DCSP-5: Fourier Transform II

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Fourier Theorem

This representation is quite general. In fact we have the following theorem due to Fourier.

Any signal \( x(t) \) of period \( T \) can be represented as the sum of a set of cosinusoidal and sinusoidal waves of different frequencies.
This week’s summary

- Introduce FT in *layman’s* language (today)
- Applications (next Monday)
This week’s summary

• Introduce FT in *layman’s* language (today)

• Applications (next monday)

You might find the world is different!
This week’s summary

• Introduce FT in layman’s language (today)

• Talk about continuous FT since it is clean and simple

• Come back to it on how to numerically calculate it later on

  • Intuition
  • Fourier theorem
  • Examples
  • Bandwidth
Intuition of FT

Two dimensional space (all points)
Intuition of FT

Two dimensional space (all points)

Two bases which are orthogonal

(1,0)  (0,1)
Intuition of FT

Two dimensional space (all points)

\[(a, b) = a (1,0) + b (0,1)\]

- \(a\): the contribution from the first basis, the load at the first basis
- \(b\): the load at the second basis
Intuition of FT

Two dimensional space (all points)

(a, b)
Intuition of FT

Two dimensional space (all points)

(a, b)

Signal space (all functions of t)

x (t)
Intuition of FT

A point in it

\[(a, b) = a (1,0) + b (0,1)\]
Intuition of FT

\[(a, b) = a (1,0) + b (0,1)\]

Signal = coef basis

\(a \text{ point in it}\)

coef basis
Intuition of FT

Two points in it

\((a, b) = a (1, 0) + b (0, 1)\)

\(x(t) = ?\)

Signal = coef basis

ccoef basis

\(a point in it\)
Intuition of FT

(a, b) = a \begin{pmatrix} 1,0 \end{pmatrix} + b \begin{pmatrix} 0,1 \end{pmatrix}

x(t) = a F_1(t) + b F_2(t)

Signal = coef basis + coef basis
Intuition of FT

• It is a branch of mathematics called **functional analysis**: treat each function as a point in a functional space

• It is the starting point of modern mathematics

• It is also the actual power of mathematics

• Branch space, Hilbert space etc.
Intuition of FT

How to calculate these coefficients?

\[(a, b) = a \ (1,0) \ + \ b \ (0,1)\]

\[x (t) = a \ \cos(\omega t) \ + \ b \ \cos(2\ \omega t)\]

\[\omega = \frac{2 \pi}{T}\]
Intuition of FT

1. Coefficient is obtained via Inner product
   \[ \langle (x, y), (m,n) \rangle = xm + yn \]

2. All bases are orthogonal
   \[ \langle (0, 1), (1,0) \rangle = 0, \quad (0,1) \quad (1, 0) \]

\[(a, b) = a \ (1,0) \quad + \ b \ (0,1)\]

Therefore

\[ \langle (a, b), (1,0) \rangle = a \langle (1,0), (1,0) \rangle + b \langle (0,1), (1,0) \rangle = a \]

\[ \langle (a, b), (0,1) \rangle = a \langle (1,0), (0,1) \rangle + b \langle (0,1), (0,1) \rangle = b \]
Intuition of FT

**Coefficient is obtained via Inner product**

$$(a, b) = a \ (1,0) + b \ (0,1)$$

$$\langle a, b, (1,0) \rangle = a \ \langle (1,0), (1,0) \rangle + b \ \langle (0,1), (1,0) \rangle = a$$

**Coefficient is obtained via inner product**

$$x(t) = a \cos(\omega t) + b \cos(2\omega t)$$

$$\langle x(t), \cos(\omega t) \rangle = \langle a \cos(\omega t), \cos(\omega t) \rangle + \langle b \cos(2\omega t), \cos(\omega t) \rangle = a$$

If $\cos(\omega t)$ and $\cos(2\omega t)$ are orthogonal
Intuition of FT

**orthogonal basis**

\[(a, b) = a \begin{pmatrix} 1,0 \end{pmatrix} + b \begin{pmatrix} 0,1 \end{pmatrix}\]

