Non-local games and verifiable delegation of quantum computation

Alex Bredariol Grilo

CWI QuSoft

joint work with Andrea Coladangelo, Stacey Jeffery and Thomas Vidick

Why verifiably delegate quantum computation?

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Why verifiably delegate quantum computation?

- *Superiorita* 😄
- But they are expensive ☹
- Online service ☺
- Can a client be sure that she is experiencing a quantum speedup? 😐

Ideal world

- Goal: Interacrive proof system for BQP where

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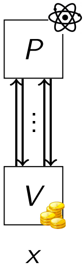

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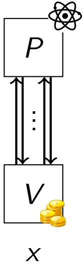

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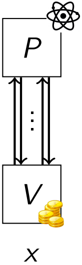

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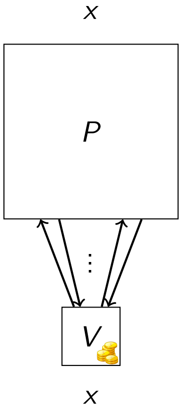
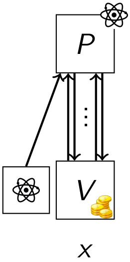
  - the verifier runs poly-time prob. computation
  - an honest prover runs poly-time quantum computation
  - the protocol is sound against any malicious prover
  - additional property: the prover does not learn the input

Relaxed models

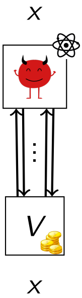
Exponential-size provers



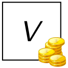
Almost-classical clients



Comput. soundness



Multiple provers



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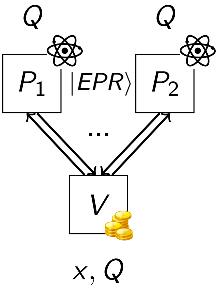

- Multiple entangled non-communicating P

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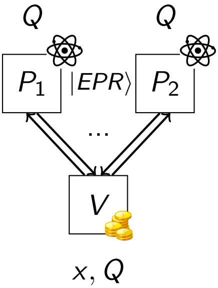

- Multiple entangled non-communicating P
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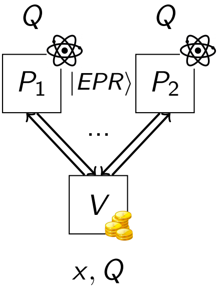

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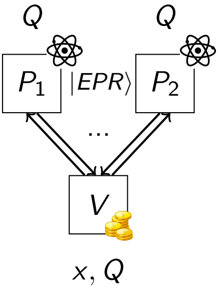

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- “Plug-and-play” ☺

Previous works

	Provers	Rounds	Total Resources	Blind
RUV 2012	2	poly(n)	poly(n)	yes

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GKW 2015	2	poly(n)	$\geq g^{2048}$	yes
HDF 2015	poly(n)	poly(n)	$\Theta(g^4 \log g)$	yes
FH 2015	5	poly(n)	$> g^3$	no
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The results

Delegate circuit Q on n qubits, with g gates and depth d, 2 provers:

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- Verifier-on-a-leash protocol: $O(d)$ rounds, $O(g \log g)$ EPR pairs, blind
- Dogwalker protocol: 2 rounds, $O(g \log g)$ EPR pairs

Comparing to previous works

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VoL	2	O(depth)	$\Theta(g \log g)$	yes
DW	2	2	$\Theta(g \log g)$	no
Relativistic	2	1	g^3	no

Very quick introduction to quantum computation

- 1 qubit
  - Unit vector in $\mathbb{C}^2$
  - Basis: $|0\rangle = \left(\begin{smallmatrix}1\\0\end{smallmatrix}\right)$ and $|1\rangle = \left(\begin{smallmatrix}0\\1\end{smallmatrix}\right)$
  - $|\psi_1\rangle = \alpha|0\rangle + \beta|1\rangle$, $\alpha, \beta \in \mathbb{C}$ and $|\alpha|^2 + |\beta|^2 = 1$

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- $|EPR\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
  - It cannot be written as a product state
  - Source of quantum "spooky actions"
  - For every orthonomal basis $\{|v\rangle, |v^\perp\rangle\}$, $|EPR\rangle = \frac{1}{\sqrt{2}}\left(|vv\rangle + |v^\perp v^\perp\rangle\right)$

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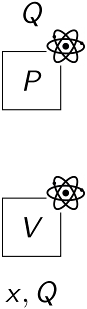
- $|EPR\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
  - If measure the first half, the second half is completely defined (independent of the chosen basis)

