# On Multiplexing Gain for Networks with Deterministic Delay Guarantees

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***Abstract*— Multiplexing gain has been studied extensively in the context of statistical characterizations of traffic streams with quality-of-service criteria such as packet loss probability, mean delay, and delay variance. In this paper, we demonstrate that multiplexing gain can also arise in the context of deterministic traffic constraint functions, service curve scheduling, and quality-of-service requirements based on deterministic delay constraints. We show how to evaluate this multiplexing gain via the use of deterministic network calculus for both worst-case and time-averaged delay constraints. We show that significant multiplexing gain can be achieved in a deterministic setting using numerical examples drawn from a number of well-known MPEG video traces. Our results have application to provisioning services with tight, real-time constraints on end-to-end delay performance.**

## I. Introduction

Current proposals for supporting network-level deterministic quality-of-service (QoS) guarantees can be distinguished by scheduling policies, traffic regulation schemes, and the degree of multiplexing of individual connections or flows. Some policies, such as IntServ [1], support traffic on a flow by flow basis, while other policies, including DiffServ [2], require that traffic from multiple flows be multiplexed together. Deterministic QoS performance can be analyzed using network calculus theory (see [3][4]).

An important result of the network calculus theory under min-plus algebra concerns the *end-to-end deterministic gain* [3]: If an input stream $A_i$ is relayed through $n$ network elements, then the maximum delays incurred at each network element are not necessarily specific to a particular packet. This fact is referred to as the "pay bursts only once" phenomenon [4]. In this theory, deterministic end-to-end delay results are obtained by convolving service curves associated with nodes on the end-to-end path.

In the present paper, we develop a new concept of *deterministic multiplexing gain*, whereby a greater number of connections can be admitted under a given deterministic delay constraint if they are multiplexed together in one aggregate stream. Although statistical multiplexing gain with stochastic traffic models has been well-studied, it is not obvious that multiplexing gains can be achieved with respect to deterministic QoS constraints. Deterministic multiplexing gain may be explained intuitively as follows. The maximum delays encountered by two packets from two input streams $A_1$, respectively $A_2$, do not necessarily occur during the same time slot. We show analytically that deterministic multiplexing gain can be achieved and propose a heuristic to maximize the achievable gain among a set of connections. We present numerical examples of MPEG video traces which demonstrate that substantial gains can be achieved. Unlike statistical multiplexing gain, the deterministic multiplexing gain applies directly to the end-to-end case via the end-to-end convolved service curve result from the network calculus theory.

The paper continues as follows: Section II provides basic background in network calculus. Section III demonstrates how to obtain tight traffic constraint functions for multiplexed traffic. Sections IV and V discuss multiplexing gain with respect to worst case delay and time-averaged delay, respectively. Section VI shows the results of our numerical experiments using different MPEG-4 trace files, and Section VII offers some conclusions and observations.

## II. Traffic envelopes and service curves

Consider a network element $E$, for example a switch or multiplexer, and an input arrival process specified by a non-negative, non-decreasing function $A(\cdot)$, where $A(t)$ denotes the cumulative arrivals from time $0$ to time $t$. The process $A$ is usually called the arrival process. The output process is denoted by $B$ and the output capacity of $E$ is assumed to be $c > 0$.

*Definition 1:* A function $f : \mathbf{N} \rightarrow \mathbf{N}$ with the property that

$$A(t) - A(s) \leq f(t-s) \ (\forall) \ 0 \leq s \leq t, \tag{1}$$

is a traffic constraint function or an *envelope* of $A$ [5]. The relation between $A$ and $f$ will be denoted by $A \prec f$.
The expression (1) may be also written as $A \leq A * f$, where $A * f$ represents the convolution between $A$ and $f$, and $(A * f)(t) = \inf_{0 \leq s \leq t} (A(s) + f(t-s)) \ (\forall) \ t \geq 0$.