**basis**

\[x(t) = a \cos(\omega t) + b \cos(2\omega t)\]
Intuition of FT

It turns out that we can define the inner product in the signal space

\[
\langle x(t), y(t) \rangle = \frac{2}{T} \int_{-T/2}^{T/2} x(t) y(t) dt
\]

\[
\langle \cos(\omega t), \cos(2\omega t) \rangle = \frac{2}{T} \int_{-T}^{T} \cos(\omega t) \cos(2\omega t) dt
\]
Intuition of FT

\[ \cos(2\pi t) \cdot \cos(2 \cdot 2\pi t) \]

\[
\langle \cos(\omega t), \cos(2\omega t) \rangle = \frac{2}{T} \int_{-T}^{T} \cos(\omega t) \cos(2\omega t) dt
\]
Intuition of FT

\[ \{ \cos(n \omega t), n=1,2,\ldots \} \text{ are orthogonal} \]

\[ \cos(2\pi t) \]
\[ \cos(2\times2\pi t) \]

\[ \cos(2\pi t) \times \cos(2\times2\pi t) \]

- \[ \{ \cos(n \omega t), n=1,2,\ldots \} \text{ are orthogonal} \]
- they form orthogonal bases
Intuition of FT

It turns out that we can define the inner product in the signal space

\[ \langle \cos(m\omega t), \cos(n\omega t) \rangle = \begin{cases} 2/T, & n = m \\ 0, & n \neq m \end{cases} \]

In particular, we have

\[ x(t) = a \cos(\omega t) + b \cos(2\omega t) \]

\[ a = \langle X(t), \cos(\omega t) \rangle = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \cos(\omega t) \, dt \]

b = ?
Intuition of FT

- In an n-dim Euclidean space, we decompose any vector $X$ in terms of orthogonal bases.
- Coefficient is obtained by inner product between a basis and $X$.
- We sometime call $x_i$ weight.

$X = x_1 (1 0 0 0 0) + x_2 (0 1 0 0 0) + ... + x_6 (0 0 0 0 1)$
Intuition of FT

\[ x(t) = A_i \cos(\omega t) + \text{more} \]
Intuition of FT

\[ x(t) = A_1 \cos(\omega t) + A_2 \cos(2\omega t) + \text{more} \]
Intuition of FT

\[ x(t) = A_1 \cos(\omega t) + A_2 \cos(2\omega t) + A_3 \cos(3\omega t) + \text{more} \]
Intuition of FT

\[ x(t) = A_1 \cos(\omega t) + A_2 \cos(2\omega t) + A_3 \cos(3\omega t) + A_4 \cos(4\omega t) + \text{more} \]
Intuition of FT

\[ X(t) = A_1 \cos(\omega t) + A_2 \cos(2\omega t) + A_3 \cos(3\omega t) + A_4 \cos(4\omega t) + \text{more} \]
$x(t) = A_1 \cos(\omega t) + A_2 \cos(2\omega t) + A_3 \cos(3\omega t) + A_4 \cos(4\omega t) + A_6 \cos(6\omega t)$
Intuition of FT

In an n-dim Euclidean space, we decompose any vector $X$ in terms of orthogonal bases.

- Coefficient is obtained by inner product between a basis and $X$.
- We sometimes call $x_i$ weight.
FT: Fourier Thm in terms of Sin and Cos

• simply a generalization of common knowledge of the Euclidean space

• \{ 1, \cos (n \omega t), \quad n=1,2\ldots \} are orthogonal

• they form orthogonal bases
FT: Fourier Thm in terms of Sin and Cos

- simply a generalization of common knowledge of the Euclidean space
- \{1, \cos(n\omega t), \sin(n\omega t), n=1,2\ldots\} are orthogonal and complete
- they form orthogonal bases
FT: Fourier Thm in terms of Sin and Cos

• simply a generalization of common knowledge of the Euclidean space

• \{ 1, \cos(n \omega t), \sin(n \omega t), n=1,2… \} are orthogonal and complete

• they form orthogonal bases

\[ x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n \omega t) + \sum_{n=1}^{\infty} B_n \sin(n \omega t) \]
FT: Fourier Thm in terms of Sin and Cos

- simply a generalization of common knowledge of the Euclidean space
- \{ 1, \cos(n\omega t), \sin(n\omega t), n=1,2... \} are orthogonal and complete
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\[ x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega t) + \sum_{n=1}^{\infty} B_n \sin(n\omega t) \]

\[
A_0 = \langle 1, x(t) \rangle = \frac{1}{T} \int_{-T/2}^{T/2} x(t) \, dt
\]
FT: Fourier Thm in terms of Sin and Cos

