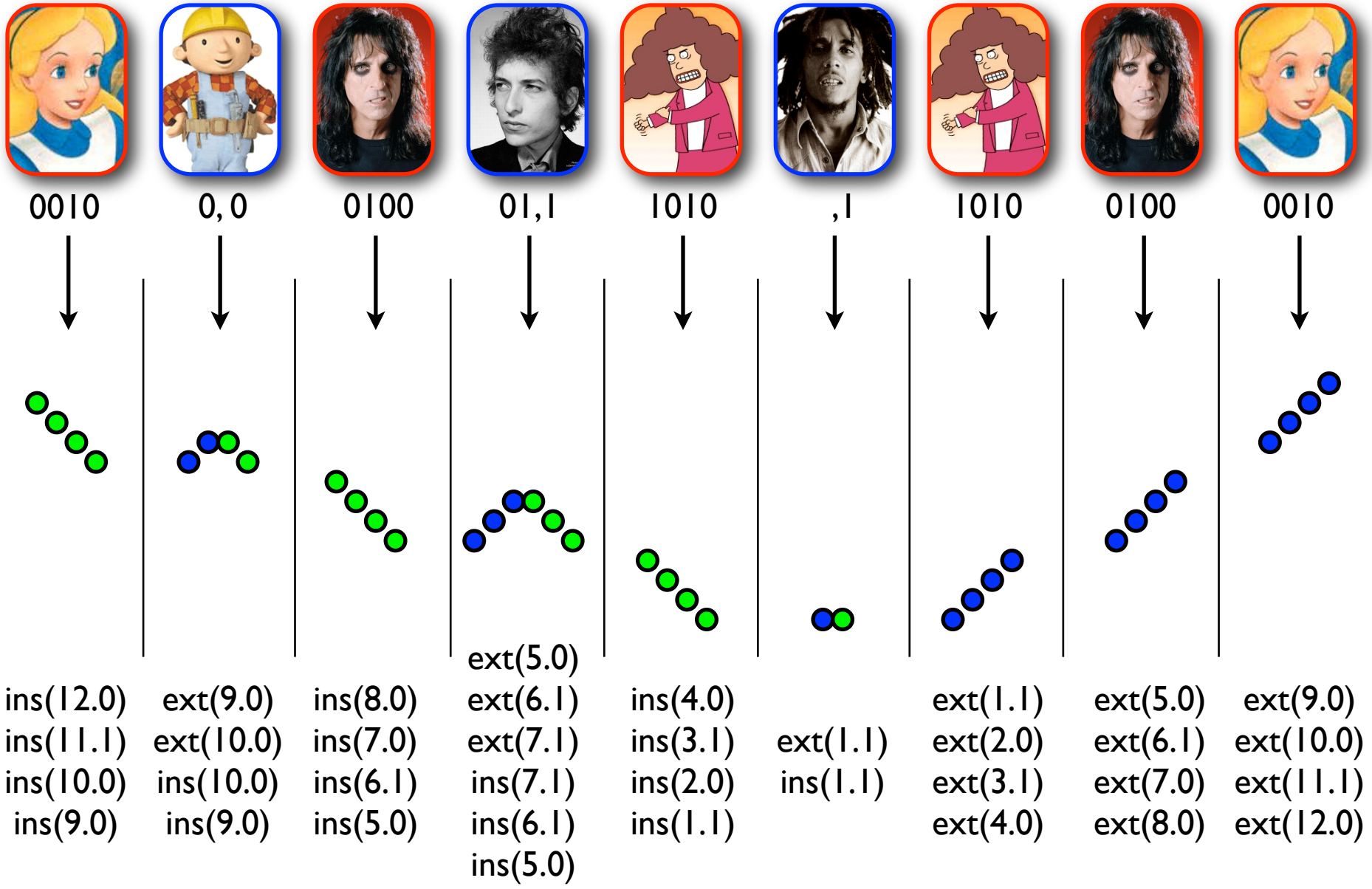
"Ascension Problem" [Magniez, Mathieu, Nayak '10]

Reduction from multi-player AI to PQ...



"Ascension Problem" [Magniez, Mathieu, Nayak '10]

Reduction from multi-player AI to PQ...



