Consistency of circuit lower bounds with bounded theories

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Talk based on joint work with Jan Bydžovský (Vienna) and Jan Krajíček (Prague).

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Status of circuit lower bounds

- Interested in **unrestricted** (non-uniform) Boolean circuits.

- Proving a lower bound such as $\text{NP} \not\subseteq \text{SIZE}[n^2]$ seems out of reach.
ZPP^{NP} \not\subseteq \text{SIZE}[n^k] \ [\text{Kobler-Watanabe’90s}]

\text{MA/1} \not\subseteq \text{SIZE}[n^k] \ [\text{Santhanam’00s}]

While we have lower bounds for larger classes, there is an important issue:

Frontier 1: Lower bounds for deterministic class P

Frontier 2: Known results only hold on infinitely many input lengths.
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▶ Frontier 1: Lower bounds for \textbf{deterministic} class P^{NP}?
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▶ Frontier 1: Lower bounds for \textit{deterministic} class \text{P}^{NP}?

While we have lower bounds for larger classes, \textit{there is an important issue}:

▶ Frontier 2: Known results only hold on \textit{infinitely many input lengths}. 
Mystery: Existence of mathematical objects of certain sizes making computations easier only around corresponding input lengths.
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Issue not restricted to complexity lower bounds:

Sub-exponential time generation of canonical prime numbers [Oliveira-Santhamam’17].
We discussed two frontiers in complexity theory:

1. Understand relation between $P^{NP}$ and say $\text{SIZE}[n^2]$.

2. Establish almost-everywhere circuit lower bounds.

This work investigates these challenges from the perspective of mathematical logic.
Investigating complexity through logic

- Theories in the standard framework of first-order logic.

- Investigation of complexity results that can be established under certain axioms.

Example: Does theory T prove that SAT can be solved in polynomial time?

- Complexity Theory that considers efficiency and difficulty of proving correctness.
Bounded Arithmetics

- Fragments of Peano Arithmetic (PA).

- Intended model is \( \mathbb{N} \), but numbers can encode binary strings and other objects.
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**Example:** Theory $I\Delta_0$ [Parikh’71].

$I\Delta_0$ employs the language $\mathcal{L}_{PA} = \{0, 1, +, \cdot, <\}$.

14 axioms governing these symbols, such as:

1. $\forall x \ x + 0 = x$
2. $\forall x \forall y \ x + y = y + x$
3. $\forall x \ x = 0 \lor 0 < x$

...
**Induction Axioms.** $I\Delta_0$ also contains the induction principle

$$
\psi(0) \land \forall x (\psi(x) \rightarrow \psi(x + 1)) \rightarrow \forall x \psi(x)
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for each **bounded formula** $\psi(x)$ (additional free variables are allowed in $\psi$).
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A **bounded formula** only contains quantifiers of the form $\forall x \leq t$ and $\exists x \leq t$, where $t$ is a term not containing $x$. 
Bounded formulas and bounded induction

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- Roughly, this shifts the perspective from computability to complexity theory.
Theories PV, $S^1_2$, and $T^1_2$

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Ex.: $T^1_2$ uses induction scheme for bounded formulas corresponding to NP-predicates.
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This does not mean that the corresponding theories prove correctness of algorithms:

$T^1_2 \vdash \forall x \ f_{AKS}(x) = 1 \leftrightarrow \text{“x is prime”}$
Theories PV, $S_2^1$, and $T_2^1$

- [Cook’75] and [Buss’86] introduced theories more closely related to levels of PH:

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\[
\text{PV} \approx T_2^0 \subseteq S_2^1 \subseteq T_2^1 \subseteq S_2^2 \subseteq T_2^2 \subseteq \ldots \subseteq \bigcup_i T_2^i \approx \text{I\textDelta}_0 + \Omega_1
\]
Resources
Many complexity results have been formalized in such theories.

Cook-Levin Theorem in PV [folklore].

PCP Theorem in PV [Pich’15].

Parity $\not\in AC^0$, $k$-Clique $\not\in mSIZE[n^{\sqrt{k}/1000}]$ in $APC^1 \subseteq T^2_2$ [Muller-Pich’19].
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Rest of the talk: Independence of complexity results from bounded arithmetic.
Using $\mathcal{L}_{PV}$, we can refer to circuit complexity:

$$\exists y \ (\text{Ckt}(y) \land \text{Vars}(y) = n \land \text{Size}(y) \leq 5n \land \forall x \ (|x| = n \rightarrow (\text{Eval}(y, x) = 1 \leftrightarrow \text{Parity}(x) = 1)))$$

$n$ is the “feasibility” parameter (formally, the length of another variable $N$).
Unprovability and circuit complexity

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Two directions: unprovability of \textbf{LOWER} bounds and unprovability of \textbf{UPPER} bounds.
Initiated by Razborov in the nineties under a different formalization.