The tightest envelope of $A$ is given by the so-called *empirical envelope* [6] or minimal envelope process [3] and is defined as $\varepsilon_A(t) = \sup_{s \geq 0}(A(s+t) - A(s))$. In practice, all traffic constraint models try to approximate $\varepsilon_A$ through an envelope $f \geq \varepsilon_A$. We will measure the accuracy of such an approximation through the following metric: $\Lambda(f, \varepsilon_A) =$

$\int_0^{\infty} |f(t) - \varepsilon_A(t)|\, dt$. Note that $\Lambda(f, \varepsilon_A)$ has the properties of a metric in an appropriate metric space.

In the following, we briefly review two well-known traffic envelope functions that have been discussed in the literature. The affine model $(\sigma, \rho)$ is the simplest among the traffic constraint functions [5]. It is defined as $f(t) = \sigma + \rho t$, where $\sigma \geq 0$ represents the maximal allowable burst of the arrival process $A$, while $\rho > 0$ represents the long term average rate of $A$. The $(\overrightarrow{\sigma}, \overrightarrow{\rho})_n$ model [6] has an envelope function defined by $f(t) = \min_{i=\overline{1,n}} (\sigma_i + \rho_i t)$ ,where $0 \leq \sigma_1 < ... < \sigma_n$ and $0 < \rho_n < ... < \rho_1$.

Besides the concept of traffic envelope, the concept of *service curve* [7] is necessary to derive performance results in the deterministic network calculus. We will adopt here the definition from [3]. Specifically, $S$ denotes a service curve for an input arrival process $A$ if $B(t) \geq (A * S)(t)\ (\forall)\ t \geq 0$. In terms of implementation, the SCED (Service Curve-based Earliest Deadline first) [7] scheduling algorithm is designed to guarantee service curves for the corresponding traffic flows.

Let us assume now that we have a set of $n$ connections $A_i$, each with the envelope $f_i$ and the service curve $S_i$. The next theorem gives a sufficient condition that must be held by a number of traffic flows in order to be accepted by the CAC (connection admission control) component.

*Theorem 1 ([8][3]):* If

$$\sum_{i=1}^{n} (f_i * S_i)(t) \leq ct\ (\forall)\ t \geq 0, \tag{2}$$

then the SCED policy guarantees connection $A_i$ the service curve $S_i$ for any $i = \overline{1,n}$.

Also, a stronger condition than (2), that is

$$\sum_{i=1}^{n} S_i(t) \leq ct\ (\forall)\ t \geq 0, \tag{3}$$

is given in [8]. If condition (3) holds, we say that the set of connections satisfies the condition $MPX$.

Suppose that the condition (2) holds and let $T \subseteq \{1, 2, ..., n\}$. One may notice that it is not generally true that the SCED policy can guarantee the service curve $\sum_{i \in T} S_i$ for the multiplexed connection $\sum_{i \in T} A_i$. Motivated by this observation, we need the following definition for the rest of the paper:

*Definition 2:* Suppose that the condition (2) holds. For a subset $T \subseteq \{1, 2, ..., n\}$, we say that the connections $A_i$, $i \in T$ may be multiplexed under condition $MPX_T$, if the following condition holds: $((\sum_{i \in T} f_i) * (\sum_{i \in T} S_i))(t) + \sum_{i \notin T} (f_i * S_i)(t) \leq ct$ $(\forall)\ t \geq 0$. Note that the other connections $A_i$, $i \notin T$ are treated individually.

In other words, if the conditions (2) and $MPX_T$ hold for $T \subseteq \{1, 2, ..., n\}$, then the SCED policy guarantees the service curves $S_i$ for the connections $A_i$ $(\forall)$ $i \notin T$, and the service curve $\sum_{i \in T} S_i(\cdot)$ for the multiplexed connection $\sum_{i \in T} A_i$.

## III. Aggregate envelope construction

This section demonstrates how to construct an envelope for a multiplexed stream, and derives some of its properties, which will be useful later in the paper. As the primary concern of this section is represented by traffic envelopes, our analysis is based on projections under min-plus algebra (for more details, see [3]).