From quantum delegation to classical delegation



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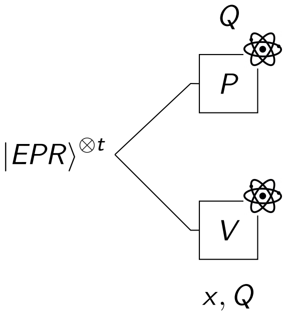

- V and P share EPR pairs

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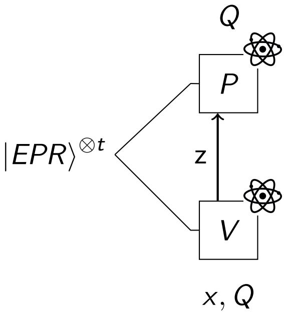

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- V sends $z_i \in_R \{0, 1\}$

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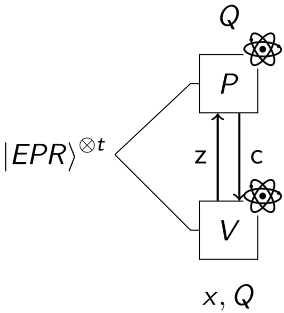

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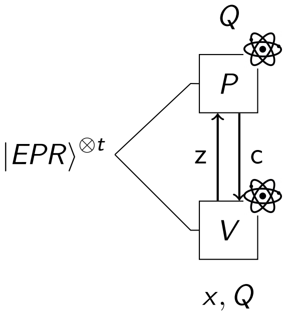

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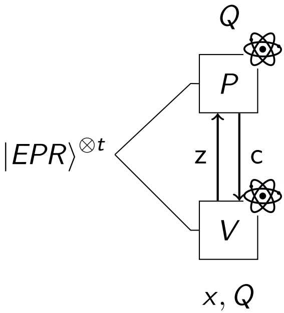

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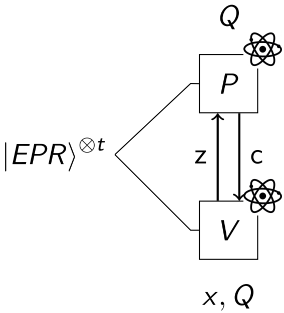

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- V measures half of EPR pairs with Clifford observables
- V performs checks
- If P passes tests, then no "harmful" errors

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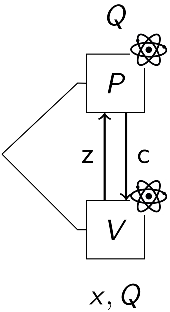

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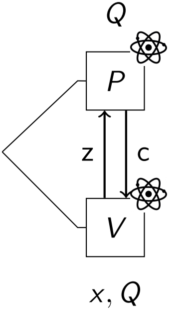

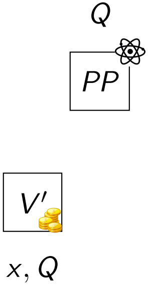

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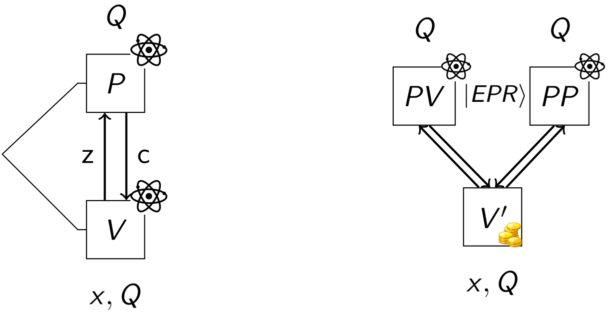

- Idea: Delegate V to a prover

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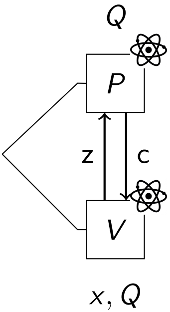

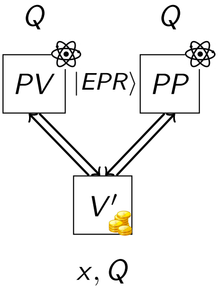

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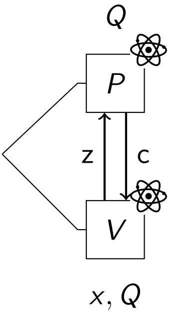

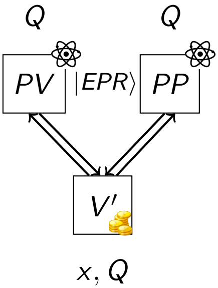

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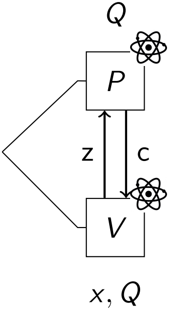

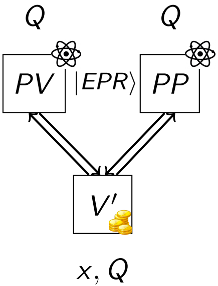

- Idea: Delegate V to a prover
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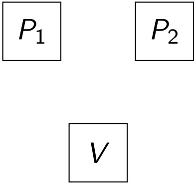
- How to test PV?

