Digital Signal Processing and the Brain

- Is the brain a digital signal processor?
- Digital vs continuous signals
- Digital signals involve streams of binary encoded numbers
- The brain uses digital, all or none, action potentials or spikes for information transmission in its neuronal axons over distances of 1 mm to several metres to avoid the uncertain decay of an analog signal in a long non-uniform cable.
- But the spikes are then converted into analog signals within a neuron, which has a threshold for the summed depolarization produced by many small synaptic currents to elicit an action potential, which is binary.
- The brain does not work by logical operations as in digital computers, but instead by the similarity of an input vector with a synaptic weight vector.
- The spikes in the brain have Poisson timing, i.e. are random in time for a given mean firing rate. This leads to stochastic processing in the brain, related to the timing of the digital inputs to a neuron.

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Single neurons in the cerebral cortex
1. Dot product similarity vs logical functions e.g. AND, NAND, OR
followed by a threshold
2. 10,000 inputs per neuron vs several inputs to a logic gate
3. Content-addressable memory vs data accessed by address
4. Fault tolerant: dot product vs no inherent fault tolerance
5. Generalization and completion vs only exact match in the hardware
6. Fast: inherently parallel update of a neuron, and of neurons in a network vs serial processing with fast components
7. Approximate, low precision vs high precision 64 bit
8. Stochastic spiking noise: probabilistic computation helps originality, creativity vs noise free because of non-linearities
9. Dynamics are parallel – an attractor network retrieves in 1 - 2 time constants of the synapses. vs inherently serial
10. Heuristics, e.g. Invariant object recognition uses temporo-spatial constraints, analyses only part of a scene. vs attempt to understand a whole scene
11. Syntax: none inherent vs syntactical operations on data at an address
12. The architecture adapts to implement different computations. vs fixed architecture with different software
13. Sparse distributed representation vs binary encoding in a computer word, e.g. 64 bits
14. Mind vs brain vs software vs hardware
Each neuron has approximately 10,000 inputs through synapses. The neuron sums the currents produced by each input firing rate $x_j$ injected through each synapse $w_{ij}$ to produce an activation $h_i$.

$$h_i = \sum_j x_j \cdot w_{ij}$$
The activations are converted into spikes when the neurons reach a threshold for firing. This is a non-linear operation and produces digital spikes, with a firing rate that depends on the strength of the activation. The output firing $y_i$ is a function of the activation

$$y_i = f(h_i)$$
Neurodynamical Modeling: Neurons, Synapses & Cortical Architecture

Spiking Neuron -> Integrate-and-Fire Model:

\[ \tau_m \frac{d}{dt} V_i(t) = -g_m (V_i(t) - V_L) - I_{syn}(t) \]

Synaptic Dynamics:

\[ I_{syn}(t) = I_{AMPA,ext}(t) + I_{AMPA,rec}(t) + I_{NMDA,rec}(t) + I_{GABA,rec}(t) \]

\[ I(t) = g(V_i(t) - V_E) f(V_i(t)) \sum_j w_{ij} s_j(t) \]

\[ \frac{d}{dt} s_j(t) = -\frac{s_j(t)}{\tau_{decay}} + \alpha x_j(t)(1 - s_j(t)) \]

\[ \frac{d}{dt} x_j(t) = -\frac{x_j(t)}{\tau_{rise}} + \sum_k \delta(t - t_j^k) \]
Pattern Association Memory

Learning Rule:
Where $x_i$ approximates $e_i$ (which is the unconditioned or external or forcing input)
\[ \delta w_{ij} = k \cdot y_i \cdot x_j \]

Recall: The activation,
\[ h_i = \sum_j x_j \cdot w_{ij} \]
where the sum is over the $C$ input axons, $x_j$
The output firing $y_i$ is a function of the activation
\[ y_i = f (h_i) \]
This activation function $f$ may be linear, sigmoid, binary threshold, etc.
Long Term Potentiation and Long Term Depression of synapses

(a) A weak → No change
   B → No change

(b) A strong → LTP
    B → Heterosynaptic LTD

(c) A weak up → Associative LTP
    B weak up → Associative LTP

(d) A medium down → Homosynaptic LTD
    B → no change

Some post synaptic activation
NMDA threshold exceeded
Auto-association Memory

Learning Rule:

Where $y_i$ approximates $e_i$ (which is the external input)

$$\delta w_{ij} = k \cdot y_i \cdot x_j \quad \text{(Hebb rule)}$$

and $y_i$ is the firing of the output neuron (post-synaptic term)

$x_j$ is the firing of input axon $j$ (presynaptic term)

$w_{ij}$ is the synapse to output neuron $i$ from input axon $j$.

Recall: The (internal) activation produced by the recurrent collateral effect (after the first iteration) is

$$h_i = \sum_j x_j \cdot w_{ij} \quad \text{where the sum is over the } C \text{ axons indexed by } j.$$

The output firing $y_i$ is a function of the activation produced by the recurrent collateral effect and by the external input ($e_i$):

$$y_i = f(h_i + e_i)$$

The activation function $f$ may be binary threshold, linear threshold, sigmoid, etc.
<table>
<thead>
<tr>
<th>stored pattern</th>
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<tbody>
<tr>
<td>cue (20% corr.)</td>
<td>..................................................................................................................</td>
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<td>iteration</td>
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<td>7</td>
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<tr>
<td>retrieval state</td>
<td>(90% corr.)</td>
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Single neuron selectivity: in this case to a face
How is information encoded by the inferior temporal visual cortex?

Rate Code
Local (Grandmother), Distributed and Sparse Distributed Coding

- **Local (Grandmother):** One cell codes for one stimulus and fires only in response to this one.

- **Fully Distributed:** Almost every cell participates in encoding all stimuli. (With binary firing rates half the cells respond to any stimulus)

- **Sparse Distributed:** The cell responds to a certain number of stimuli. Intermediate between Grandmother and Fully distributed.

Inferior Temporal Visual Cortex neuron firing rate response to a set of 68 visual stimuli
How does the information scale with the number of neurons?

Approximately linearly.

Optimal vs Dot Product decoding.
How does the number of stimuli encoded scale with the number of neurons? (Information uses a log scale.)
Dot Product Multiple Cell Decoding

\[ h_i = \sum_j w_j x_j \]
An attractor network for probabilistic decision-making, with lambda 1 and 2 inputs, and noise from the neuronal spiking influencing which decision attractor, D1 or D2, wins.

This is also a model for short-term memory.


Deco, Rolls et al 2013 Progress in Neurobiology 103
Integrate-and-fire simulations predict earlier and higher neuronal responses on easy vs difficult trials.

Confidence is reflected in the higher firing on easy trials.

The decision stimuli start at $t=2 \text{ s}$

Rolls, Grabenhorst and Deco 2010 Neuroimage
The theoretical framework based in statistical mechanics predicts higher and faster neuronal responses as Delta I, the difference between the two stimuli, increases.

Delta I is a measure of the easiness of the decision, and subjective confidence ratings correlate with this.
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Brain computation vs Digital computer computation

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Further reading:

On differences between signal processing in the brain and in digital computers:

On the signal processing performed by neuronal networks in the brain:

On the representation of information in the brain:


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