Consider a network element that consists of two arrival channels $C_1$, $C_2$ and a scheduler $S$. Each $C_i$ consists of an input arrival process $I_i$, a traffic regulator $R_i$, and a regulated arrival process denoted by $A_i$. The empirical envelope $\varepsilon_{I_i}$ of $I_i$ is given by $\varepsilon_{I_i}(t) = \sup_{s \geq 0} (I(s+t) - I(s))$ and is usually determined experimentally. Lastly, the traffic regulator $R_i$ is specified through an approximation of $\varepsilon_{I_i}$, that is the envelope function $f_i$ of $A_i$. Since the actual inputs $I_i$ may not conform to $\varepsilon_{I_i}$, they will have to go through $R_i$ in order to yield a regulated output $A_i$.

An important property of the envelope functions is expressed as follows:

*Proposition 1:* If $A_1 \prec f_1$ and $A_2 \prec f_2$, then $A_1 + A_2 \prec f_1 + f_2$.

*Proof:* Since the $\inf(\cdot)$ functional is superadditive, the following holds: $\inf_{0 \leq s \leq t} (A_1(t-s) + f_1(s) + A_2(t-s) + f_2(s)) \geq \inf_{0 \leq s \leq t} (A_1(t-s) + f_1(s)) + \inf_{0 \leq s \leq t} (A_2(t-s) + f_2(s))$. ■

In view of the definition of a traffic envelope (1), we will show how to construct a better traffic envelope for $A_1 + A_2$ rather than $f_1 + f_2$. We now give some useful definitions pertinent to our study.

### A. Projections under the min-plus algebra

We denote by $\mathcal{F}$ the set of envelope functions $\{f \mid f \geq 0\}$. Two operations are introduced on $\mathcal{F}$: $f \oplus g = \min(f, g)$ and $(f * g)(t) = \min_{0 \leq s \leq t} (f(t-s) + g(s))$. Define $\overline{\mathcal{R}}^+ = \{x \in \mathcal{R} \mid x \geq 0\} \cup \{\infty\}$. On this set, two operations are introduced: $\alpha \oplus \beta = \min(\alpha, \beta)$ and $\alpha \otimes \beta = \alpha + \beta$. The elements of $\mathcal{F}$ are also called vectors while the elements of $\overline{\mathcal{R}}^+$ are called scalars. We relate these sets with the operation $\otimes$ such that $\alpha \otimes f = \alpha + f\ (\forall)\ \alpha \in \overline{\mathcal{R}}^+$, $f \in \mathcal{F}$. The set $\mathcal{F}$ can be treated now as a moduloid (a vector space without inverse elements) over the semiring $(\overline{\mathcal{R}}^+, \oplus, \otimes)$. For two vectors $f, g \in \mathcal{F}$, the scalar projection of $f$ onto $g$ is defined by $\langle f, g \rangle = \sup_{t \geq 0} (f(t) - g(t))^+$, where $(x)^+ = \sup\{x, 0\}$. The vector projection of $f$ onto $g$ is denoted by $\langle f, g \rangle \otimes g$.

Consider a generic input arrival process $I$ having the empirical envelope $\varepsilon_I$. As it is usually hard to implement directly a regulator for $\varepsilon_I$, we will give an upper $(\overrightarrow{\sigma}, \overrightarrow{\rho})_n$ approximation of it. Consider a subset $\{g_1, ..., g_n\} \subset \mathcal{F}$ with $g_i(t) = \rho_i t$ and $\rho_1 > \rho_2 > ... > \rho_n \geq 0$. The $(\overrightarrow{\sigma}, \overrightarrow{\rho})_n$ envelope of $I$ is given by $f = \bigoplus_{i=1}^{n} (\langle \varepsilon_I, g_i \rangle \otimes g_i)$ .We call $\{g_1, ..., g_n\}$ a *base characterization* set of $I$.

Recalling the metric $\Lambda$, we introduce an $f$*−pseudo-norm* on the moduloid $\mathcal{F}$, where $f \in \mathcal{F}$: $\|\varepsilon_I\|_f = \Lambda(\langle \varepsilon_I, f \rangle \otimes f, \varepsilon_I) = \int_0^{\infty} |\langle \varepsilon_I, f \rangle \otimes f - \varepsilon_I|\, dt$, $\varepsilon_I \in \mathcal{F}$