Non-local games



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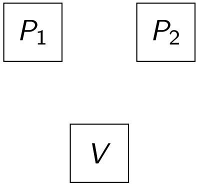

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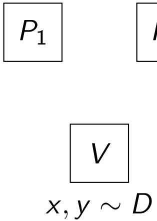

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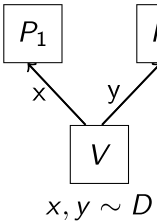

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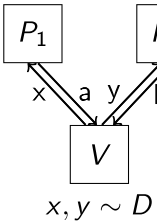

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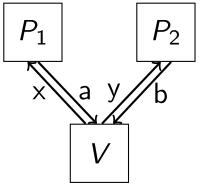

$x, y \sim D$

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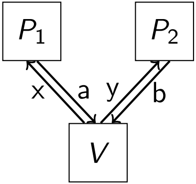

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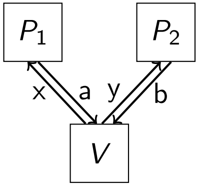

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Bell inequalities and rigidity theorems - Example CHSH

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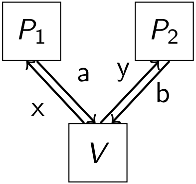

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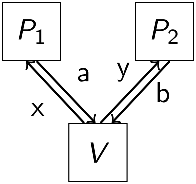

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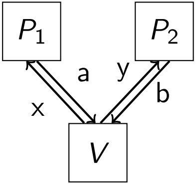

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	0	1
P_1	X	Z
P_2	$\frac{X+Z}{\sqrt{2}}$	$\frac{Z-X}{\sqrt{2}}$

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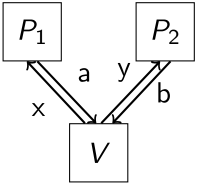

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P_2	$\frac{X+Z}{\sqrt{2}}$	$\frac{Z-X}{\sqrt{2}}$

- Rigidity: if acceptance prob. is $\omega^*(CHSH) - \varepsilon$, then strategy is $O(\sqrt{\varepsilon})$ close to the previous one

Our rigidity results

Our game

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- $\mathcal{G}$ is a set of one-qubit Clifford observables
- Game where a constant fraction of the questions are in a random $\mathcal{G}^m$
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Honest strategy

Share m EPR pairs and on question of the form $W \in \mathcal{G}^m$ the prover measures the "correct" observable W.

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Theorem

The honest strategy succeeds with prob. $1 - e^{-\Omega(m)}$ *in the game.*

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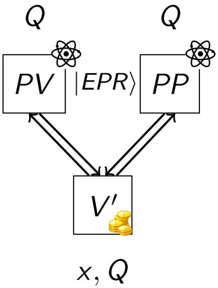
The honest strategy succeeds with prob. $1 - e^{-\Omega(m)}$ in the game.

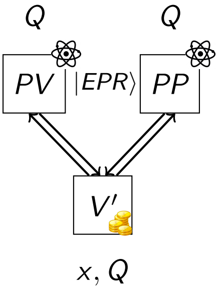
Theorem

For any $\varepsilon > 0$, any strategy for the provers that succeeds with prob. $1 - \varepsilon$ must be $O(\sqrt{\varepsilon})$-close to the honest strategy.

From quantum delegation to classical delegation

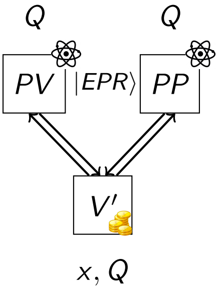


From quantum delegation to classical delegation



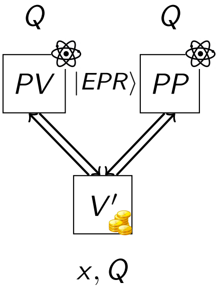
- Protocol

From quantum delegation to classical delegation



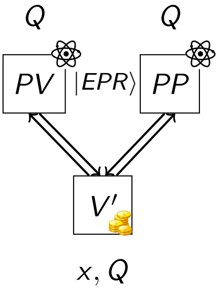
- Protocol
  - With prob. p, play non-local game

