Unambiguous DNFs
&
Alon-Saks-Seymour

Balodis, Ben-David, Göös, Jain, Kothari

Takehome message: NEW!

Boolean Function

Hex with draws

Input: $x \in \{0,1\}^{n \times n}$

$$f(x) = \begin{cases} 1, & \text{if vertical 1-path of length} \leq 2n \\ 0, & \text{if horizontal 0-path of length} \leq 2n \\ *, & \text{o/w} \end{cases}$$

Question:

- Easy to certify $f(x)=0$ or $f(x)=1$

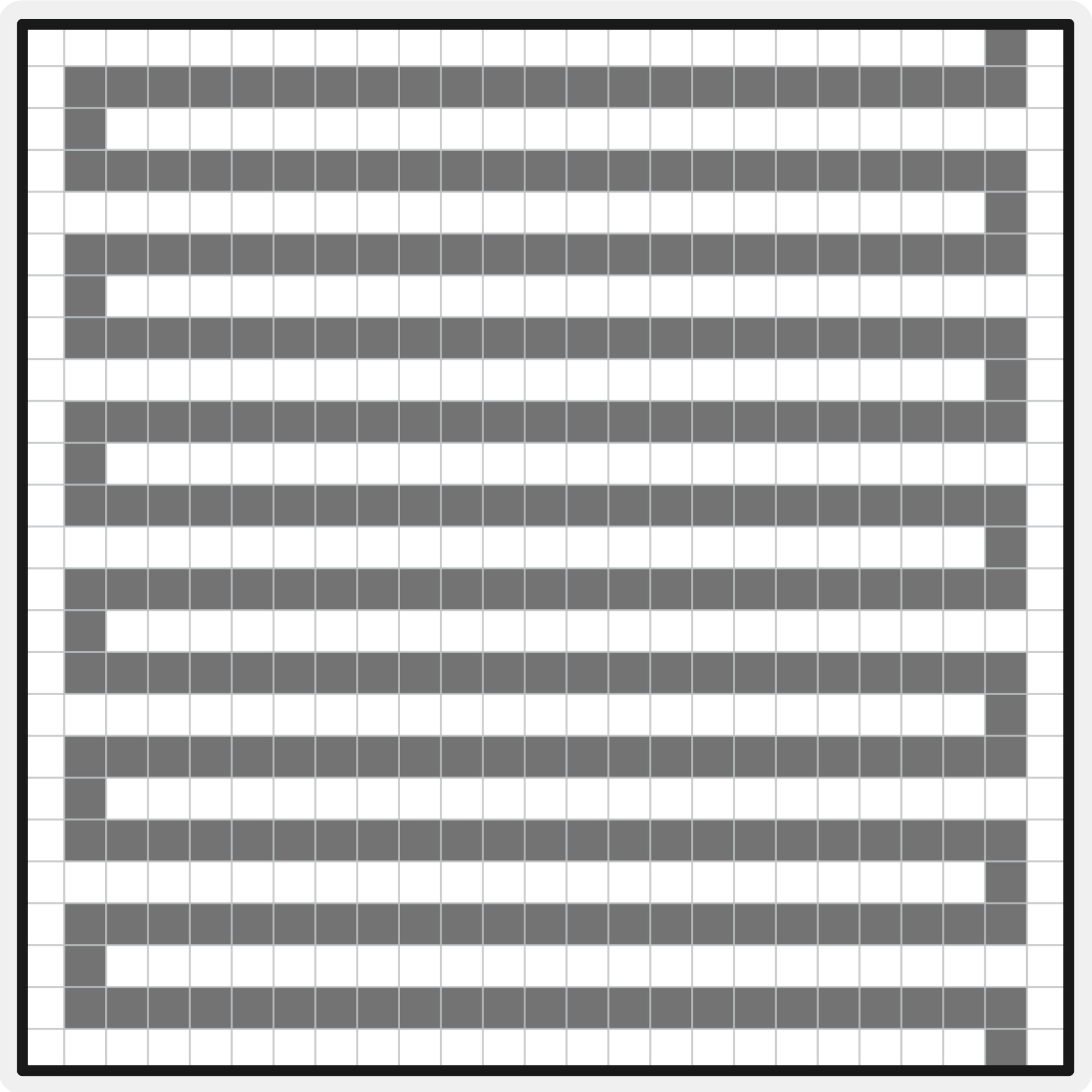
$$C_0(f) \leq 2n$$

$$C_1(f) \leq 2n$$

- Can we show

$$C_*(f) \geq n^{1+\varepsilon} \ ?$$

Mildly hard input x



$$f(x) = *$$

$$C_* = \Omega(n^2)$$

Note:

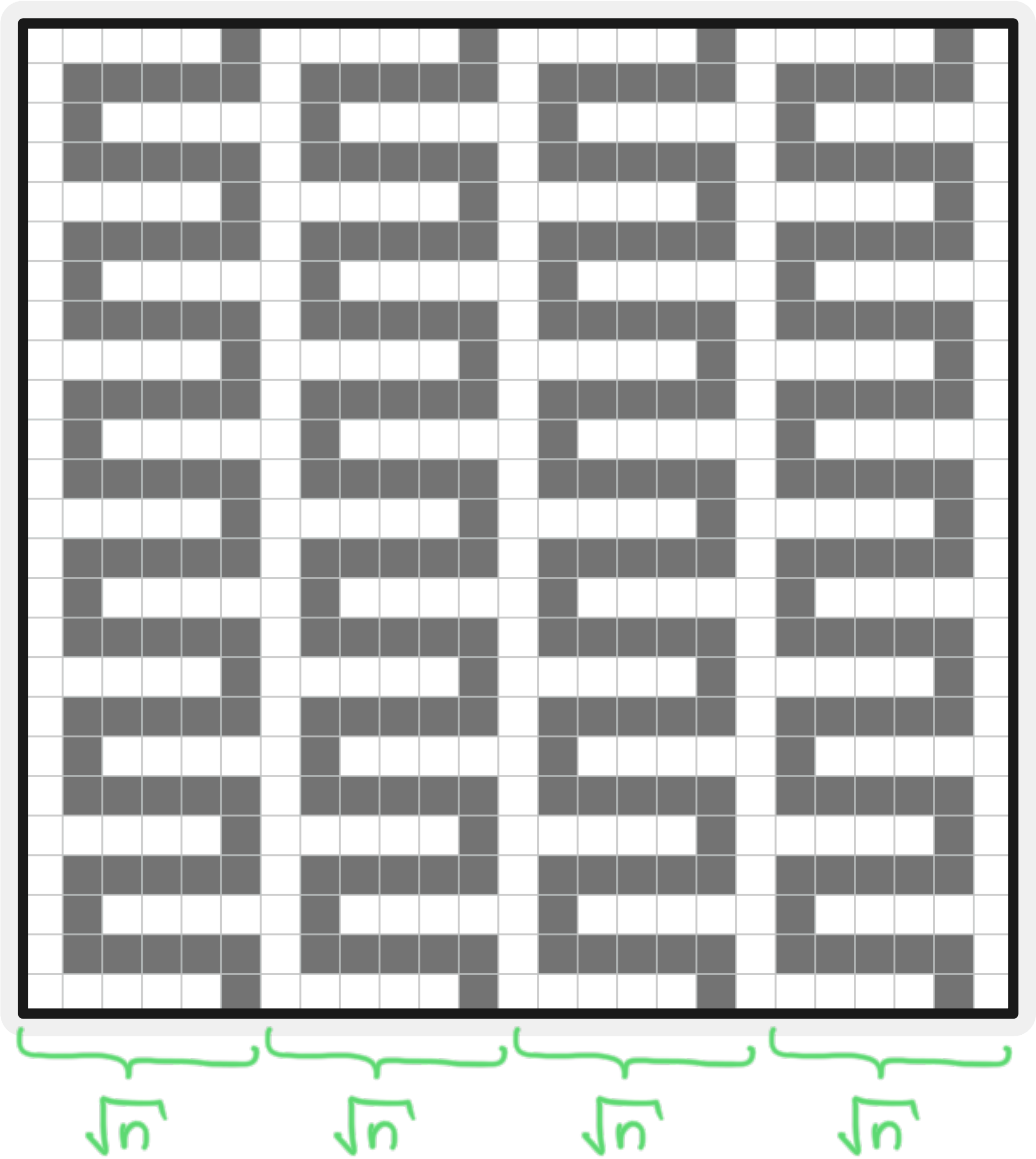
$$C_{\bar{0}} = \Omega(n^2)$$

$$C_{\bar{1}} = O(n)$$

Real Question

$\exists x \in f^{-1}(*)$: both $\begin{cases} C_{\bar{0}}(x) \\ C_{\bar{1}}(x) \end{cases}$ are $\geqslant n^{1+\varepsilon}$?