### B. $(\sigma, \rho)$-envelopes

Let us suppose that the base characterization sets for $I_1$, $I_2$ are $\{g_1 = \rho_1 t\}$ and $\{g_2 = \rho_2 t\}$, respectively. Let $f_i = \langle \varepsilon_{I_i}, g_i \rangle \otimes g_i$ $(\forall)$ $i = 1, 2$, and $f = f_1 + f_2$. Let $C$ be the aggregated arrival channel $C_1 + C_2$ and let $I = I_1 + I_2$. One may notice that the simplest way to constrain $I$ is through $f$, while an alternative way to do it would be through $\tilde{f} = \langle \varepsilon_{I_1} + \varepsilon_{I_2}, g_1 + g_2 \rangle \otimes (g_1 + g_2)$ (see Fig. 1).

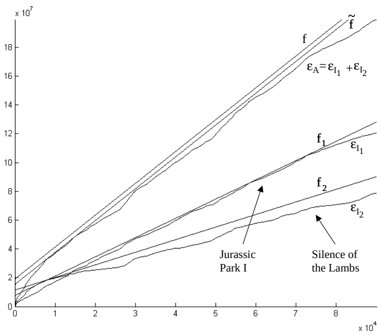


Fig. 1. Two trace files

The following two results reflect the properties of aggregate envelope $\tilde{f}$.

*Proposition 2:* For $f$ and $\tilde{f}$ defined above, we have that $\tilde{f} \leq f$.

*Proof:* Since the $\sup\{\cdot\}^+$ functional is subadditive, we have that $\langle \varepsilon_{I_1} + \varepsilon_{I_2}, g_1 + g_2 \rangle \leq \langle \varepsilon_{I_1}, g_1 \rangle + \langle \varepsilon_{I_2}, g_2 \rangle$. Then, $\tilde{f} = \langle \varepsilon_{I_1} + \varepsilon_{I_2}, g_1 + g_2 \rangle \otimes (g_1 + g_2) \leq \langle \varepsilon_{I_1}, g_1 \rangle \otimes g_1 + \langle \varepsilon_{I_2}, g_2 \rangle \otimes g_2 = f_1 + f_2 = f$. ■

The intuition behind the fact that $\tilde{f} \leq f$ is that it is possible that the burst times of $\varepsilon_{I_1}$ differ from the burst times of $\varepsilon_{I_2}$. Hence, by aggregating $C_1$ and $C_2$, the bursts of $\varepsilon_{I_1}$ and $\varepsilon_{I_2}$ will be interleaved, rather than summed up. As we will see in the next section, this property leads to possibly tighter bounds for the worst case delay of the aggregated flow $C$. That is, the worst case delay for $C$ may be less than the minimum between the worst case delays of $C_1$ and $C_2$.

*Theorem 2:* $\|\varepsilon_{I_1} + \varepsilon_{I_2}\|_{g_1+g_2} \leq \|\varepsilon_{I_1}\|_{g_1} + \|\varepsilon_{I_2}\|_{g_2}$.

*Proof:* We have that:

$$\int_0^\infty \left(\langle \varepsilon_{I_1} + \varepsilon_{I_2}, g_1 + g_2 \rangle \otimes (g_1 + g_2) - (\varepsilon_{I_1} + \varepsilon_{I_2})\right) dt \leq$$
$$\int_0^\infty \left(\langle \varepsilon_{I_1}, g_1 \rangle \otimes g_1 - \varepsilon_{I_1}\right) dt + \int_0^\infty \left(\langle \varepsilon_{I_2}, g_2 \rangle \otimes g_2 - \varepsilon_{I_2}\right) dt$$
$$\Longleftarrow \sup_{t \geq 0} \{\varepsilon_{I_1}(t) - g_1(t) + \varepsilon_{I_2}(t) - g_2(t)\}^+ \leq$$
$$\sup_{t \geq 0} \{\varepsilon_{I_1}(t) - g_1(t)\}^+ + \sup_{t \geq 0} \{\varepsilon_{I_2}(t) - g_2(t)\}^+,$$

which is true because the $\sup\{\cdot\}^+$ functional is subadditive. ■

### C. Generalization

A key feature of $(\overrightarrow{\sigma}, \overrightarrow{\rho})_n$ envelopes is the ability to capture the property that the arrival rate of a traffic stream decreases as function of time. Since the $(\sigma, \rho)$ model cannot capture this property, we will generalize the previous results to the $(\overrightarrow{\sigma}, \overrightarrow{\rho})_n$ model.

