The Stochastic Network Calculus: A Modern Approach to Queueing using Inequalities

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ABSTRACT

Over the last two decades the stochastic network calculus has been evolving as a modern approach to analyzing queueing systems, especially in the context of communication networks. The goal of this talk is to motivate the necessity for such a calculus, and to give insight into its fundamental concepts and mathematical apparatus.

Categories and Subject Descriptors

C.4 [**Computer System Organization**]: Performance of Systems

General Terms

Performance

Keywords

Deterministic network calculus, stochastic network calculus

1. INTRODUCTION

In the early 1990s, Cruz proposed an alternative approach [3] to the classical queueing networks theory for analyzing backlog and delays in networks, which evolved in what is currently known as the deterministic network calculus [2, 1]. The novelty of this approach is that, instead of using exact distribution functions to characterize arrivals and service as in queueing theory, to use a more relaxed characterization. Concretely, arrival processes are characterized using upper bounds, whereas service processes are characterized using lower bounds. Consequently, the calculus yields bounds, instead of exact results, on network performance metrics such as backlogs and delays.

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The fundamental feature of the produced bounds is that they are deterministic, i.e., they hold with probability one. In contrast, in queuing theory, such bounds generally do not exist: for instance, in an M/M/1 queue, any finite delay value can be actually realized with non-zero probability. Therefore, the deterministic network calculus is often regarded as a deterministic or worst-case theory for queueing systems.

From a more quantitative point of view, the bounds are tight in the sense that there exist worst-case arrival and service patterns for which the bounds become realizable. A valid concern, however, is that such worst-case realizations are very unlikely to happen, especially in scenarios with many arrival processes, and consequently that the produced bounds may be too loose if violation probabilities are allowed. This observation largely motivates the extension of the calculus in a probabilistic framework where much tighter bounds can be derived at the expense of small violation probabilities.

Using this perspective, supported by numerical results, this talk introduces the stochastic network calculus [2, 4]. Its main concepts, i.e., statistical envelopes and service curves, are presented as probabilistic extensions of their deterministic counterparts. The formulation of these concepts is illustrated in relationship to how much information is known about the statistical independence of the arrival processes and the scheduling algorithms used. The immediate advantage is that traditionally difficult issues in queueing networks theory, such as non-Poisson arrivals or scheduling algorithms, become much more approachable with the calculus.

Furthermore, the talk dedicates a special attention to the main mathematical techniques used in the stochastic network calculus to derive probabilistic backlog and delay bounds. On one hand stand manipulations of arrival and service processes characteristic to the deterministic network calculus. On the other hand stand basic probability inequalities, such as those due to Boole and Chernoff, which make the connection between the deterministic and probabilistic domains. Complete derivation of probabilistic bounds are carried out in order to illustrate the elegance of the calculus.

The talk also addresses a common criticism of the stochastic network calculus, i.e., related to the tightness of the derived bounds. To this end the classical M/M/1 and M/D/1 queue are modelled within the framework of the calculus and it is shown that the obtained bounds are very tight relative to the exact results.

2. REFERENCES

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