- simply a generalization of common knowledge of the Euclidean space
- \{ 1, \cos (n\omega t), \sin(n\omega t), n=1,2…\} are orthogonal and complete
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\[
x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega t) + \sum_{n=1}^{\infty} B_n \sin(n\omega t)
\]

\[
\begin{aligned}
A_0 &= \langle 1, x(t) \rangle = \frac{1}{T} \int_{-T/2}^{T/2} x(t) dt \\
A_n &= \langle \cos(n\omega t), x(t) \rangle = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \cos(n\omega t) dt
\end{aligned}
\]
FT: Fourier Thm in terms of Sin and Cos

- simply a generalization of common knowledge of the Euclidean space
- \{ 1, \cos (n\omega t), \sin(n\omega t), n=1,2\ldots \} are orthogonal and complete
- they form orthogonal bases

\[ x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega t) + \sum_{n=1}^{\infty} B_n \sin(n\omega t) \]

\[
\begin{cases}
A_0 = \langle 1, x(t) \rangle = \frac{1}{T} \int_{-T/2}^{T/2} x(t) \, dt \\
A_n = \langle \cos(n\omega t), x(t) \rangle = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \cos(n\omega t) \, dt \\
B_n = \langle \sin(n\omega t), x(t) \rangle = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \sin(n\omega t) \, dt \\
\omega = \frac{2\pi}{T}
\end{cases}
\]

where \( A_0 \) is the d.c. term, and \( T \) is the period of the waveform.
Example 1

\[ x(t) = 1, \quad 0 < t < \pi, \quad 2\pi < t < 3\pi, \quad 0 \text{ otherwise} \]

Hence \( x(t) \) is a signal with a period of \( 2\pi \)
Example 1

\[
\begin{align*}
A_0 &= \frac{1}{2\pi} \int_0^\pi dt = \frac{1}{2} \\
A_n &= \frac{2}{2\pi} \int_0^\pi \cos(nt) dt = \frac{1}{n\pi} \sin(n\pi) = 0, n = 1, 2, \ldots \\
B_n &= \frac{2}{2\pi} \int_0^\pi \sin(nt) dt = \frac{1}{n\pi} (1 - \cos(n\pi)), n = 1, 2, \ldots
\end{align*}
\]

Finally, we have

\[
x(t) = \frac{1}{2} + \frac{2}{\pi} \left[ \sin(t) + \frac{1}{3} \sin(3t) + \frac{1}{5} \sin(5t) + \ldots \right]
\]
The description of a signal in terms of its constituent frequencies is called its frequency (power) spectrum.

\[ x(t) = \frac{1}{2} + \frac{2}{\pi} \left[ \sin(t) + \frac{1}{3} \sin(3t) + \frac{1}{5} \sin(5t) + \ldots \right] \]
Example 1

Time domain

Frequency domain

\[
|X(F)| = \frac{2}{\pi}
\]
Example I

- A periodic signal is uniquely determined by its coefficients \(\{A_n, B_n\}\).

- If we truncated the series into finite term, the signal can be approximated by a finite sines as shown below (compression, MP3, MP4, JPG, … )

![Graph showing approximation of a periodic signal with one, two, three, and five terms.](image-url)
Example II (understanding music)
Example II

a. Pure tone:

This confirms our earlier belief that it is a signal with a finite bandwidth (N-S sampling Thm)
Digital Music
DCSP-6: Applications

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Happy Chinese New Year 2020
Recap

In general, a signal with $T = 2\pi / \omega$ can be represented as follows

$$x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(\omega nt) + \sum_{n=1}^{\infty} B_n \sin(\omega nt)$$

$$\sim A_0 + \sum_{n=1}^{N} A_n \cos(\omega nt) + \sum_{n=1}^{N} B_n \sin(\omega nt)$$

$$x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega t) + \sum_{n=1}^{\infty} B_n \sin(n\omega t)$$

$$\left\{ \begin{array}{l}
A_0 = \langle 1, x(t) \rangle = \frac{1}{T} \int_{-T/2}^{T/2} x(t) \, dt \\
A_n = \langle \cos(n\omega t), x(t) \rangle = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \cos(n\omega t) \, dt \\
B_n = \langle \sin(n\omega t), x(t) \rangle = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \sin(n\omega t) \, dt \\
\omega = \frac{2\pi}{T} \end{array} \right. $$
Example II

b. Different waveforms

This confirms our earlier belief that it is a signal without bandlimit (N-S sampling Thm)
Bandwidth can be properly defined