Consider two arrival channels $C_1$ and $C_2$ with the base characterization sets $\{g_1, ..., g_n\}$ and $\{h_1, ..., h_n\}$, respectively, with $n > 1$. The envelopes $g$ and $h$ corresponding to $C_1$ and $C_2$, respectively, are constructed as before. The envelope for the aggregated channel $C = C_1 + C_2$ is given by $\tilde{f} = \bigoplus_{i=1}^{n} \left(\langle \varepsilon_{I_1} + \varepsilon_{I_2}, g_i + h_i \rangle \otimes (g_i + h_i)\right)$.

We now extend the definition of the norm $\|\cdot\|_{\cdot}$ in the following way. For any $g_1, ..., g_n \in \mathcal{F}$, we define $\|\varepsilon_I\|_{(g_1,...,g_n)} = \int_0^\infty \left[\bigoplus_{i=1}^{n} \left(\langle \varepsilon_I, g_i \rangle \otimes g_i\right) - \varepsilon_I\right] dt$. The generalization of Proposition 2 is the following:

*Proposition 3:*

$$\tilde{f} \triangleq \bigoplus_{i=1}^{n} \left(\langle \varepsilon_{I_1} + \varepsilon_{I_2}, g_i + h_i \rangle \otimes (g_i + h_i)\right) \leq$$
$$\bigoplus_{i=1}^{n} \left(\langle \varepsilon_{I_1}, g_i \rangle \otimes g_i + \langle \varepsilon_{I_2}, h_i \rangle \otimes h_i\right) \triangleq f.$$

The proof is based on the following lemma:

*Lemma 1:* Let $g_1, ..., g_n \in \mathcal{F}$ and $h_1, ..., h_n \in \mathcal{F}$ two sequences of non-decreasing real functions such that $g_i \leq h_i$ $(\forall)$ $i$. Then, $\bigoplus_{i=1}^{n} g_i \leq \bigoplus_{i=1}^{n} h_i$.

*Proof:* Let $t \geq 0$, $i_g = \sup\{j \mid g_j(t) \leq g_l(t)\ (\forall)\ l = \overline{1,n}\}$ and $i_h = \sup\{j \mid h_j(t) \leq h_l(t)\ (\forall)\ l = \overline{1,n}\}$. We have the following: $\bigoplus_{i=1}^{n} g_i(t) = g_{i_g}(t) \leq g_{i_h}(t) \leq h_{i_h}(t) = \bigoplus_{i=1}^{n} h_i(t)$. ■

Theorem 2 can be generalized as follows:

*Theorem 3:* If $\|\varepsilon_{I_1} + \varepsilon_{I_2}\|_{g_n+h_n} < \|\varepsilon_{I_1}\|_{g_n} + \|\varepsilon_{I_2}\|_{h_n}$, then

$$\|\varepsilon_{I_1} + \varepsilon_{I_2}\|_{(g_1+h_1,...,g_n+h_n)} < \|\varepsilon_{I_1}\|_{(g_1,...,g_n)} + \|\varepsilon_{I_2}\|_{(h_1,...,h_n)}.$$

The proof is omitted here. Note that the theorem's condition $\|\varepsilon_{I_1} + \varepsilon_{I_2}\|_{g_n+h_n} < \|\varepsilon_{I_1}\|_{g_n} + \|\varepsilon_{I_2}\|_{h_n}$ gives a sufficient condition in order to consider the envelope multiplexing of $C_1$ and $C_2$ for the general case.