"Ascension Problem" [Magniez, Mathieu, Nayak '10]

Reduction from multi-player AI to PQ...

Reduction from multi-player AI to PQ...

- *Thm:* Any algorithm for recognizing PQ with probability at least 2/3 requires $\Omega(\sqrt{N})$ space.

Reduction from multi-player AI to PQ...

- *Thm:* Any algorithm for recognizing PQ with probability at least 2/3 requires $\Omega(\sqrt{N})$ space.
- *Proof:*

  i. Let $\mathfrak{A}$ be a stream algorithm using s bits of space.

Reduction from multi-player AI to PQ...

- *Thm:* Any algorithm for recognizing PQ with probability at least 2/3 requires $\Omega(\sqrt{N})$ space.
- *Proof:*

  i. Let $\mathfrak{A}$ be a stream algorithm using s bits of space.

  ii. Use $\mathfrak{A}$ to construct a protocol with s bit messages: Players run $\mathfrak{A}$ on their input and send memory state to next player.

Reduction from multi-player AI to PQ...

- *Thm:* Any algorithm for recognizing PQ with probability at least 2/3 requires $\Omega(\sqrt{N})$ space.
- *Proof:*

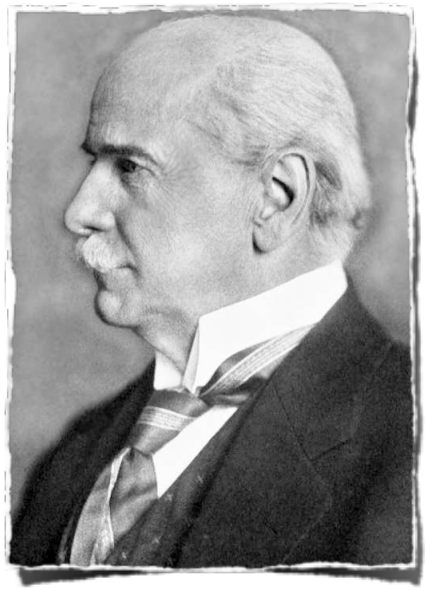
  i. Let $\mathfrak{A}$ be a stream algorithm using s bits of space.

  ii. Use $\mathfrak{A}$ to construct a protocol with s bit messages: Players run $\mathfrak{A}$ on their input and send memory state to next player.

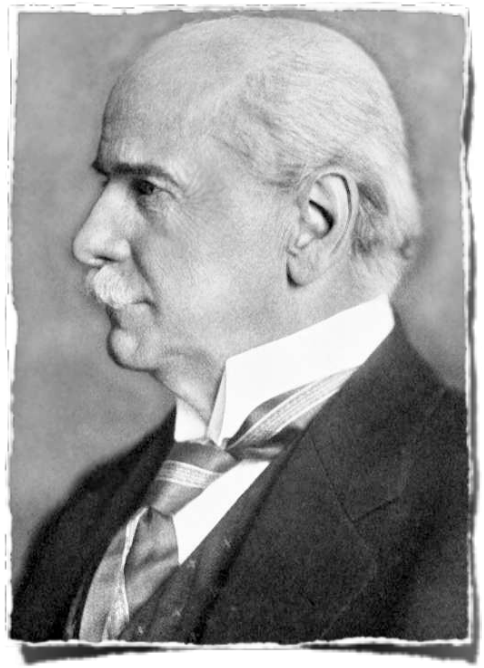
  iii. Therefore, $s = \Omega(\min m,n)$ for length mn sequence.