Solution: $x \in f^{-1}(*)$



Open: Is x extremal?

We claim:

$$C_{\bar{0}} = \Omega(n^{1.5})$$

$$C_{\bar{1}} = \Omega(n^{1.5})$$

What we solved:

Puzzle I: $\exists f: \{0,1\}^n \to \{0,1,*\}$, $x \in f^{-1}(*)$:

$$\min\{C_{\bar{0}}, C_{\bar{1}}\} \geq \max\{C_0, C_1\}^{1.5}$$

$$\Updownarrow$$

Puzzle II: $\exists f: \{0,1\}^n \to \{0,1\}$: $C_0 \geq UC_1^{1.5}$

Previosly:

[G'15] exponent 1.128

[BHT'17] exponent 1.22

arXiv.org > cs > arXiv:2102.08348

arXiv.org > cs > arXiv:2103.05593

arXiv.org > cs > arXiv:2102.08348

Search... All fields Search

Help | Advanced Search

Computer Science > Computational Complexity

[Submitted on 16 Feb 2021 (v1), last revised 5 Jun 2021 (this version, v2)]

Unambiguous DNFs and Alon–Saks–Seymour

Kaspars Balodis, Shalev Ben-David, Mika Göös, Siddhartha Jain, Robin Kothari

We exhibit an unambiguous k-DNF formula that requires CNF width $\tilde{\Omega}(k^2)$, which is optimal up to logarithmic factors. As a consequence, we get a near-optimal solution to the Alon--Saks--Seymour problem in graph theory (posed in 1991), which asks: How large a gap can there be between the chromatic number of a graph and its biclique partition number? Our result is also known to imply several other improved separations in query and communication complexity.

Comments:	v1: 12 pages, 2 figures. v2: Added an author; improved result; 15 pages
Subjects:	**Computational Complexity (cs.CC)**
Cite as:	arXiv:2102.08348 **[cs.CC]**
	(or arXiv:2102.08348v2 **[cs.CC]** for this version)

Submission history

From: Robin Kothari [view email]

[v1] Tue, 16 Feb 2021 18:37:37 UTC (31 KB)

[v2] Sat, 5 Jun 2021 18:32:11 UTC (344 KB)

Download:

- PDF
- Other formats

(license)

Current browse context:
cs.CC

< prev | next >

new | recent | 2102

Change to browse by:
cs

References & Citations

- NASA ADS
- Google Scholar
- Semantic Scholar

DBLP – CS Bibliography

listing | bibtex

Shalev Ben-David
Mika Göös
Siddhartha Jain
Robin Kothari

Export Bibtex Citation

Puzzle II

$\exists$ total f: $C_0(f) \geq UC_1(f)^2$

Can be lifted to communication world

$\Rightarrow$ $\tilde{\Omega}(\log^2 n)$ conondet. of "clique vs ind. set"

[Yannakakis'88]

$\Rightarrow$ $\exists G$: $\chi(G) \geq \exp(\tilde{\Omega}(\log^2 bp(G)))$

[Alon-Saks-Seymour'80s]

biclique packing #

Alon-Saks-Seymour / Clique vs Ind. Set

Reference			$\chi(G)$	CIS_G
Yannakakis	[Yan91]	$\forall G$:		$O(\log^2 n)$
Mubayi and Vishwanathan	[MV09]	$\forall G$:	$\exp(O(\log^2 \mathrm{bp}(G)))$	
Huang and Sudakov	[HS12]	$\exists G$:	$\Omega(\mathrm{bp}(G)^{6/5})$	$\geqslant\ 6/5 \cdot \log n$
Amano	[Ama14]	$\exists G$:		$\geqslant\ 3/2 \cdot \log n$
Shigeta and Amano	[SA15]	$\exists G$:		$\geqslant\ 2 \cdot \log n$
Göös	[Göö15]	$\exists G$:	$\exp(\Omega(\log^{1.12} \mathrm{bp}(G)))$	$\Omega(\log^{1.12} n)$
Ben-David et al.	[BHT17]	$\exists G$:	$\exp(\Omega(\log^{1.22} \mathrm{bp}(G)))$	$\Omega(\log^{1.22} n)$
This work		$\exists G$:	$\exp(\tilde{\Omega}(\log^2 \mathrm{bp}(G)))$	$\tilde{\Omega}(\log^2 n)$

Alon-Saks-Seymour problem

Crazy many reductions!