## IV. Multiplexing gain under worst-case delay

Consider two regulated arrival processes $A_1$, $A_2$ with $A_i \prec f_i$ and worst case delay $d_i$ for any $i = 1, 2$. Without loss of generality, suppose that $d_1 \leq d_2$ and assume that the condition (2) holds. The minimum service curve corresponding to $A_i$ such that the server guarantees the delays $d_i$ is $S_i(t) = f_i((t - d_i)^+)$ $(\forall)$ $i = 1, 2$. [3]

Let us suppose that the condition $MPX_{\{1,2\}}$ holds, that is $S_1 + S_2$ is a service curve for $A := A_1 + A_2$. Also, let $\tilde{f}$, constructed as in the previous section, be the envelope of $A$. The worst case delay corresponding to the multiplexed connection $A$ is simply: $d = \inf\left\{d \geq 0 \mid \tilde{f}(t) \leq S_1(t+d) + S_2(t+d)\ \ (\forall)\ \ t \geq 0\right\}$.

*Proposition 4:* In the current context, we have that $d \le d_2$.

*Proof:* From Propositions 2 and 3 we have that $\tilde{f}(t) \le f_1(t)+f_2(t) \le S_1(t+d_2)+S_2(t+d_2)$ $(\forall)$ $t \ge 0$. The result then follows immediately. ■

Informally, the above result states the following: Suppose that we have a server that guarantees the worst case delay $d_1$ for an input $A_1$. Although it may be impossible to accept a new input $A_2$ requiring a worst case delay $d_2$, it may be possible to accept $A_2$, but at the expense of a possibly higher worst case delay for $A_1$, if the inputs $A_1$ and $A_2$ are treated as the multiplexed input $A := A_1 + A_2$. Also, note the fact that the delay $d$ for the multiplexed connection $A$ may be less than $d_1$!

*Definition 3 (Deterministic Multiplexing Gain):* Assume that we have a server and $n$ input connections $A_i$, each requiring a worst case delay. Let $N$ be the maximum number of connections accepted by a given CAC (see for example Equation (2)) when no multiplexing is done. Let $M$ be the maximum number of connections accepted by the CAC when multiplexing is done according to a partition $\{T_k\}$ on a subset $T$ of the connections as follows. The connections $A_j \in T_k$ are multiplexed together and treated by the CAC as a single aggregate connection with a worst case delay requirement less than or equal to the minimum of the worst case delays for the connections $A_j \in T_k$. Multiplexing according to the partition $\{T_k\}$ is called "good multiplexing" since the worst case delay requirements of all connections in $T$ are satisfied. Then, the *deterministic multiplexing gain* is defined as:

$$G = \frac{M}{N}. \tag{4}$$

The previous definition is similar to the definition for statistical multiplexing gain given in [9].

In general, the problem of finding a partition of the set of connections to maximize the deterministic multiplexing gain $G$ belongs to the class of NP-complete problems. We now give a heuristic, called $H^{MG}$, for this problem:

*Algorithm 1 ($H^{MG}$):* Loop among the connections $A_i$, $i = 1, \cdots, n$. Assume that before each step $i$, there is a partition $\{T_k\}$ of the set $\{A_1, ..., A_{i-1}\}$. At step $i$, the connection $A_i$ is multiplexed sequentially with each set $T_k$ in the partition, and it is checked if this multiplexing is "good". If yes, add $A_i$ to the current set $T_k$ and proceed to the algorithm's next step. If $A_i$ cannot be multiplexed in a "good" way with any of the sets $T_k$, create a new set consisting just of $A_i$, add this set to the partition $\{T_k\}$, and then proceed to the next step.

One may deduce that the computational complexity for this algorithm is $O(n^2)$. The main intuition behind this algorithm can be expressed as follows: If $A_i$ cannot be multiplexed in a "good" way with either $T_{k_1}$ or $T_{k_2}$, it is less likely that a "good multiplexing" can be realized with the set $T_{k_1} \cup T_{k_2}$.

The following example shows a case when the worst case delay for $A$ is less than the minimum between the worst case delay for the independent connections $A_1$ and $A_2$. In other words, a multiplexed gain of $G > 1$ can be obtained.

