Signal Processing and Computations in the Brain

Lecture by Professor E T Rolls, Dept Computer Science.

- Is the brain a digital signal processor?
- Digital vs continuous signals
- Digital signals involve streams of binary encoded numbers
- The brain uses digital, all or none, action potentials or spikes for information transmission in its neuronal axons over distances of 1 mm to several metres to avoid the uncertain decay of an analog signal in a long non-uniform cable.
- But the spikes are then converted into analog signals within a neuron, which has a threshold for the summed depolarization produced by many small synaptic currents to elicit an action potential, which is binary.
- The brain does not work by logical operations as in digital computers, but instead by the similarity of an input vector of firing rates with a synaptic weight vector to produce a post-synaptic potential.
- The spikes in the brain have Poisson timing, i.e. are random in time for a given mean firing rate. This leads to stochastic processing in the brain, related to the timing of the digital inputs to a neuron.

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Recommended reading. See also www.oxcns.org/publications

Brain computation vs Digital computer computation

1. Dot product similarity followed by a threshold vs logical functions e.g. AND, NAND, OR
2. 10,000 inputs per neuron vs several inputs to a logic gate
3. Content-addressable memory vs data accessed by address
4. Fault tolerant: dot product vs no inherent fault tolerance
5. Generalization and completion vs only exact match in the hardware
6. Fast: inherently parallel update of a neuron, and of neurons in a network vs serial processing with fast components
7. Approximate, low precision vs high precision 64 bit
8. Stochastic spiking noise: probabilistic computation vs noise free because of non-linearities helps originality, creativity
9. Dynamics are parallel – an attractor network retrieves in 1 - 2 time constants of the synapses. vs inherently serial
10. Heuristics, e.g. Invariant object recognition vs attempt to understand a whole scene analyses only part of a scene.
11. Syntax: none inherent vs syntactical operations on data at an address
12. The architecture adapts vs fixed architecture with different software to implement different computations.
13. Sparse distributed representation vs binary encoding in a computer word, e.g. 64 bits vs software vs hardware
14. Mind vs brain

Each neuron has approximately 10,000 inputs though synapses. The neuron sums the currents produced by each input firing rate \( x_j \) injected through each synapse \( w_{ij} \) to produce an activation \( h_i \).

\[
h_i = \sum_j x_j \cdot w_{ij}
\]

The activations are converted into spikes when the neurons reaches a threshold for firing. This is a non-linear operation and produces digital spikes, with a firing rate that depends on the strength of the activation. The output firing \( y_j \) is a function of the activation \( h_j \).
Neurodynamical Modeling: Neurons, Synapses & Cortical Architecture

Spiking Neuron -> Integrate-and-Fire Model:

\[ \tau \frac{dv}{dt} = -v + g_e(v,v_t,v_e) - g_i(v,v_t,v_i) \]

Synaptic Dynamics:

\[ I_{syn} = \sum_j \delta(t - t_j) \cdot w_{ij} \]

Dynamical Cooperation and Competition

Pattern Association Memory

Learning Rule:

Where \( y_j \) approximates \( e_i \) (which is the unconditioned or external or forcing input)

\[ \delta w_{ij} = k \cdot y_j \cdot x_j \]

Recall: The activation,

\[ h_i = \sum_j x_j \cdot w_{ij} \]

where the sum is over the \( C \) input axons, \( x_j \)

The output firing \( y_j \) is a function of the activation

\[ y_j = f(h_j) \]

This activation function \( f \) may be linear, sigmoid, binary threshold, etc.

Learning: Long Term Potentiation and Long Term Depression of synapses

![Learning Diagram](image)

Auto-association Memory

Learning Rule:

Where \( y_j \) approximates \( e_i \) (which is the external input)

\[ \delta w_{ij} = k \cdot y_i \cdot x_j \]

(Hebb rule)

and \( y_j \) is the firing of the output neuron (post-synaptic term)

\( x_j \) is the firing of input axon \( j \) (presynaptic term)

\( w_{ij} \) is the synapse to output neuron \( i \) from input axon \( j \).

Recall: The (internal) activation produced by the recurrent collateral effect (after the first iteration) is

\[ h_i = \sum_j x_j \cdot w_{ij} \]

where the sum is over the \( C \) axons indexed by \( j \).

The output firing \( y_j \) is a function of the activation produced by the recurrent collateral effect and by the external input (\( e_i \)):

\[ y_j = f(h_i + e_i) \]

The activation function \( f \) may be binary threshold, linear threshold, sigmoid, etc.

Cortical visual pathways

stored pattern
cue (20% corr.) iteration retrieved state (90% corr.)

Temporal ‘What’

Hippocampal ‘Memory’
Single neuron selectivity: in this case to a face

Stimulus 1
Stimulus 2
Stimulus 3
Stimulus 4

How is information encoded by the inferior temporal visual cortex?

Rate Code

Local (Grandmother), Distributed and Sparse Distributed Coding

- Local (Grandmother): One cell codes for one stimulus and fires only in response to this one.
- Fully Distributed: Almost every cell participates in encoding all stimuli. (With binary firing rates half the cells respond to any stimulus)
- Sparse Distributed: The cell responds to a certain number of stimuli. Intermediate between Grandmother and Fully distributed.

Inferior Temporal Visual Cortex neuron firing rate response to a set of 68 visual stimuli

How does the information scale with the number of neurons?

Approximately linearly.
Optimal vs Dot Product decoding.

How does the number of stimuli encoded scale with the number of neurons? (Information uses a log scale.)

Dot Product Multiple Cell Decoding

\[ I_3 = \sum_j w_j x_j \]
An attractor network for probabilistic decision-making, with lambda 1 and 2 inputs, and noise from the neuronal spiking influencing which decision attractor, D1 or D2, wins. This is also a model for short-term memory.


Integrate-and-fire simulations predict earlier and higher neuronal responses on easy vs difficult trials. Confidence is reflected in the higher firing on easy trials.

The decision stimuli start at t=2 s

Rolls, Grabenhorst and Deco 2010 Neuroimage

• The theoretical framework based in statistical mechanics predicts higher and faster neuronal responses as Delta I, the difference between the two stimuli, increases.
• Delta I is a measure of the easiness of the decision, and subjective confidence ratings correlate with this.

Rolls, Grabenhorst and Deco 2010a Neuroimage

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The hippocampal episodic memory system
### Signal Processing and the Brain.

Further reading:

- On differences between signal processing in the brain and in digital computers:
  - On the representation of information in the brain:
- Papers, books, and contact information:
  - [https://www.oxcns.org](https://www.oxcns.org)
  - Edmund.Rolls@warwick.ac.uk

### Brain computation vs Digital computer computation

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**Invariant visual object recognition in the brain**

Right – convergence in the ventral stream cortical hierarchy for object recognition. LGN, lateral geniculate nucleus; V1, visual cortex area V1; TEO, posterior inferior temporal cortex; TE, anterior inferior temporal cortex (IT).

Left – convergence as implemented in VisNet, the model of invariant visual object recognition described here. Convergence through the hierarchical feedforward network is designed to provide Layer 4 neurons with information from across the entire input retina, by providing an increase of receptive field size of 2.5 times at each stage. Layer 1 of the VisNet model corresponds to V2 in the brain, and Layer 4 to the anterior inferior temporal visual cortex (TE).

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