Predicates and Propositions

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CS252.HACD: Fundamentals of Relational Databases
Section 3: Predicates and Propositions

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Recommended Book

Highly recommended if you like this sort of thing (but definitely not a course requirement):

Wilfrid Hodges: **"Logic"**.

Pelican books, 1977. ISBN: 0 14 02.1985 4

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What Is A Predicate?

Strictly, the *meaning* of a certain kind of **sentence**, but often used (conveniently) to refer to the sentence itself.

Example: " Student S1 is enrolled on course C1."

Note that the meaning is language-independent; the sentence is not!

Note also that we get a very similar sentence, with very similar meaning, if we just change either of the designators, S1 and C1 (e.g., replace C1 by C2).

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What Kind of Sentence?

A sentence having the grammatical form of a *statement* — something that can be *believed,* or *not believed.*

In English, if "Is it true that *x*?" is a grammatical English *question*, then *x* is a statement (having the form of a *declarative* sentence).

Might need to paraphrase *x*. E.g. (from Shakespeare):
"O for a muse of fire" ≡ "I wish for a muse of fire".
"To be or not to be, that is the question" ≡
"The question is whether to be or not to be"

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Some Counterexamples

Sentences that are not declarative:

- "Let's all get drunk."
- "Will you marry me?"
- "Please pass me the salt."
- "If music be the food of love, play on."

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Some Examples

Sentences that *are* declarative (and so *denote* predicates):

- "Student S1 is enrolled on course C1."
- "I will marry you."
- "The king of France is bald"
- "2 + 2 = 5"
- "*x* ≤ *y*"
- "*a* + *b* = *c*"
- "Student *s* is enrolled on course *c*"
- "*P*(*x*)" (notation for the general form)

Propositions

Parameters

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Deriving Predicates from Predicates (1)

Substitution: of a *designator* for a parameter

Given an *n*-adic predicate, yields an (*n*-1)-adic predicate.

E.g., in "$a < b$" substitute 10 for *b* to give "$a < 10$".

Now substitute 5 for *a*, and we get "5 < 10", a proposition.

Instantiation: substitution of *all* the parameters, yielding a proposition.

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Intension and Extension

Of a predicate:

Intension: its meaning (loosely speaking).

Extension: all the instantiations that are (believed to be) *true*.

The concept of extension is crucially important in relational theory. Note that it is a *set* of propositions. Alternatively, it is a single proposition formed by connecting all the members of that set together using "and".

Note in passing that the extension of a niladic predicate is either itself (if it is *true*) or the empty set (if it is *false*).

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Deriving Predicates from Predicates (2)

The familiar *logical* operators:

conjunction: "Student *s* is enrolled on course *c* **and** *s* is called *name*."

disjunction: "$a < b$ **or** $c < d$"

negation: "It is **not** the case that I will marry you."

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Deriving Predicates from Predicates (3)

Conditionals:

implication: "**If** you ask me nicely, **then** I will marry you."

only if: "I will marry you **only if** you ask me nicely."

biconditional: "I will marry you **if and only if** you ask me nicely." **(equivalence)**

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Deriving Predicates from Predicates (4)

Quantifiers:

existential: "There exists *s* such that *s* is a student and *s* is enrolled on course *c*." (≡ "At least one student is enrolled on course *c*.")

universal: "For all *s*, if *s* is a student then *s* is enrolled on course *c*." (≡ "All the students are enrolled on course *c*.")

Quantification, like substitution, *binds* a parameter.

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Sets

Let $P(x)$ be a predicate. If object *a* is such that $P(a)$ is true, then *a* is said to *satisfy P*. And $P(x)$ is called a *membership predicate* for the *set* consisting of all such objects *a*.

Example: "*x* is an integer such that $1 < x < 4$"

"*x* is an integer such that $1 < x < 4$" is a membership predicate for the set consisting of the elements 2 and 3, denoted by the expression **{ 2, 3 }** (an enumeration).

This set is also denoted by $\{ x : x \in Z$ **and** $1 < x$ **and** $x < 4 \}$.

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The Language of Sets (1)

Let A and B be sets with membership predicates $PA(x)$ and $PB(x)$, respectively. Let a be an element. Then we have the following comparisons:

membership: $a \in A$ (a is a member of A)
containment: $B \subseteq A$ (B is a subset of A)
$A \supseteq B$ (A is a superset of B)
$B \subset A$ (B is a proper subset of A)
$A \supset B$ (A is a proper superset of B)
equality: $A = B$ ($A \subseteq B$ **and** $B \subseteq A$)
disjointness: A and B are *disjoint* (have no members in common)

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The Language of Sets (2)

And the following operations on sets that yield sets:

union: $A \cup B = \{x : x \in A$ **or** $x \in B\}$ (disjunction)
intersection: $A \cap B = \{x : x \in A$ **and** $x \in B\}$ (conjunction)
complement: (of A) = $\{x :$ **not** $x \in A\}$ (negation)
difference: $A - B = \{x : x \in A$ **and not** $x \in B\}$

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EXERCISES

Assume that the membership predicate for the following relation is "Student *StudentId,* named *Name,* is enrolled on course *CourseId.*"

StudentId	Name	CourseId
S1	Anne	C1
S1	Anne	C2
S2	Boris	C1
S3	Cindy	C3
S4	Devinder	C1

(The exercises are in the Notes)

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