

# Password Authenticated Key Exchange by Juggling

## A key exchange protocol without PKI

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# Outline

- 1 Introduction
- 2 Related work
- 3 Our Solution
- 4 Evaluation
- 5 Summary

# Password Authenticated Key Exchange

## Definition

Password Authenticated Key Exchange (PAKE) studies how to establish a secure communication channel between two parties **solely** based on a shared low-entropy password.

## A concrete example

- Alice and Bob share a four-digit code.
- Now Alice wants to send Bob a private message.
- But, eavesdropping is everywhere.
- And attackers may intercept a message, and change it at will.
- Bootstrap a **high-entropy** key from a **low-entropy** secret. Is it possible?

# The PAKE problem

## Research development

- In 1992, Bellare and Merrit proposed EKE, a milestone.
- In 1996, Jablon designed SPEKE, another well-known scheme.
- In 2000, IEEE formed P1363.2 Working Group to standardize PAKE solutions. Still on-going ...

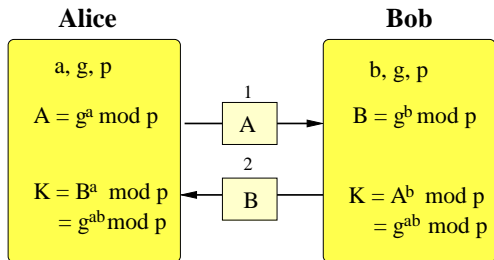
To date, the problem remains unsolved.

- Patent is one big issue; many schemes were patented.
- Also there are technical issues.

# Patent is one big issue

- EKE (1992) was patented by Lucent.
- SPEKE (1996) was patented by Phoenix.
- SRP-6 (1998) was patented by Stanford.
- OPAKE (2005) was patented by DoCoMo.
- In practice, many applications have no choice but to use SSL/TLS, which relies on a PKI in place.

# A Basic Diffie-Hellman protocol



- EKE modifies the protocol by encrypting  $A$  and  $B$  with a password, hence the name “Encrypted Key Exchange”.
- SPEKE modifies the protocol by replacing  $g$  with a password-derived variable, say a hash of password.

# Reported Weaknesses with EKE

## Practical attack

- An attacker can narrow down the password range [Jaspan'96].

## Theoretical analysis

- Proving the EKE security requires a strong “ideal-cipher model” assumption [Bellare et al'00]

## Efficiency

- EKE uses long exponents by definition.

# Reported Weaknesses with SPEKE

## Practical attack

- An attacker can guess multiple passwords in one try [Zhang'04].

## Theoretical analysis

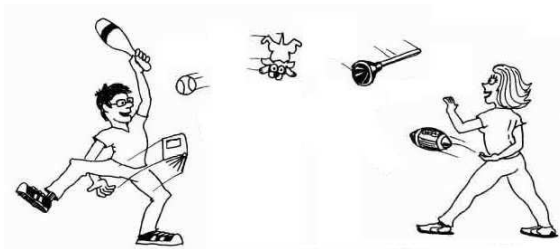
- Proving the security of SPEKE requires relaxing the requirements – allow multiple guesses of password in on try [MacKenzie'01].

## Efficiency

- SPEKE uses long exponents by definition.



# A New Approach – Public Key Juggling



- A technique first devised to solve the Dining Cryptographers problem [Hao,Zielinski'06], in a multi-party case.
- We apply the juggling technique to the two-party case.

# Password Authenticated Key Exchange by Juggling

## Round 1:

- ① Alice sends  $g^{x_1}, g^{x_2}$  and Know-Proof  $\{x_1, x_2\}$ .
- ② Bob sends  $g^{x_3}, g^{x_4}$  and Know-Proof  $\{x_3, x_4\}$ .

## Round 2:

- ① Alice sends  $\mathcal{A} = g^{(x_1+x_3+x_4) \cdot x_2 \cdot s}$  and Know-Proof  $\{x_2 \cdot s\}$ .
- ② Bob sends  $\mathcal{B} = g^{(x_1+x_2+x_3) \cdot x_4 \cdot s}$  and Know-Proof  $\{x_4 \cdot s\}$ .

## To get a common key:

- Alice compute  $K = (\mathcal{B}/g^{x_2 \cdot x_4 \cdot s})^{x_2} = g^{(x_1+x_3) \cdot x_2 \cdot x_4 \cdot s}$ .
- Bob computes  $K = (\mathcal{A}/g^{x_2 \cdot x_4 \cdot s})^{x_4} = g^{(x_1+x_3) \cdot x_2 \cdot x_4 \cdot s}$ .
- In this juggling game, each public key is a ball.

# Why the Knowledge Proofs in the protocol?

## The sixth principle in designing robust protocols

- “Do not assume that a message you receive has a particular form (such as  $g^r$  for known  $r$ ) unless you can check this.”  
[Anderson, Needham'95]

## Schnorr's signature

- A well-established technique
- Non-interactive, efficient.
- It allows one to prove knowledge of  $r$  for  $g^r$  without leaking it.

# Computation cost of J-PAKE - Java, 2.33-GHz

Item	Description	No of Exp	Time (ms)
1	Compute $\{g^{x_1}, g^{x_2}\}$ and KPs for $\{x_1, x_2\}$	4	23
2	Verify KPs for $\{x_3, x_4\}$	4	24
3	Compute $\mathcal{A}$ and KP for $\{x_2 \cdot s\}$	2	9
4	Verify KP for $\{x_4 \cdot s\}$	2	10
5	Compute $\kappa$	2	9
	Total	<b>14</b>	<b>75</b>

- It requires 14 exponentiations, while EKE and SPEKE only 2.
- But, both EKE and SPEKE require long exponents.
- One exp is 6-7 times more expensive than in J-PAKE for a typical  $p, q$  setting.
- Overall, the cost is actually about the same.

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# Security analysis of J-PAKE

Modules	Security property	Attacker type	Assumptions
Schnorr signature	leak 1-bit: whether sender knows discrete logarithm	passive/active	DL and random oracle
Password encryption	indistinguishable from random	passive/active	DDH
Session key	incomputable	passive	CDH
	incomputable	passive (know $s$ )	CDH
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# Summary of J-PAKE security proofs

1. **Off-line dictionary attack resistance:** It does not leak any password verification information to a passive attacker.
2. **Known-key security:** It prevents a disclosed session key from affecting the security of other sessions.
3. **Forward secrecy:** It produces session keys that remain secure even when the password is later disclosed.
4. **On-line dictionary attack resistance:** It strictly limits an active attacker to test only one password per protocol execution.

By comparison, EKE does not fulfill 1, and SPEKE does not fulfill 4.

# Comparison to the de facto standard SSL/TLS

Alice: "I know the password to Prince William's email."

Bob: "No, you don't."

Alice: "Yes, I do."

Bob: "Prove it."

Alice: "All right, I tell you." (She whispers in Bob's ear)

Bob: "Interesting. Now I'm going to tell the *Daily Mirror*."

Alice: "Oops!"

## Using J-PAKE: a Zero Knowledge Proof protocol

- Reveal only one-bit: whether or not one knows the password.
- Much more resistant against phishing attacks.
- In addition, it requires no PKI deployments ★★★

# Summary

## We proposed the J-PAKE protocol

- Authenticate password? Let's play a game.
- If juggling successful, both sides obtain a session key.
- If failure, it leaks nothing more than “wrong password”.
- Better security than EKE and SPEKE, with comparable efficiency.

## Compare with SSL/TLS

- It prevents leaking passwords, say to a fake bank website.
- It requires no PKI developments.

# Thank you!