I. Memory Checking

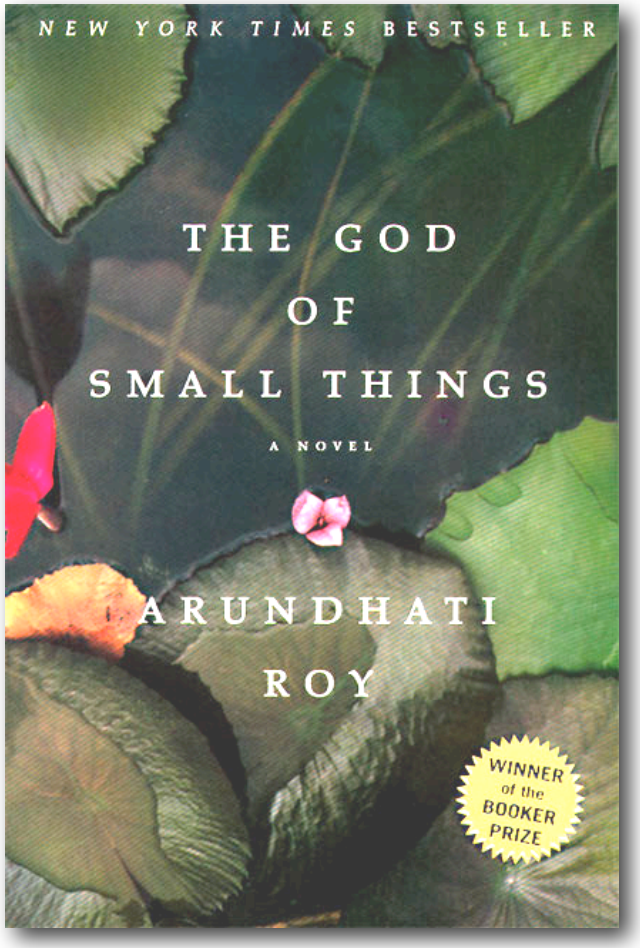
II. Lower Bounds

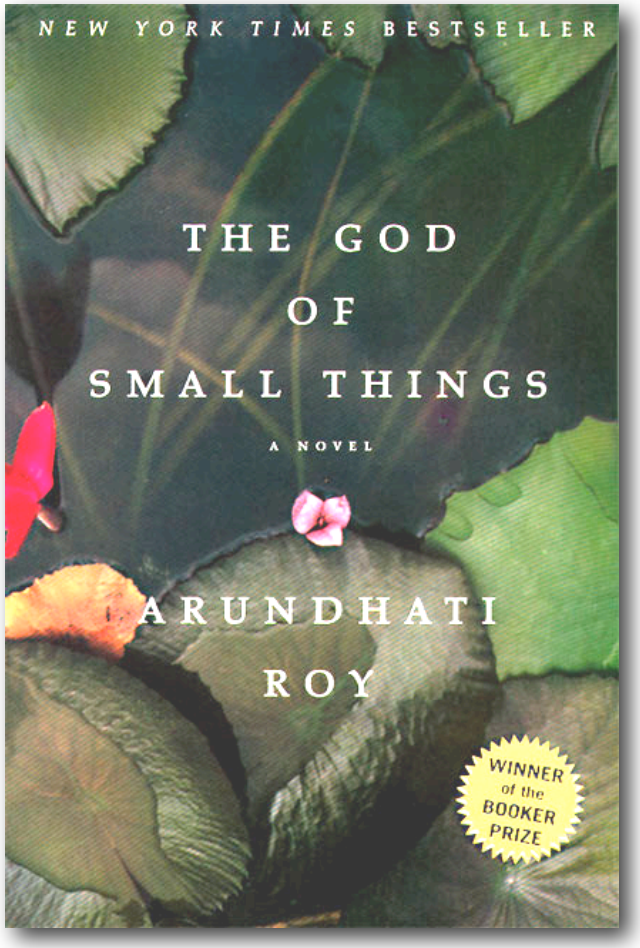


III. Parenthesis and Passes



III. Parenthesis and Passes


NEW YORK TIMES BESTSELLER
THE GOD OF SMALL THINGS
A NOVEL
ARUNDHATI ROY
WINNER of the BOOKER PRIZE



Fictional Quote:

"After Ammu died (after the last time she came back to Ayemenem (she had been swollen with cortisone and a rattle in her chest that sounded like a faraway man shouting), Rahel drifted."

DYCK Language Problem

DYCK Language Problem

- $DYCK_2$ is the set of strings of properly nested brackets when there are two different types of brackets:

$(([])()[]) \in DYCK_2$ $([([]])[]) \notin DYCK_2$

DYCK Language Problem

- $DYCK_2$ is the set of strings of properly nested brackets when there are two different types of brackets:

$(([])()[]) \in DYCK_2$ $([([]])[]) \notin DYCK_2$

- DYCK *Problem:* Given *streaming* access to length N string, determine if it's in $DYCK_2$ using o(N) space.

