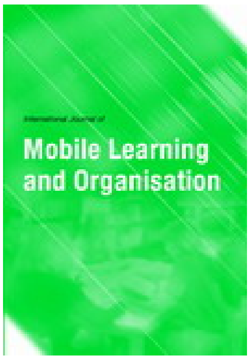

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Integrating problem-based learning and augmented reality for enhancing problem-solving and computational thinking skills

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Abstract: The study investigated the effect of problem-based learning (PBL) with augmented reality (AR) on problem-solving and computational thinking (CT) skills, along with students' satisfaction with AR in mathematics learning. Ninety eighth-grade students participated in a quasi-experiment with three groups: experimental (PBL with AR and standard PBL) and control (conventional learning) groups, each with thirty students. The geometry learning using augmented reality experience (GLARE) application covered 3D geometry, including cubes, cuboids, prisms, pyramids, cones, cylinders, and spheres. MANOVA was used to examine the problem-solving and CT skills, while the satisfaction was evaluated as percentages. The findings demonstrated all groups improved significantly in CT and problem-solving ($p < 0.05$), with the PBL+AR group achieving the highest improvement. Most students found AR engaging, motivating, and effective for understanding complex geometry concepts. They also appreciated its ease of use and support for independent learning. However, some encountered technical problems, such as device incompatibility and poor internet connectivity.

Keywords: augmented reality; computational thinking; problem-based learning; PBL; problem-solving; satisfaction.

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1 Introduction

The development of student problem-solving skills is essential to mathematics learning across all content areas and grades. Students can use any strategy they can devise, draw from anything that they know already, and support their arguments in a way that they feel is convincing as they work to solve problems (Cai and Lester, 2010). Every problem is challenging, and a student's learning performance is recognised when they can solve a new problem that is comparable to a previous one but has an alternative solution (Rézio et al., 2022).

Unfortunately, among the participating countries in Programme for International Student Assessment (PISA) 2018, Indonesian students performed relatively lower compared to their neighbouring countries in mathematics. PISA has developed into a widely used tool for evaluating the effectiveness, equity, and quality of learning outcomes across countries for the past two decades and is a significant factor for educational reform. The PISA test evaluates students' understanding of and skills in reading, mathematics, and science. The result in Table 1 reveals that students in ASEAN countries, except Singapore, achieved a score below the OECD average of 489 in mathematics and 379 for Indonesian students. One of the fundamental mathematics capabilities in the PISA 2018 framework is problem-solving. Problem-solving has always been an essential component of studying mathematics (Charles, 2009) and should be emphasised in mathematics education for K-12 students (Cai and Lester, 2010). Problem-solving is a cognitive skill that combines analytical and creative skills, relying on systematic approaches to address and resolve problems (Hodhod et al., 2014). The important points in problem-solving are objectives and strategies for interacting with the problems, eliciting their thoughts, and encouraging mathematical thinking (Liljedahl et al., 2016).

Table 1 The result of PISA 2018

	Reading	*Mathematics*	*Science*
OECD average	487	489	489
Malaysia	415	440	438
Brunei Darussalam	408	430	431
Thailand	393	419	426
Indonesia	371	379	396
Philippines	340	353	357

Source: Schleicher (2019)

Problem-solving itself problem-based learning (PBL) are extremely similar (Sahid, 2011). PBL is an instructional approach where learning is mainly influenced by problems (Roh, 2003). Since PBL begins with a problem that requires to be solved, students who participate in PBL must develop their problem-solving, creative thinking, and critical thinking skills (Roh, 2003). PBL is an approach that emphasises teaching mathematics through problem-solving activities while also giving students the opportunity to think critically, deliver original ideas, and interact mathematically with peers (Rézio et al., 2022). The student is the focus of the learning and teaching in the student-centred learning environment that PBL is targeting. The learning objective is not to reproduce, remember, and retain information that was passively learned; rather, it is to transfer skills and knowledge through students engaging in group projects and individual study (Tick, 2007). Individual, autonomous self-directed learning allows the student to practice adapting and to consciously choose the learning technique and time frame he or she desires to follow (Abdullah et al., 2010; Roh, 2003; Tick, 2007). As a result, learning becomes more engaging and effective, and students are motivated and self-directed learners (Tick, 2007).