- **Bandwidth** is the difference between the upper and lower frequencies in a set of frequencies. It is typically measured in hertz.
• **Bandwidth** is the difference between the upper and lower frequencies in a continuous set of frequencies. It is typically measured in **hertz**.
Example II

C. Approximation (compression)

Script1_3.m

Without doing much, we can compress the original data now, as in Example I.
In general, a signal with $T=\frac{2\pi}{\omega}$ can be represented as follows:

$$X(t) \sim A_0 + A_1 \cos(\omega t)$$
Recap

In general, a signal with $T = \frac{2\pi}{\omega}$ can be represented as follows:

$$X(t) \sim A_0 + A_1 \cos(\omega t) + A_2 \cos(2\omega t)$$
Recap

In general, a signal with $T=\frac{2\pi}{\omega}$ can be represented as follows:

$$X(t) \sim A_0 + A_1 \cos(\omega t) + A_2 \cos(2\omega t) + A_3 \cos(3\omega t)$$
In general, a signal with $T=\frac{2\pi}{\omega}$ can be represented as follows

$$X(t) \sim A_0 + A_1 \cos(\omega t) + A_2 \cos(2\omega t) + A_3 \cos(3\omega t) + A_4 \cos(4\omega t) + A_5 \cos(5\omega t) + A_6 \cos(6\omega t) + A_7 \cos(7\omega t) + A_8 \cos(8\omega t) + A_9 \cos(9\omega t)$$
Recap

In general, a signal with $T=\frac{2\pi}{\omega}$ can be represented as follows

$$X(t) = A_0 + A_1 \cos(\omega t) + A_2 \cos(2\omega t) + A_3 \cos(3\omega t) + A_4 \cos(4\omega t) + A_5 \cos(5\omega t) + A_6 \cos(6\omega t) + A_7 \cos(7\omega t) + A_8 \cos(8\omega t) + A_9 \cos(9\omega t) + \ldots$$
In general, a signal with $T=2\pi/\omega$ can be represented as follows:

$$X(t) = A_0 + A_1 \cos(\omega t) + A_2 \cos(2\omega t) + A_3 \cos(3\omega t) + A_4 \cos(4\omega t) + A_5 \cos(5\omega t) + A_6 \cos(6\omega t) + A_7 \cos(7\omega t) + A_8 \cos(8\omega t) + A_9 \cos(9\omega t) + \ldots$$
Recap

In general, a signal with $T=\frac{2\pi}{\omega}$ can be represented as follows

\[ X(t) = A_0 + A_1 \cos(\omega t) + A_2 \cos(2\omega t) + A_3 \cos(3\omega t) + A_4 \cos(4\omega t) + A_5 \cos(5\omega t) + A_6 \cos(6\omega t) + A_7 \cos(7\omega t) + A_8 \cos(8\omega t) + A_9 \cos(9\omega t) + \ldots \]
Recap

In general, a signal with $T=2\pi/\omega$ can be represented as follows

$$x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(\omega nt) + \sum_{n=1}^{\infty} B_n \sin(\omega nt)$$

$$\sim A_0 + \sum_{n=1}^{N} A_n \cos(\omega nt) + \sum_{n=1}^{N} B_n \sin(\omega nt)$$

- The more terms we have, the more accurate: compression
- The plot of $\{A_n, B_n\}$ against $\{n\}$ is called frequency spectrum
Today’s Summary

• General form of FT

• A few applications (6 more precisely)

• Noise
In general, a signal can be represented as follows:

\[ x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(\omega nt) + \sum_{n=1}^{\infty} B_n \sin(\omega nt) \]

\[ = A_0 + \sum_{n=1}^{\infty} A_n \left[ \exp(-j\omega nt) + \exp(j\omega nt) \right] / 2 + \]
\[ + \sum_{n=1}^{\infty} B_n \left[ \exp(-j\omega nt) - \exp(j\omega nt) \right] / (2j) \]

\[ = A_0 + \sum_{n=1}^{\infty} \left[ A_n / 2 + B_n / (2j) \right] \exp(-j\omega nt) + \]
\[ + \sum_{n=1}^{\infty} \left[ A_n / 2 - B_n / (2j) \right] \exp(j\omega nt) \]
In general, a signal can be represented as follows

\[ x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(\omega nt) + \sum_{n=1}^{\infty} B_n \sin(\omega nt) \]