*Example 1:* Let $c = 4$, $\varepsilon_{A_1} = \{0, 12, 12, 12\}$ $(\varepsilon_{A_1}(0) = 0, ..., \varepsilon_{A_1}(3) = 12)$, and $\varepsilon_{A_2} = \{0, 8, 12, 16\}$. Also, let $S_1(t) = 2t$, $S_2(t) = 2t$ and the base characterization sets for $A_1$ and $A_2$ be $\{g_1(t) = 2t\}$, respectively $\{g_2(t) = 2t\}$. We simply obtain the expressions for the corresponding envelopes of $A_1$ and $A_2$ as $f_1(t) = 2t+10$, respectively $f_2(t) = 2t+10$. We obtain that the worst case delays $d_1$ and $d_2$ that can be guaranteed for the connections $A_1$, respectively $A_2$, are $d_1 = 5$ and $d_2 = 5$, respectively. Observe now that the condition $MPX$ holds. Hence, the multiplexed input $A_1 + A_2$ is guaranteed the service curve $S(t) = 4t$ $(\forall)$ $t \ge 0$. On the other hand, the envelope for the multiplexed input $A_1 + A_2$ is $\tilde{f}(t) = 4t + 16$. Now, since $\tilde{f}(t) \le S(t+4)$ $(\forall)$ $t \ge 0$, a worst case delay of 4 for packets from both $A_1$ and $A_2$ can be guaranteed!

## V. Multiplexing gain under time-averaged delay

For various network traffic, such as data transfer, one may request the required QoS parameter to be the time-averaged delay, instead of the worst case delay. Motivated by this, we extend the results from the previous section to time-averaged deterministic multiplexing gain.

Let us denote the average arrival rate of the process $A$ by $\lambda = \lim_{t\to\infty} \frac{1}{t} A(t)$,and the average delay by $d^f_{avg} = \lim_{t\to\infty} \frac{1}{t} \int_0^t d_f(s) ds$. One can show that

$$d^f_{avg} \le \lim_{t\to\infty} \frac{1}{t} \int_0^t (f(s) - (A * S)(s)) ds \tag{5}$$

Let us define $H(f, A, S) = \lim_{t\to\infty} \frac{1}{t} \int_0^t (f(s) - (A*S)(s)) ds$. A simple analysis of (5) shows that if $\int_0^\infty (f(t) - (A * S)(t)) dt < \infty$, then $\lambda d^f_{avg} = 0$; this is the intuitive case when the arrival envelope and the output curve tend to each other asymptotically. If this is not the case, then we obtain that $\lambda d^f_{avg} \le \lim_{t\to\infty} (f(t) - (A * S)(t))$.

Let us consider now two input connections $A_1$ and $A_2$ with average arrival rates of $\lambda_1$ and $\lambda_2$, respectively. Also, assume that $A_i \prec f_i$ and that the service curve $S_i$ are given for $i = 1, 2$. As in the previous section, the condition $MPX_{\{1,2\}}$ must hold, while $A$ represents the multiplexed connection $A_1 + A_2$. Let $\tilde{f}$ be the envelope of $A$ constructed as in Section III and let $S := S_1 + S_2$ be the service curve for $A$. Assume that $f_i$ belongs to the $(\sigma, \rho)$-envelope model.

*Proposition 5:* In the current context, we have that $\frac{H(\tilde{f}, A, S)}{\lambda_1 + \lambda_2} \le \max\left(\frac{H(f_1, A_1, S_1)}{\lambda_1}, \frac{H(f_2, A_2, S_2)}{\lambda_2}\right)$.

The proof is omitted. Note that a similar result may be obtained if $f_i$ belong to the $(\overrightarrow{\sigma}, \overrightarrow{\rho})_n$-envelope model. The consequence of the last result is exactly that of Proposition 4. That is, by aggregating two connections, one likely obtains a smaller time-averaged delay than the smallest between the time-averaged delays of the two flows, if those were obtained without aggregation.

## VI. Experimental results

In this section, we consider several trace files for MPEG-4 encoded videos with a duration of 1 hour and a rate of 25 frames/sec (see [10]). The time unit in our analysis is $\tau = 40$ms, and the data unit is 1 byte.