DYCK Language Problem

- DYCK_2 is the set of strings of properly nested brackets when there are two different types of brackets:

  $(([])()[]) \in \text{DYCK}_2$ $([([]])[]) \notin \text{DYCK}_2$

- DYCK *Problem:* Given *streaming* access to length N string, determine if it's in DYCK_2 using o(N) space.

- *Previous result:* $O(\sqrt{N})$ space suffices. [Magniez, Mathieu, Nayak '10]

DYCK Language Problem

- DYCK_2 is the set of strings of properly nested brackets when there are two different types of brackets:

  $(([])()[]) \in \text{DYCK}_2$ $([([]])[]) \notin \text{DYCK}_2$

- DYCK *Problem:* Given *streaming* access to length N string, determine if it's in DYCK_2 using o(N) space.

- *Previous result:* $O(\sqrt{N})$ space suffices. [Magniez, Mathieu, Nayak '10]

- *But...* If you're allowed a forward pass followed by a backwards pass, space can be reduced to O(log N)!

DYCK Language Problem

- DYCK_2 is the set of strings of properly nested brackets when there are two different types of brackets:

$$(([])()[]) \in \text{DYCK}_2 \qquad ([([]])[]) \notin \text{DYCK}_2$$

- DYCK *Problem:* Given *streaming* access to length N string, determine if it's in DYCK_2 using o(N) space.
- *Previous result:* $O(\sqrt{N})$ space suffices. [Magniez, Mathieu, Nayak '10]
- *But...* If you're allowed a forward pass followed by a backwards pass, space can be reduced to O(log N)!

"How useful is reading backwards? Do we also get a space saving if you can take multiple forward passes?"

Space/Pass Trade-Offs

Space/Pass Trade-Offs

- If you increase the number of passes p, for some problems the space required can be dramatically reduced...

Space/Pass Trade-Offs

- If you increase the number of passes p, for some problems the space required can be dramatically reduced...
- *Example 1:* Necessary and sufficient space to find the median of n values $\Theta(n^{1/p})$.

Space/Pass Trade-Offs

- If you increase the number of passes p, for some problems the space required can be dramatically reduced...
- *Example 1:* Necessary and sufficient space to find the median of n values $\Theta(n^{1/p})$.
- *Example 2:* Necessary and sufficient space to find an increasing subsequence of length k is $\Theta(k^{1+\frac{1}{2^p-1}})$.

$\mathrm{DYCK}_2 \Rightarrow \mathrm{PQ}$

$\mathrm{DYCK}_2 \Rightarrow \mathrm{PQ}$

- There's a reduction from DYCK_2 to PQ and our bounds extend to multi-pass algorithms.

$DYCK_2 \Rightarrow PQ$

- There's a reduction from $DYCK_2$ to PQ and our bounds extend to multi-pass algorithms.
- *Thm:* Any p pass algorithm for $DYCK_2$ that only uses forward passes requires $\Omega(\sqrt{N/p})$.

$DYCK_2 \Rightarrow PQ$

- There's a reduction from $DYCK_2$ to PQ and our bounds extend to multi-pass algorithms.
- *Thm:* Any p pass algorithm for $DYCK_2$ that only uses forward passes requires $\Omega(\sqrt{N/p})$.

"Reading backwards can be very helpful!"

$DYCK_2 \Rightarrow PQ$

- There's a reduction from $DYCK_2$ to PQ and our bounds extend to multi-pass algorithms.
- *Thm:* Any p pass algorithm for $DYCK_2$ that only uses forward passes requires $\Omega(\sqrt{N/p})$.

"Reading backwards can be very helpful!"

? *Open Problem:* Stream complexity of recognizing other languages and examples of backwards phenomena?

Summary

Memory Checking: Sub-linear space recognition of various data-structure transcript languages!