**Motivation:** Why are complexity lower bounds so difficult to prove?

**Also:** potential source of hard tautologies; self-referential arguments and implications.
Unprovability of circuit LOWER bounds

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**Example:** Is it the case that $T_2^2 \nvdash k$-Clique $\not\in$ SIZE[$n^{\sqrt{k}/100}$]?
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Example: Is it the case that $T_2^2 \not\prec k$-Clique $\notin$ SIZE[$n^{\sqrt{k}/100}$]?

Extremely interesting, but not much is known in terms of unconditional unprovability results for strong theories such as PV.
Unprovability of circuit \textbf{UPPER} bounds

► We currently cannot rule out that \( \text{SAT} \in \text{SIZE}[10n] \). Can we at least show that easiness of SAT doesn’t follow from certain axioms?

\textbf{At least as interesting as previous direction:}
Unprovability of circuit UPPER bounds

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1. **Necessary** before proving in the standard sense that \( \text{SAT} \notin \text{SIZE}[10n] \). Rules out algorithmic approaches in a principled way.

2. **Formal evidence** that SAT is computationally hard:
   
   – By Godel’s completeness theorem, there is a model \( M \) of \( T \) where SAT is hard.
   – \( M \) satisfies many known results in algorithms and complexity theory.
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2. \textbf{Formal evidence} that SAT is computationally hard:

   – By Godel’s completeness theorem, there is a model $M$ of $T$ where SAT is hard.
   – $M$ satisfies many known results in algorithms and complexity theory.

3. \textbf{Consistency of lower bounds:} Adding to $T$ axiom stating that SAT is hard will never lead to a contradiction. We can develop a theory where circuit lower bounds exist.
Some works on unprovability of circuit upper bounds

Cook-Krajicek, 2007: “Consequences of the provability of $\text{NP} \subseteq \text{P}/\text{poly}$”.

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Krajicek-Oliveira, 2017: “Unprovability of circuit upper bounds in Cook’s theory PV”.

Established unconditionally that PV does not prove that $\text{P} \subseteq \text{SIZE}[n^k]$. 

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- Bydzovsky-Muller, 2018: “Polynomial time ultrapowers and the consistency of circuit lower bounds”.
  
  Model-theoretic proof of a slightly stronger statement.
Weaknesses of previous results

1. We would like to show unprovability results for theories believed to be stronger than PV.
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2. Infinitely often versus almost everywhere results:

PV might still show that every \( L \in P \) is infinitely often in \( \text{SIZE}[n^k] \).
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2. Infinitely often versus almost everywhere results:

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➤ Recall issue mentioned earlier in the talk:

We lack techniques to show hardness with respect to every large enough input length.
This work

- T_2^1 and weaker theories cannot establish circuit upper bounds of the form n^k for classes contained in P^{NP}.

- Unprovability of infinitely often upper bounds, i.e., models where hardness holds almost everywhere.

- All results are unconditional.
Our main result

**Theorem 1 (Informal):** For each $k \geq 1$,

\[
T_2^1 \nvdash P^{\text{NP}} \subseteq \text{i.o.SIZE}[n^k]
\]

\[
S_2^1 \nvdash \text{NP} \subseteq \text{i.o.SIZE}[n^k]
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\[
\text{PV} \nvdash \text{P} \subseteq \text{i.o.SIZE}[n^k]
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Our main result

**Theorem 1 (Informal):** For each $k \geq 1$,

$$T_2^1 \not\models \text{P}^{NP} \subseteq \text{i.o.SIZE}[n^k]$$

$$S_2^1 \not\models \text{NP} \subseteq \text{i.o.SIZE}[n^k]$$

$$\text{PV} \not\models \text{P} \subseteq \text{i.o.SIZE}[n^k]$$

**Extensions.** True$_1 \overset{\text{def}}{=} \forall \Sigma_1^b(\mathcal{L}_{PV})$-sentences true in $\mathbb{N}$ can be included in first item.

**Example:** $\forall x (\exists y (1 < y < x \land y | x) \leftrightarrow f_{\text{AKS}}(x) = 0)$

$T_2^1 \cup \text{True}_1$ proves that Primes $\in \text{SIZE}[n^c]$ for some $c \in \mathbb{N}$, but not that $\text{P}^{NP} \subseteq \text{i.o.SIZE}[n^k]$. 
A more precise statement

- $\mathcal{L}_{PV}$-formulas $\varphi(x)$ interpreted over $\mathbb{N}$ can define languages in P, NP, etc.