The need for students to possess ‘soft’ skills, such as logic and problem-solving abilities, has arisen out of technological developments and computer science (Sunendar et al., 2020). According to Voskoglou and Buckley (2012), using new technologies strategically significantly impacts how students improve their problem-solving skills. Moreover, traditional teaching methods, such as remote classrooms or chalkboard lectures during the COVID-19 pandemic, have limited visualisation capabilities, which results in difficult concepts in mathematics classes being presented in a highly abstract manner (Schutera et al., 2021). Nevertheless, as secondary school students’ cognitive development moves from concrete to abstract, they learn concepts easily when they are presented in a concrete form (Piaget, 1976). Therefore, it is essential to offer students access to resources and learning activities that will enable them to overcome their mathematical obstacles (Schutera et al., 2021). Better mathematics instruction also will help students enhance their mathematical skills as well as their problem-solving skills (Bataluna et al., 2021). Within this setting, augmented reality (AR) has shown to be an extensively effective technology that improves students' academic performance while engaging them and teaching them about the importance of technology (Ab Halim et al., 2020).

AR refers to technological innovations that dynamically collaborate digital information based on context and real-world surroundings (Ibáñez and Delgado-Kloos, 2018; Sommerauer and Müller, 2014). This technology provides an innovative approach to teaching secondary school students who often struggle with abstract ideas due to their stage of cognitive development (Sirakaya and Kiliç Çakmak, 2018). Since learning at this level strongly relies on visualisation, AR's powerful visual capabilities simplify complex concepts and enhance the overall learning process (Akçayır and Akçayır, 2017). By utilising AR, teachers can assist students more effectively in understanding difficult ideas and procedures, while simultaneously improving the learning environment and experience (Gargrish et al., 2021; Singh et al., 2019). Moreover, AR provides a fun and safe learning environment that blends digital and physical elements, enabling students to practice and develop critical cognitive skills such as critical thinking, problem-solving, and communication (Dunleavy et al., 2009; Hodhod et al., 2014), and computational thinking (CT) skills (Hanid et al., 2022). Beyond improving test scores or attitudes, AR fosters deep learning of content and techniques, making it a valuable addition to classrooms (Cai et al., 2019).

The interactive and immersive nature of AR significantly enhances student motivation and engagement by promoting active participation and exploration, which ultimately improves their problem-solving skills (Voulgari et al., 2024). For example, Guntur and Setyaningrum (2021) found that implementing an AR module (ARM) had a positive impact on students' problem-solving performance. AR-based learning environments encourage students to explore freely, learn through trial and error, and solve ill-defined problems that allow for multiple solutions (Wasko, 2013). These features are particularly beneficial in mathematics education, where AR transforms abstract concepts, such as geometric shapes, into tangible, visual experiences (Cetintav and Yilmaz, 2023). For instance, integrating AR tools to teach the surface areas of geometric shapes has been shown to reduce cognitive effort and improve students' understanding and performance in three-dimensional tasks (Koparan et al., 2023). By bridging the gap between theoretical knowledge and real-world application, AR reinforces mathematical concepts and helps students tackle practical problems effectively (İslim et al., 2024). AR's hands-on, interactive structure further enhances learning by immersing students in experiences that encourage them to explore concepts more deeply (İslim et al., 2024). This hands-on approach not only supports cognitive development but also enhances students' ability to apply theoretical knowledge in real-world scenarios, making problem-solving more intuitive and effective (Voulgari et al., 2024). The engaging nature of AR fosters enthusiasm for subjects like geometry, as students become more motivated to interact with learning materials and invest greater effort in achieving their academic goals (Cetintav and Yilmaz, 2023). As a result, students not only learn problem-solving processes but also develop a deeper understanding of concepts, moving beyond rote memorisation (İslim et al., 2024).