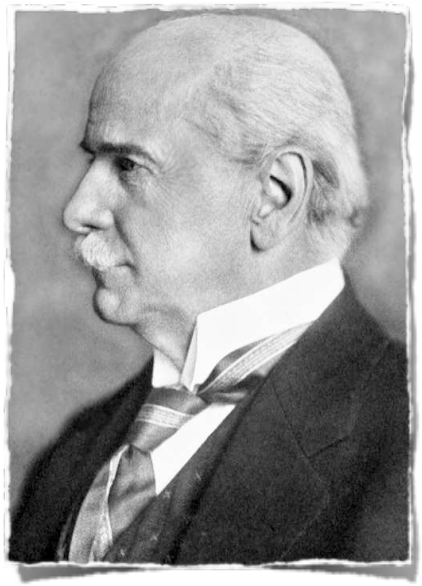
Theory of Stream Computation: Forward and backward pass better than many forward passes!

Further Work: Annotations, stream language recognition, ...

Thanks!

I. Memory Checking

II. Lower Bounds



III. Parenthesis and Passes

IV. Augmented Index Bound

Information Complexity

[Chakrabarti, Shi, Wirth, Yao '01]

Information Complexity

[Chakrabarti, Shi, Wirth, Yao '01]

- *Entropy and Mutual Information:*

$$H(X) = -\Sigma \Pr[X = x] \lg \Pr[X = x]$$

$$H(X|Y) = -\Sigma \Pr[X = x, Y = y] \lg \Pr[X = x|Y = y]$$

Information Complexity

[Chakrabarti, Shi, Wirth, Yao '01]

- *Entropy and Mutual Information:*

$$
\begin{aligned}
H(X) &= -\Sigma \Pr[X = x] \lg \Pr[X = x] \\
H(X|Y) &= -\Sigma \Pr[X = x, Y = y] \lg \Pr[X = x | Y = y] \\
I(X;Y) &= H(X) - H(X|Y) = H(Y) - H(Y|X) \\
I(X;Y|Z) &= H(X|Z) - H(X|Y,Z)
\end{aligned}
$$

Information Complexity

[Chakrabarti, Shi, Wirth, Yao '01]

- *Entropy and Mutual Information:*

$$
\begin{aligned}
H(X) &= -\Sigma \Pr[X = x] \lg \Pr[X = x] \\
H(X|Y) &= -\Sigma \Pr[X = x, Y = y] \lg \Pr[X = x | Y = y] \\
I(X;Y) &= H(X) - H(X|Y) = H(Y) - H(Y|X) \\
I(X;Y|Z) &= H(X|Z) - H(X|Y,Z)
\end{aligned}
$$

- *Information cost method:* Consider mutual information between random input for a communication problem and the communication transcript:

$$I(\text{transcript}; \text{input})$$

Information Complexity

[Chakrabarti, Shi, Wirth, Yao '01]

- *Entropy and Mutual Information:*

$$\begin{aligned} H(X) &= -\Sigma \Pr[X = x] \lg \Pr[X = x] \\ H(X|Y) &= -\Sigma \Pr[X = x, Y = y] \lg \Pr[X = x | Y = y] \\ I(X;Y) &= H(X) - H(X|Y) = H(Y) - H(Y|X) \\ I(X;Y|Z) &= H(X|Z) - H(X|Y,Z) \end{aligned}$$

- *Information cost method:* Consider mutual information between random input for a communication problem and the communication transcript:

$$I(\text{transcript}; \text{input}) \leq \text{length of transcript}$$

Information Complexity

[Chakrabarti, Shi, Wirth, Yao '01]

- *Entropy and Mutual Information:*

$$\begin{aligned} H(X) &= -\Sigma \Pr[X = x] \lg \Pr[X = x] \\ H(X|Y) &= -\Sigma \Pr[X = x, Y = y] \lg \Pr[X = x | Y = y] \\ I(X;Y) &= H(X) - H(X|Y) = H(Y) - H(Y|X) \\ I(X;Y|Z) &= H(X|Z) - H(X|Y,Z) \end{aligned}$$

- *Information cost method:* Consider mutual information between random input for a communication problem and the communication transcript:

$$I(\text{transcript}; \text{input}) \leq \text{length of transcript}$$

- Can restrict to partial transcript and subsets of input: useful for proving direct-sum arguments.