- The sentence $\text{UB}^i_{k, o}(\varphi)$ expresses that the corresponding $n$-bit boolean functions are computed infinitely often by circuits of size $n^k$:

\[
\forall 1^{(\ell)} \exists 1^{(n)} (n \geq \ell) \exists C_n (|C_n| \leq n^k) \forall x (|x| = n), \varphi(x) \equiv (C_n(x) = 1)
\]

**Theorem**

For any of the following pairs of an $\mathcal{L}_{PV}$-theory $T$ and a uniform complexity class $C$:

(a) $T = T_2^1$ and $C = P^{NP}$,

(b) $T = S_2^1$ and $C = NP$,

(c) $T = PV$ and $C = P$,

there is an $\mathcal{L}_{PV}$-formula $\varphi(x)$ defining a language $L \in C$ such that $T$ does not prove the sentence $\text{UB}^i_{k, o}(\varphi)$. 
High-level ideas

Two approaches (forget the “i.o.” condition for now):

\[
T_2^1 \not\subseteq P^{NP} \subseteq \text{i.o.SIZE}[n^k],
\]

\[
S_2^1 \not\subseteq \text{NP} \subseteq \text{i.o.SIZE}[n^k].
\]

Main ingredient is the use of "logical" Karp-Lipton theorems.

\[
\text{PV} \not\subseteq P \subseteq \text{i.o.SIZE}[n^k]
\]

Extract from (non-uniform) circuit upper bound proofs a "uniform construction".
Parikh’s Theorem. Let $A(\vec{x}, y)$ be a bounded formula.

If $I \Delta_0 \vdash \forall \vec{x} \exists y A(\vec{x}, y)$ then $I \Delta_0 \vdash \forall \vec{x} \exists y \leq t(\vec{x}) A(\vec{x}, y)$.
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We use similar results to “tame” i.o. upper bounds in bounded arithmetic.

Example: If $T_2^1 \vdash \text{SAT} \in \text{i.o.}\text{SIZE}[n^k]$ then $T_2^1 \vdash \text{SAT} \in \text{SIZE}[n^{k'}]$. 
Parikh’s Theorem. Let $A(\vec{x}, y)$ be a bounded formula.

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Not every language is paddable, and more delicate arguments are needed.
A major question is to establish the unprovability of \( P = NP \):

For a function symbol \( f \in \mathcal{L}_{PV} \), consider the universal sentence

\[
\forall x \forall y \psi_{SAT}(x, y) \rightarrow \psi_{SAT}(x, f(x))
\]

Conjecture. For no function symbol \( f \) in \( \mathcal{L}_{PV} \) theory PV proves the sentence \( \varphi_{P=NP}(f) \).

Reduces to the study of unprovability of circuit lower bounds (Theorem 2 in our work).

Motivates both research directions (unprovability of upper and lower bounds).
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Motivates **both** research directions (**unprovability of upper and lower bounds**).
Thank you
Approach 1: “Logical” Karp-Lipton theorems

A few unconditional circuit lower bounds in complexity theory use KL theorems. For instance, $\text{ZPP}^\text{NP} \not\subseteq \text{SIZE}[n^k]$ can be derived from:

[Kobler-Watanabe’98] If $\text{NP} \subseteq \text{SIZE}[\text{poly}]$ then $\text{PH} \subseteq \text{ZPP}^\text{NP}$.
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- Stronger collapses provide better lower bounds. It is not known how to collapse to $P^{\text{NP}}$. Better KL theorems in fact necessary in this case [Chen-McKay-Murray-Williams’19].
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Better KL theorems in fact necessary in this case [Chen-McKay-Murray-Williams’19].

[Cook-Krajicek’07] If \( \text{NP} \subseteq \text{SIZE}[\text{poly}] \) and this is provable in a theory \( T \in \{ \text{PV}, S^1_2, T^1_2 \} \), then \( \text{PH} \) collapses to a complexity class \( C_T \subseteq \text{P}^\text{NP} \).
Approach 2: A “bridge” between uniform and non-uniform circuits

If \( PV \vdash P \subseteq \text{SIZE}[n^k] \), try to extract from PV-proof a “uniform” circuit family for each \( L \in P \).

This would contradict known separation \( P \not\subseteq P\text{-uniform-SIZE}[n^k] \) [Santhanam-Williams’13].

Complications appear because Santhanam-Williams doesn’t provide a.e. lower bounds.
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- Theorem 1 (c) strengthens Krajicek-Oliveira to rule out \( PV \vdash P \subseteq \text{i.o.SIZE}[n^k] \).
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Tábor, Czech Republic
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