\[ = A_0 + \sum_{n=1}^{\infty} A_n \left[ \exp(j\omega nt) + \exp(-j\omega nt) \right] / 2 + \]

\[ + \sum_{n=1}^{\infty} B_n \left[ \exp(j\omega nt) - \exp(-j\omega nt) \right] / (2j) \]

\[ = A_0 + \sum_{n=1}^{\infty} \left[ \frac{A_n}{2} - \frac{B_n}{(2j)} \right] \exp(-j\omega nt) + \]

\[ + \sum_{n=1}^{\infty} \left[ \frac{A_n}{2} + \frac{B_n}{(2j)} \right] \exp(j\omega nt) \]

\[ = \sum_{n=-\infty}^{\infty} c_n \exp(j\omega nt) \]
FT : Neat Form

- Which is the exponential form of the Fourier series.

- In this expression the values $C_n$ are complex number, we have

$$x(t) \xrightarrow{\text{FT}} \{C_n, n=\ldots,-1,0,1,2,\ldots\}$$

$$C_{-1}^2 = C_1^2 = \left( A_1^2 + B_1^2 \right) / 4$$

$$C_{-2}^2 = C_{-2}^2 = \left( A_2^2 + B_2^2 \right) / 4$$
FT : Neat Form

• Which is the exponential form of the Fourier series.

• In this expression the values $C_n$ are complex number, we have

$$X(n\omega) = X(F) = c_n = \frac{1}{T} \int_{-T/2}^{T/2} \exp(-jn\omega t)x(t)dt$$

$$x(t) = \sum_F X(F) \exp(jFt)$$

where $\omega = (2\pi) / T$
FT in complex exponential

If the periodic signal is replace with an aperiodic signal \textbf{(general case)}

FT is given by

\[
\begin{align*}
X(F) &= FT(x) = \int_{-\infty}^{\infty} x(t) \exp(-2\pi j Ft) \, dt \\
n(t) &= FT^{-1}(X) = \int_{-\infty}^{\infty} X(F) \exp(2\pi j Ft) \, dF
\end{align*}
\]

\textit{Is that deadly simple?}
Fourier’s song

- Integrate your function times a complex exponential
- It's really not so hard you can do it with your pencil  
  yes, I agree
Example I

Consider the case of a rectangular pulse. In particular, let us define

\[ x(t) = \begin{cases} 
1, & \text{if } -0.5 < t < 0.5 \\
0, & \text{otherwise} 
\end{cases} \]

Its FT is given by

\[ X(F) = \int_{-0.5}^{0.5} x(t) \exp(-j2\pi F t) dt = \frac{\sin(\pi F)}{\pi F} \]

Which is called the Sinc function.
There are a number of features to note:

1. The bandwidth of the signal is only approximately finite. Most of the energy is contained in a limited regions called the main-lobe. However, some energy is found at all frequencies.

2. The spectrum has positive and negative frequencies. These are symmetric about the origin. This may seem non-intuitive, but can be seen from equations in periodic case.

3. Note that the spectrum is continuous now:

\[ X(F) = \text{Sinc function} \]
Fourier's Song

Integrate your function times a complex exponential  It's really not so hard you can do it with your pencil  And when you're done with this calculation You've got a brand new function - the Fourier Transformation  What a prism does to sunlight, what the ear does to sound  Fourier does to signals, it's the coolest trick around  Now filtering is easy, you don’t need to convolve All you do is multiply in order to solve.

From time into frequency - from frequency to time  Every operation in the time domain  Has a Fourier analog - that’s what I claim  Think of a delay, a simple shift in time  It becomes a phase rotation - now that's truly sublime!

And to differentiate, here's a simple trick  Just multiply by J omega, ain't that slick? Integration is the inverse, what you gonna do?  Divide instead of multiply - you can do it too.

From time into frequency - from frequency to time  Let’s do some examples... consider a sine  It's mapped to a delta, in frequency - not time  Now take that same delta as a function of time  Mapped into frequency - of course - it's a sine!