We conducted the experiments as follows. First, we took $n$ traces $A_i$ and computed the empirical envelopes $\varepsilon_{A_i}$. Second, for a given $n$, we built a $(\overrightarrow{\sigma_i}, \overrightarrow{\rho_i})_n$ envelope for $A_i$, denoted by $f_i$, as discussed in Section III. Third, we assigned various service curves $S_i(t) = c_i t$ for the connections $A_i$ and set the server capacity to be $c = \sum c_i$. Then, we computed the delays $d_i$ that could be guaranteed by the server for each $A_i$. Finally, we aggregated all the connections $A_i$ into one connection $A$ and computed the $(\overrightarrow{\sigma}, \overrightarrow{\rho})_n$ envelope $\tilde{f}$ of $A$. Note that the condition $MPX$ holds. We then computed the guaranteed delay $d$ for $A$ and compared it with the guaranteed delays $d_i$.

Recall Fig. 1 which shows the corresponding empirical envelopes and the $(\sigma, \rho)$ envelopes for two video traces. Fig. 2 presents an experiment involving the results obtained for three trace files.

| Trace file | Avg. rate | # leaky buckets | S(·) | Worst case delay | Time-averaged delay |
|---|---|---|---|---|---|
| Silence of the Lambs Medium quality (SLM) | 876 | 10 | 1500*t | 4672 | 7720 |
| Jurassic Park I Medium quality (JPM) | 1339 | 10 | 2500*t | 2425 | 1352 |
| Soccer High Quality (SOH) | 5533 | 10 | 7000*t | 3890 | 1139 |
| SLM+JPM+SOH | 7792 | 10 | 11000*t | 1190 | 849 |

Fig. 2. Three trace files

Observe that the last row represents the actual multiplexed stream. The most notable fact is that the bound for the worst case delay under the multiplexed analysis dramatically improves upon the bound for the worst case delay for the independent connections. The same is true for the bound for the time-averaged delay. An interesting observation is that the time-averaged delay for the first trace file is larger than the worst case delay for the same trace, which indicates that the bound (5) may be very loose. We observed that this fact stems from the backlog bound, used in the derivation of (5), which may be loose. Another observation to be specified here is that the heuristic $H^{MG}$ presented in Section IV generated the partition $\{\{SLM, JPM, SOH\}\}$.

The same conclusion can be drawn from Fig. 3, with the observation that no gain is obtained with respect to the time-averaged delay. Note that the heuristic $H^{MG}$ generated, as expected, the partition $\{\{SLM, MBM, DHH, VVM\}\}$.

We also point out that for some sets of streams we analyzed, less dramatic multiplexed gains than shown in these examples have been observed. This was the case for the streams that lacked a sufficient degree of burstiness such as "Silence of the Lambs" (see Fig. 1). Moreover, no multiplexing gain occurs when identical streams are multiplexed together. Since the bursts for two identical streams occur at the same moments of time, the envelope $\tilde{f}$ for the multiplexed stream does not differ from the sum $f$, of the envelopes corresponding to the considered streams.

| Trace file | Avg. rate | # leaky buckets | S(·) | Worst case delay | Time-averaged delay |
|---|---|---|---|---|---|
| Silence of the Lambs Medium quality (SLM) | 876 | 15 | 1500*t | 4632 | 6924 |
| Mister Bean Medium quality (MBM) | 920 | 15 | 2000*t | 5133 | 2647 |
| Die Hard III High quality (DHH) | 3487 | 15 | 5000*t | 10486 | 2362 |
| VIVA TV Music Medium quality (VVM) | 1738 | 15 | 3000*t | 4088 | 2244 |
| SLM+MBM+DHH+VVM | 7021 | 15 | 11500*t | 2853 | 2658 |

Fig. 3. Four trace files

## VII. Conclusions

We have shown that multiplexing gain can be achieved in a deterministic network framework which involves traffic envelope functions, service curves, and the SCED scheduling algorithm. Our analysis is based on min-plus algebra. In particular, multiplexing gain arises when projections are done after multiplexing. In addition to worst-case delay as a deterministic QoS parameter, we showed analytically that multiplexing gain can also be achieved with respect to time-averaged delay. We point out that the results for deterministic multiplexing gain obtained in this paper apply directly to the end-to-end case via the end-to-end service curve convolution results from network calculus theory.

Using MPEG video traces, we presented numerical results demonstrating that multiplexing gains could be achieved under the worst-case delay criteria. We found that the (averaged) delay bounds obtained for the multiplexed streams were significantly smaller than those obtained for the individual streams when no multiplexing was done.

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