Information Complexity of AI_n

Information Complexity of AI_n

- *Defn:* Let P be a protocol for AI_n using public random string R. Let T be the transcript and (X, K, C)~ξ. Define

$$\begin{aligned} \mathrm{icost}^{\mathrm{A}}_{\xi}(\mathrm{P}) &= I(T : X \mid K, C, R) \\ \mathrm{icost}^{\mathrm{B}}_{\xi}(\mathrm{P}) &= I(T : K, C \mid X, R) \end{aligned}$$

Information Complexity of AI_n

- *Defn:* Let P be a protocol for AI_n using public random string R. Let T be the transcript and (X, K, C)~ξ. Define

$$\begin{aligned} \text{icost}^A_\xi(P) &= I(T : X \mid K, C, R) \\ \text{icost}^B_\xi(P) &= I(T : K, C \mid X, R) \end{aligned}$$

- *Thm:* Let P be a randomized protocol for AI_n with error 1/3 under the uniform distribution μ. Then,

$$\text{icost}^A_{\mu_0}(P) = \Omega(n) \quad \text{or} \quad \text{icost}^B_{\mu_0}(P) = \Omega(1)$$

where μ_0 is μ conditioned on X_K=C.

MULTI-$Al_{m,n}$ versus Al_n

MULTI-AI$_{m,n}$ versus AI$_n$

- *Defn:* Let Q be a protocol for MULTI-AI$_{m,n}$ using public random string R. Let T be transcript and $(X^i,K^i,C^i)_{i\in[m]}\sim\xi$.

$$\text{icost}_\xi(Q) = I(T_m : K^1, C^1, \ldots, K^m, C^m \mid X^1, \ldots, X^m, R)$$

where T_m is the set of messages sent by B_m.

MULTI-$\mathrm{AI}_{m,n}$ versus AI_n

- *Defn:* Let Q be a protocol for MULTI-$\mathrm{AI}_{m,n}$ using public random string R. Let T be transcript and $(X^i,K^i,C^i)_{i\in[m]}\sim\xi$.

$$\mathrm{icost}_\xi(Q) = I(T_m : K^1, C^1, \ldots, K^m, C^m \mid X^1, \ldots, X^m, R)$$

  where T_m is the set of messages sent by B_m.

- *Thm (Direct Sum):* If there exists a p-round, s-bit, ε-error protocol Q for MULTI-$\mathrm{AI}_{m,n}$ then there exists a p-round, ε-error randomized protocol P for AI_n where

  i. Alice sends at most ps bits

  ii. $m \cdot \mathrm{icost}^B_{\mu_0}(P) \leq \mathrm{icost}_{\mu_0^{\otimes m}}(Q)$

Putting it all together...

Putting it all together...

- *Thm:* Any p-round, s-bit, 1/3-error protocol Q for MULTI-$AI_{m,n}$ requires $ps=\Omega(\min m,n)$.

Putting it all together...

- *Thm:* Any p-round, s-bit, 1/3-error protocol Q for MULTI-$\text{AI}_{m,n}$ requires $ps=\Omega(\min m,n)$.
- *Proof:*

  i. By direct sum theorem, there exists ε-error, p-pass protocol P for AI_n such that:

$$p \cdot s \geq \text{icost}_{\mu_0^{\otimes m}}(Q) \geq m \cdot \text{icost}^B_{\mu_0}(P)$$

$$p \cdot s \geq \text{icost}^A_{\mu_0}(P)$$

Putting it all together...

- *Thm:* Any p-round, s-bit, 1/3-error protocol Q for MULTI-$\mathrm{AI}_{m,n}$ requires $ps=\Omega(\min m,n)$.
- *Proof:*

  i. By direct sum theorem, there exists ε-error, p-pass protocol P for AI_n such that:

$$p \cdot s \geq \mathrm{icost}_{\mu_0^{\otimes m}}(Q) \geq m \cdot \mathrm{icost}^B_{\mu_0}(P)$$

$$p \cdot s \geq \mathrm{icost}^A_{\mu_0}(P)$$

  ii. By information complexity of AI_n

$$\max(m \cdot \mathrm{icost}^B_{\mu_0}(P), \mathrm{icost}^A_{\mu_0}(P)) = \Omega(\min(m, n))$$

Summary

Memory Checking: Sub-linear space recognition of various data-structure transcript languages!

Theory of Stream Computation: Forward and backward pass better than many forward passes!

Further Work: Annotations, stream language recognition, ...

Thanks!