Sine x on x is handy, let’s call it a sinc.  Its Fourier Transform is simpler than you think.  You get a pulse that's shaped just like a top hat...  Squeeze the pulse thin, and the sinc grows fat.
The world has changed

This module and a few following ones will make use of the frequency information.
This module is about practical applications

• How to use this idea?

\[
\begin{align*}
X(F) &= FT(x) = \int_{-\infty}^{\infty} x(t) \exp(-2\pi j Ft) dt \\
x(t) &= FT^{-1}(X) = \int_{-\infty}^{\infty} X(F) \exp(2\pi j Ft) dF
\end{align*}
\]
**Time domain**

\[ S_1(t) = 10 \cos(2\pi t) + \cos(10 \cdot 2\pi t) \]

for \( i = 1:10000 \)

\[ x(i) = 10 \cos(i \cdot 0.01) + \cos(i \cdot 10 \cdot 0.01); \]

End

Sound(x)

**Frequency domain**
\[ S_2(t) = \cos(2\pi t) + 10 \cos(10 \times 2\pi t) \]

for \( i = 1:10000 \)
\[ x(i) = \cos(i \times 0.01) + 10 \cos(i \times 10 \times 0.01); \]
End

Sound(s)
In frequency domain, the height of the spectrum indicates the energy of the corresponding signal.

For example,

- for $S_1$, energy concentrating on the signal $\cos(2\pi t)$, a low frequency signal
- for $S_2$, energy concentrating on the signal $\cos(10 \times 2\pi t)$, a high frequency signal
App II: Touch-tone dialing
App II: Touch-tone dialing

- Freqs 1209 Hz 1336 Hz 1477 Hz 1633 Hz
- 697 Hz 1 2 3 A
- 770 Hz 4 5 6 B
- 852 Hz 7 8 9 C
- 941 Hz * 0 # D

\[
(1) = A_{n1} \cos(n1 \omega_0 t) + A_{n2} \cos(n2 \omega_0 t)
\]

1 Hz = \(\omega_0\)

Your phone is a section (two terms) of a FT
• In bluetooth, for example, to improve resistance to radio frequency interference by avoiding using crowded frequencies in the *hopping* sequence.

• In military use, it is a simple idea (radio guided torpedo, for example)
App IV: instruments

• We all know about pitch…
• It is really about frequency, or cycles per seconds
• How about harmonics?
• This is what gives the timbre of an instrument!
App IV: instruments

- The played note is the frequency of the first peak: 220Hz (note A) in this case
- Other peaks are called harmonics: they define the typical sound of the instrument
- Without Fourier, we could have been lost
- When you play an instrument, it is a section of a FT
How can an MP3 song sound so well, while being so compressed?

Compression in a sense introduces noise.

a combination of our hearing system + FT
App V: MP3+MP4

• Original noiseless music
• Original music + noise 2 \(\rightarrow\) SNR = 13 dB
• Same amount of noise
• Impressive difference of sound
• What is the secret?

\[\text{load handel.mat}\]
\[\text{filename} = 'handel.wav';\]
\[\text{audiowrite(filename,y,Fs);}\]
\[\text{clear y Fs}\]
App V: MP3+MP4

![Graphs showing music and noise levels](image-url)
App V: MP3+MP4

- MP3: complex compression algorithm that introduces errors
- MP3 minimizes the perceived quality decay by shaping the compression errors
App VI: Mind Reading

• MRI is a medical imaging technique to visualize your internal body structure and/or activity in detail.

• Using a physics term: it measures the oscillations of hydrogen nuclei when stimulated by different radio-frequency magnetic fields.

• Reading your mind with FT.

My brain activity during one scan.
App VI: Mind Reading

- MRI is a medical imaging technique to visualize your internal body structure and/or activity in detail.

- Using a physics term: it measures the oscillations of hydrogen nuclei when stimulated by different radio-frequency magnetic fields.

- Reading you mind with FT.
• Using a DCSP term: it measures the FT of the internal structure and/or activity of interest

• How does the collected data looks like?

```matlab
img=(T1_Image(:,:,50));
img = fftshift(img(:,:,50));
F = fft2(img);
figure;
imagesc(100*log(1+abs(fftshift(F)))); colormap(gray)
title('magnitude spectrum');
figure;
imagesc(angle(F)); colormap(gray);
title('phase spectrum');
```
App VI: Mind Reading

- Little information about the subject can be gathered in the original picture.
- An inverse Fourier transform of the data reveals the slice of a human head activity.
App VI: Mind Reading

- Human Brain Project was one of the two projects with a budget of 1B Euros for ten years each (another is graphene) in 2013.

- The main purpose is to simulate the Brain.

- USA matched with 4.5 B US Dollars in 2014.

- China will do as well.
App VI: Mind Reading

Spinnaker can simulate 1% of our human brain cells (Steve Furber 21st Jan 2020)