In contrast, traditional instruction often emphasises rote memorisation and procedural knowledge, which can limit students' ability to apply mathematical concepts or CT skills in real-life situations (Angraini et al., 2024). Mathematics and CT are closely connected, as CT frequently involves problem-solving processes like designing algorithms and heuristics (Kong, 2019). CT includes skills like recognising patterns within problem structures, breaking problems into manageable components (decomposition), designing logical algorithms, and generalising principles from multiple examples (Lee and Chan, 2019). These skills are essential for creative and efficient problem-solving (Shute et al., 2017), enabling learners to approach challenges systematically while generating innovative solutions (Lee and Chan, 2019). Furthermore, CT extends beyond programming and computer science, supporting problem-solving in a wide range of real-world contexts (Gardeli and Vosinakis, 2019; Voskoglou and Buckley, 2012). By integrating CT as a fundamental skill across the curriculum, K-12 students can develop abstract, computational, and logical thinking abilities, preparing them to handle complex, open-ended problems (Wing, 2006). Students proficient in CT skills can formulate problems systematically, create logical solutions using step-by-step processes, and evaluate potential outcomes at each stage of problem-solving (Sunendar et al., 2020).

AR offers a promising solution by actively supporting the CT process through engaging with visually interactive approaches that maintain students' motivation (Saraiva et al., 2021). AR's immersive nature enhances the learning process, particularly for students who thrive in active engagement, by creating realistic experiences that captivate their interest (Herpich et al., 2019). By bridging the gap between abstract ideas and real-world applications, AR enables students to explore mathematical concepts intuitively and practically, using simulations to visualise and manipulate complex scenarios that require problem-solving (Cevikbas et al., 2023). Supporting this, Hanid et al. (2022) found significant differences in student levels of CT, visualisation skills, and achievement in geometry topics following instruction using the AR application integrated with CT, as compared to traditional learning methods. The real-time simulation offered by AR fosters consistent practice, helping students improve their CT skills over time (Huang et al., 2023).

In conclusion, integrating PBL with AR offers a dynamic and interactive framework for fostering deep learning, enhancing problem-solving and CT, and bridging the gap between abstract concepts and real-world applications. By promoting active engagement and creative problem-solving, AR transforms PBL into a more effective, motivating, and immersive experience. As the role of technology in education continues to evolve, AR has the potential to revolutionise how students develop CT and problem-solving skills, preparing them for the complex challenges of the future.

2 Literature review

2.1 Problem-based learning

PBL is a teaching strategy that focuses on complex problems with multiple possible solutions and encourages student problem-solving (Rézio et al., 2022). This method assists students to enhance their critical thinking and problem-solving abilities by allowing them to actively participate in real-world scenarios while learning important facts and ideas from their course material (Darhim et al., 2020).

PBL begins with some prerequisite knowledge. Before the PBL stages can begin, the tutor should recognise that the students do not have any prior knowledge of the theory or profession and should do a ‘minilecture’ to fill in any gaps (Tick, 2007). The process of learning starts when a problem arises that requires resolution (Rézio et al., 2022). This problem represents particular phenomena that need explanation (Dolmans et al., 2005) and requires students to gather new knowledge before they can solve it (Rézio et al., 2022). When students attempt to explain the phenomena in the problem, they learn what they already know about the topic but also what they do not know or which questions still need to be investigated (Dolmans et al., 2005). Students also actively create explanatory models based on their existing knowledge, assisting in the processing and understanding of new information (Dolmans and Schmidt, 1996).

The teacher’s role in PBL is to act as a facilitator of the learning process, promote active group participation, and monitor and evaluate the complexity and depth of discussion, learning process, and learning objectives (Tick, 2007). The teacher acts as a cognitive trainer rather than a conventional instructor, serving as a coach who inspires students to become independent, self-directed learners capable of thinking creatively, resulting in more effective training (Tick, 2007).

2.2 Augmented reality

The integration of AR technologies in the field of educational technology is promising due to their immersive properties, capability to deliver knowledge in innovative and engaging ways, and potential to create virtual experiences that can overcome financial or geographical limitations (Dick, 2021). The capacity of AR to immerse students in realistic experiences, giving significance to those who interact more actively, enhances the ability of AR resources to engage students in learning processes and help them develop their visualisation skills (Herpich et al., 2019). According to Cai et al. (2020), AR provides a more engaging learning experience than traditional virtual learning environments and could assist students in comprehending concepts and building them step by step. AR has the potential to improve students’ success in understanding and remembering the content of a subject by increasing engagement and motivation (Mystakidis et al., 2022; Wójcik, 2020). The interactivity and visually appealing design of AR can encourage learning in students and help them develop pleasant educational experiences (Wójcik, 2020).