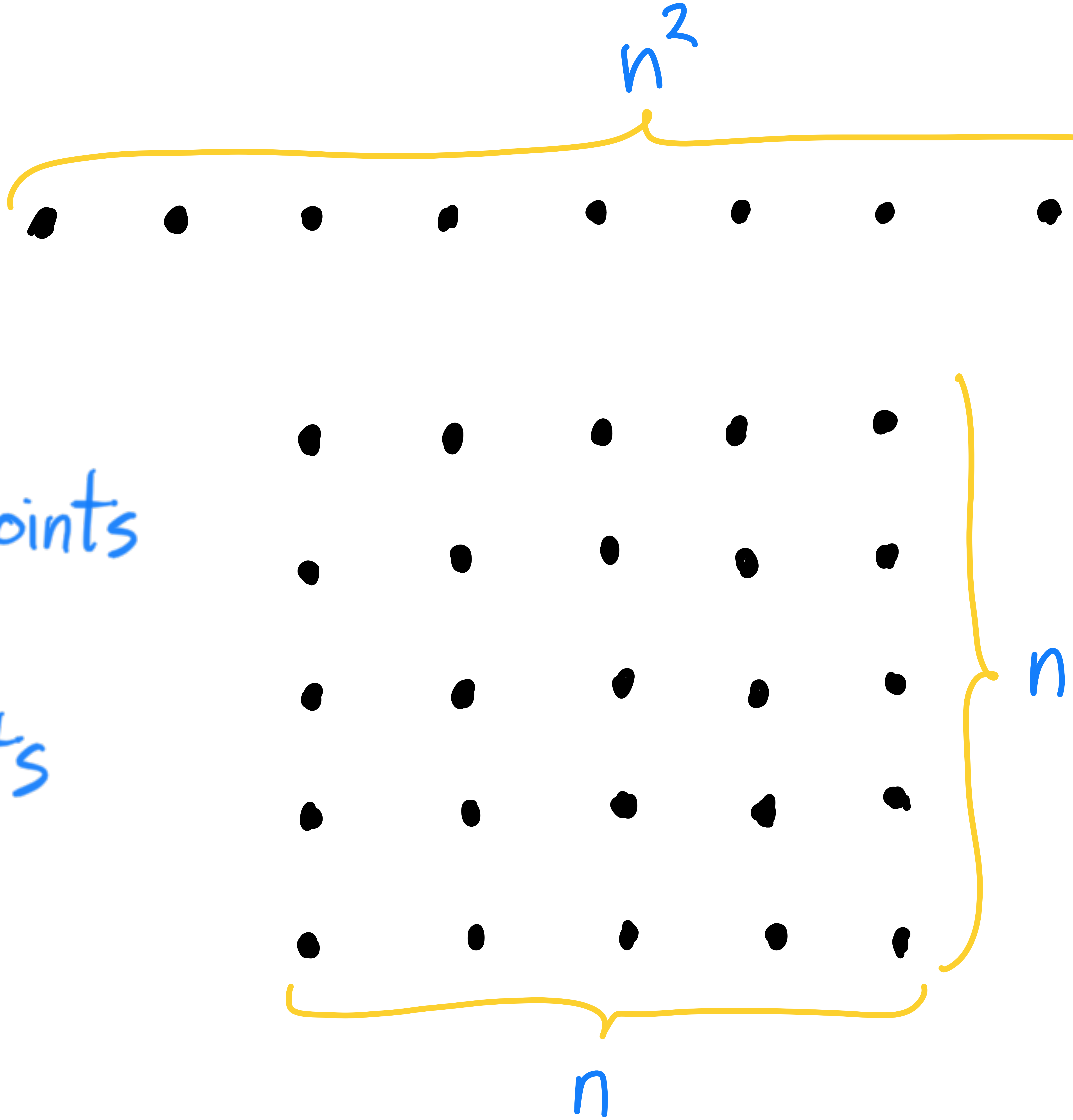
1. Alon–Saks–Seymour problem, reduces to
2. clique vs. independent set problem, reduces to
3. Puzzle I: separation $\mathrm{C}_0 \gg \mathrm{UC}_1$ in query complexity, reduces to
4. Puzzle II: separation $\mathrm{C}_{\overline{0}}, \mathrm{C}_{\overline{1}} \gg \mathrm{C}$ for a partial function, reduces to
5. Puzzle III: 2-colouring an intersecting hypergraph.