From quantum delegation to classical delegation



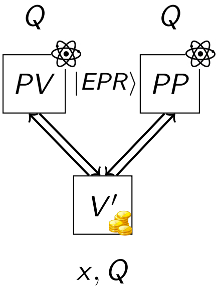
- Protocol
  - With prob. p, play non-local game
  - With prob. $1 - p$, execute original protocol

From quantum delegation to classical delegation



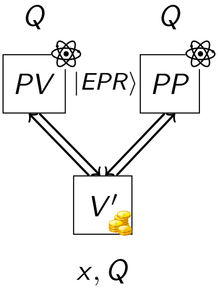
- Protocol
  - With prob. p, play non-local game
  - With prob. $1 - p$, execute original protocol
- Two tests are indistinguishable for PV

From quantum delegation to classical delegation



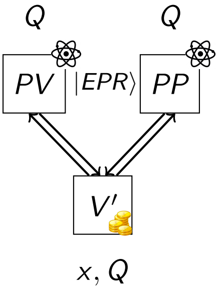
- Protocol
  - With prob. p, play non-local game
  - With prob. $1 - p$, execute original protocol
- Two tests are indistinguishable for PV
- PV is tested with the game

From quantum delegation to classical delegation



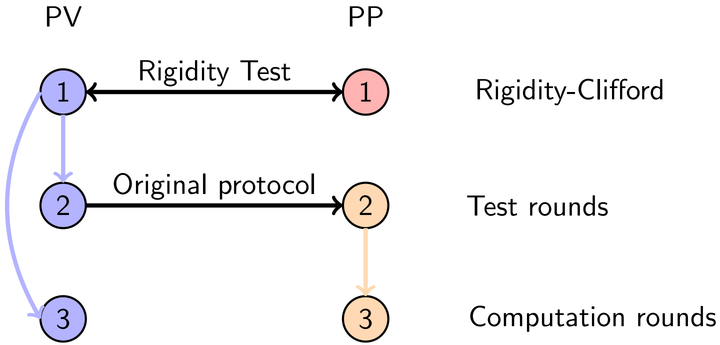
- Protocol
  - With prob. p, play non-local game
  - With prob. $1-p$, execute original protocol
- Two tests are indistinguishable for PV
- PV is tested with the game
- PP is tested in the original protocol

From quantum delegation to classical delegation



- Protocol
  - With prob. p, play non-local game
  - With prob. $1 - p$, execute original protocol
- Two tests are indistinguishable for PV
- PV is tested with the game
- PP is tested in the original protocol
- If both pass the tests, they perform the computation

Verifier-on-a-leash protocol



DogWalker protocol

- In Verifier-on-a-leash protocol

DogWalker protocol

- In Verifier-on-a-leash protocol
  - Rounds of communication for blindness

DogWalker protocol

- In Verifier-on-a-leash protocol
  - Rounds of communication for blindness
- In DogWalker protocol

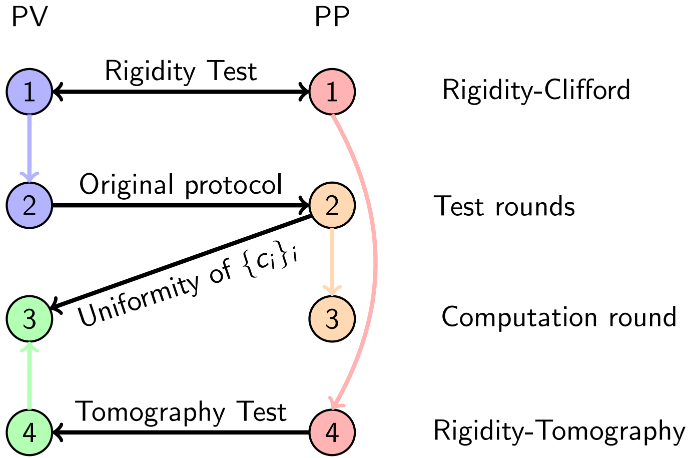
DogWalker protocol

- In Verifier-on-a-leash protocol
  - Rounds of communication for blindness
- In DogWalker protocol
  - Reveal x to PV

DogWalker protocol

- In Verifier-on-a-leash protocol
  - Rounds of communication for blindness
- In DogWalker protocol
  - Reveal x to PV
  - Extra tests to check if PV is honest

DogWalker protocol



Open problems

- More efficient 1-round schemes ($\tilde{O}(g)$ resources)

Open problems

- More efficient 1-round schemes ($\tilde{O}(g)$ resources)
- Blind $O(1)$-round protocols

Open problems

- More efficient 1-round schemes ($\tilde{O}(g)$ resources)
- Blind $O(1)$-round protocols
- Delegation protocol with non-entangled provers

Thank you for your attention!