Quadratic Gap

$2n^2$ points

=

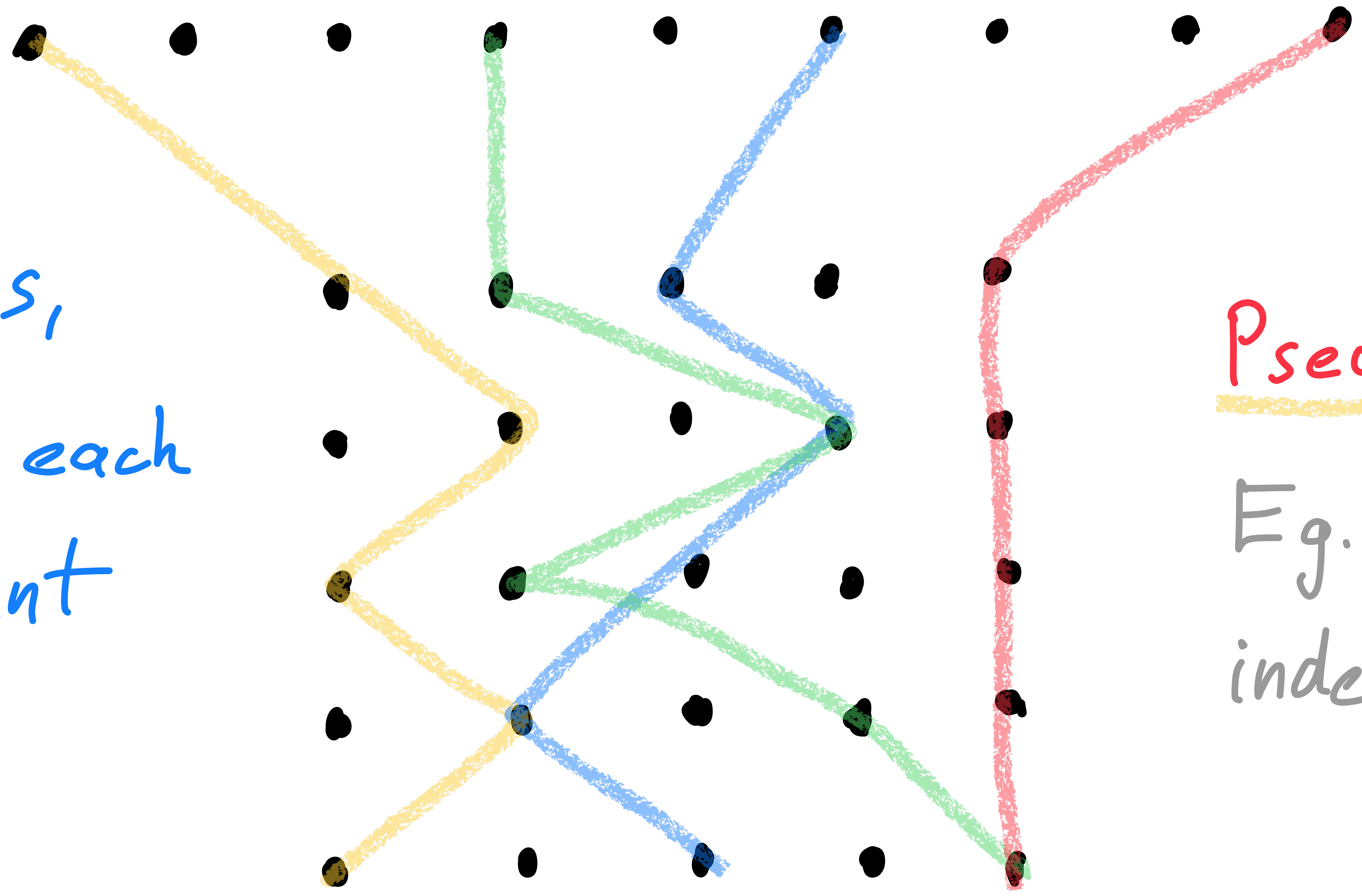
input bits



n^2 paths,

one for each

top point

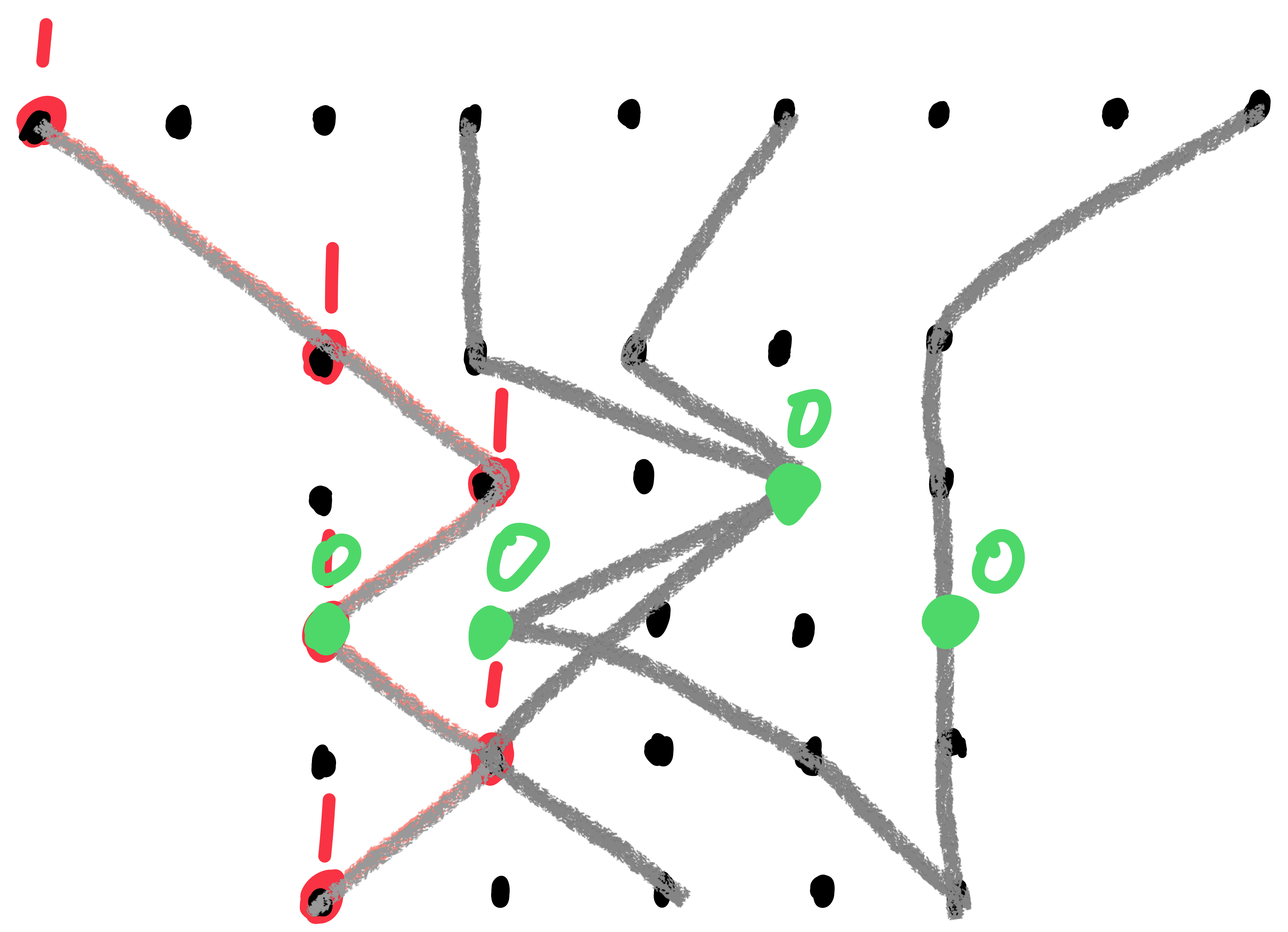


Pseudorandom

Eg. pairwise

independent

$$f(x) = \begin{cases} 1, & \text{if } x \text{ has all-1 path} \\ 0, & \text{if } x \text{ has } \tilde{O}(n)\text{-size } \mathcal{I}\text{-cert.} \\ *, & \text{o/w} \end{cases}$$

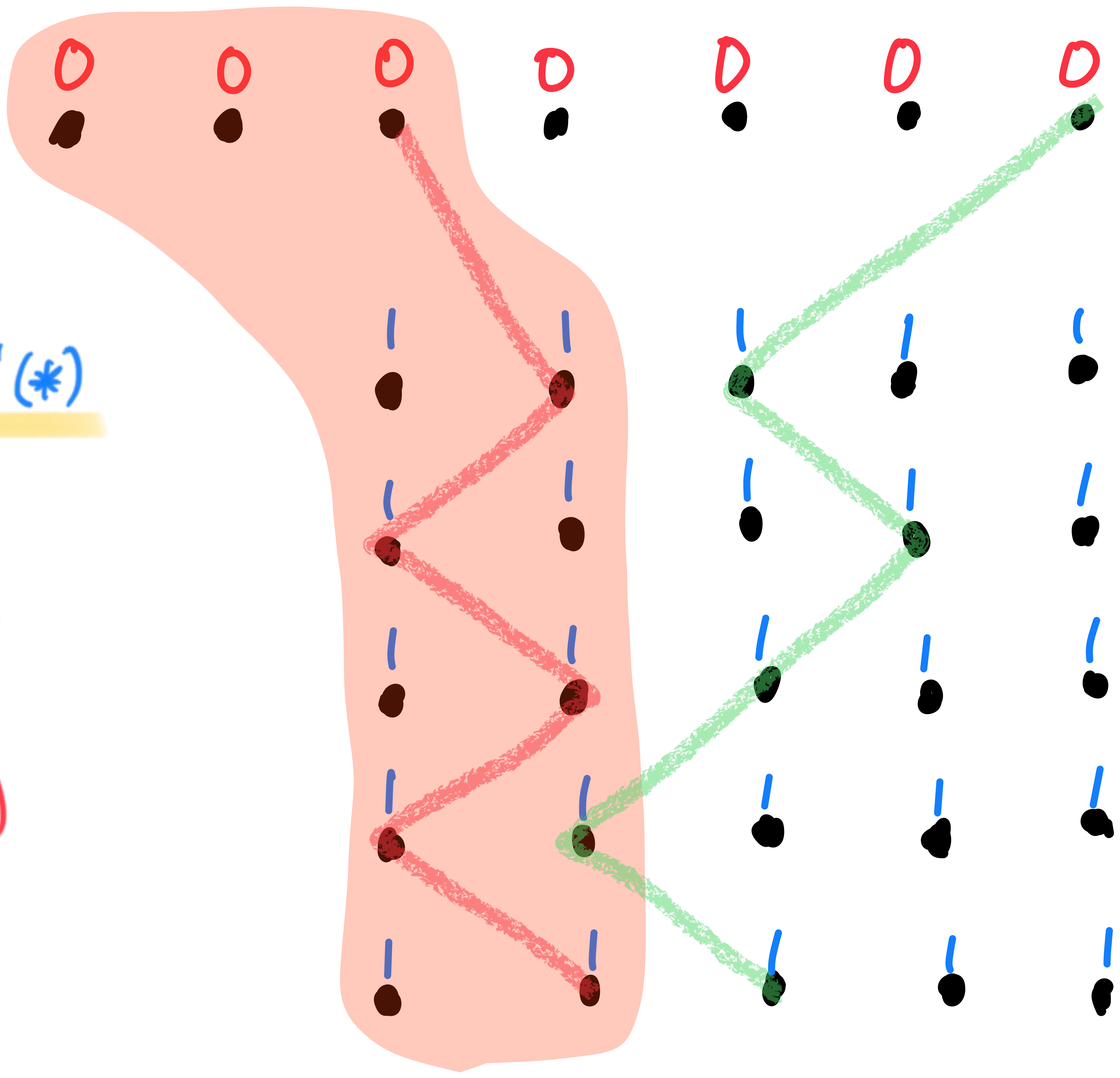


$$C_0, C_1 \leq \tilde{O}(n)$$

Hard input $x \in f^{-1}(*)$

$C_{\mathcal{I}} = \Omega(n^2)$

$C_{\bar{O}} = \Omega(n^2)$



1% read

Two path types

Mostly unread: block with random $\tilde{O}(n)$ – subset

Mostly read block in top row

$\leq n$ many

More separations

$\exists f: \quad C(f) \geq \deg(f)^2$ [NKW'95]: 1.63

$\exists f: \quad C(f) \geq s(f)^3$

$\exists f: \quad C(f) \geq \widetilde{\deg}(f)^3$ (uses Hex!)

Thanks