Most research on AR in mathematics education has used it as a method to present virtual content that is difficult to visualise in the real world through presentations (Cai et al., 2019). An ideal AR learning environment makes it easier for students to understand challenging or complex subject matter, improves topic comprehension, and allows for the 3D representation of invisible and difficult-to-visualised occurrences (Cai et al., 2020). Students may explore different perspectives of a three-dimensional object using an AR that has capabilities for rotating, transforming, and illustrating objects in a real environment in addition to animated representations of both two- and three-dimensional objects (Özçakır and Çakıroğlu, 2022; Wang et al., 2024). For example, through the presentation of real-time events and the demonstration of the practical implementations of these concepts, AR has helped students make the connection between geometric examples from textbooks or classrooms and their environment (Wang et al., 2024). Students were able to manipulate the objects by sliding, flipping, turning, and rotating them to change their perspective and scale (Bujak et al., 2013; Gecu-Parmaksiz and Delialioğlu, 2019). This helped to clarify abstract concepts by linking textbook illustrations to practical, everyday contexts (Wang et al., 2024).

2.3 Problem-solving

The US National Council of Teachers of Mathematics ‘Principles and Standards for School Mathematics’ defined problem-solving as the process of completing a task for which there is no identified method of solution (NCTM, 2000). Students must use their knowledge to come up with a solution; in the process, they frequently learn new mathematical concepts (O’Brien et al., 2011). The fundamental processes of problem-solving phases involve a variety of fundamental thinking processes, such as an individual becoming aware of and understanding a known or defined problem, gathering data required to overcome this problem, searching for solutions, evaluating the suitability of these solutions, selecting the most suitable solution, and evaluating the problem’s solution (Ince, 2018).

The term ‘problem-solving’ in the mathematics context refers to mathematical tasks that have the potential to present an intellectual challenge to enhance students’ understanding and development of mathematics (Cai and Lester, 2010). The common problem-solving process developed by Pólya emphasises the importance of prior experiences and knowledge and communicates these concepts in a highly concise, understandable, and teachable manner (Liljedahl et al., 2016). According to Pólya, students learn how to solve problems through both direct problem-solving experiences and observation and imitation of how teachers or other people solve problems. In this process, he suggests that teachers use questioning to guide their students through all stages, while students themselves should develop and actively explore questions as a means to discover and organise resources and strategies for problem-solving (Santos-Trigo, 2020). Palanisamy and Nor (2021) highlighted four steps in Pólya’s mathematics problem-solving process as follows:

1. Understanding the problem involves determining what is known and what is being asked, assessing whether the information provided is sufficient, and finding any requirements that make it simpler to solve.
2. Making a solution plan involves identifying or recalling previously solved issues with similar properties, searching for patterns or rules, and developing resolution procedures.
3. Solving the problem based on the plan involves implementing the procedures developed in the previous step.
4. Concluding involves examining and evaluating if the procedures and results that were used were accurate, determining whether other procedures may be more effective, identifying whether the procedures that were created can solve similar problems, and considering whether the procedures could be generalised.

2.4 Computational thinking

CT is essentially a way of thinking that is practiced across all subject areas and fields rather than only for computer science or technological procedures (Alyahya and Alotaibi, 2019). CT is defined as a cognitive ability that enables the formulation and expression of problem-solving ideas in a format similar to that which a computer is capable of (Wing, 2014). Rodríguez del Rey et al. (2021) noted that CT refers to problem-solving using fundamental concepts, strategies, and the creation of computer science programs and algorithms that can contribute to the development of skills like creativity, problem-solving, abstract thinking, recursion, iteration, collaborative methods, and patterns, among others. CT as a skill includes understanding a problem, defining it, and providing solutions, as well as reasoning with certain abstractions, comprehending automation, and implementing it (Lee et al., 2011). With CT, a problem that initially appears to be difficult is reformulated possibly through reduction, embedding, transformation, or simulation (Wing, 2006).

Angeli et al. (2016) highlighted an operational definition of CT was created by the Computer Science Teacher Association (CSTA) and International Society for Technology in Education (ISTE) in partnership with experts from higher education, industry, and K-12 education as a problem-solving process that includes but is not limited to, the following elements:

a formulating problems that can be solved with the help of computers and other tools

b logically organising and analysing data

c representing data through abstractions, such as models and simulations

d automating solutions through algorithmic thinking (i.e., a series of ordered steps)

e identifying, analysing, and implementing potential solutions to find the most effective and efficient combination of steps and resources.

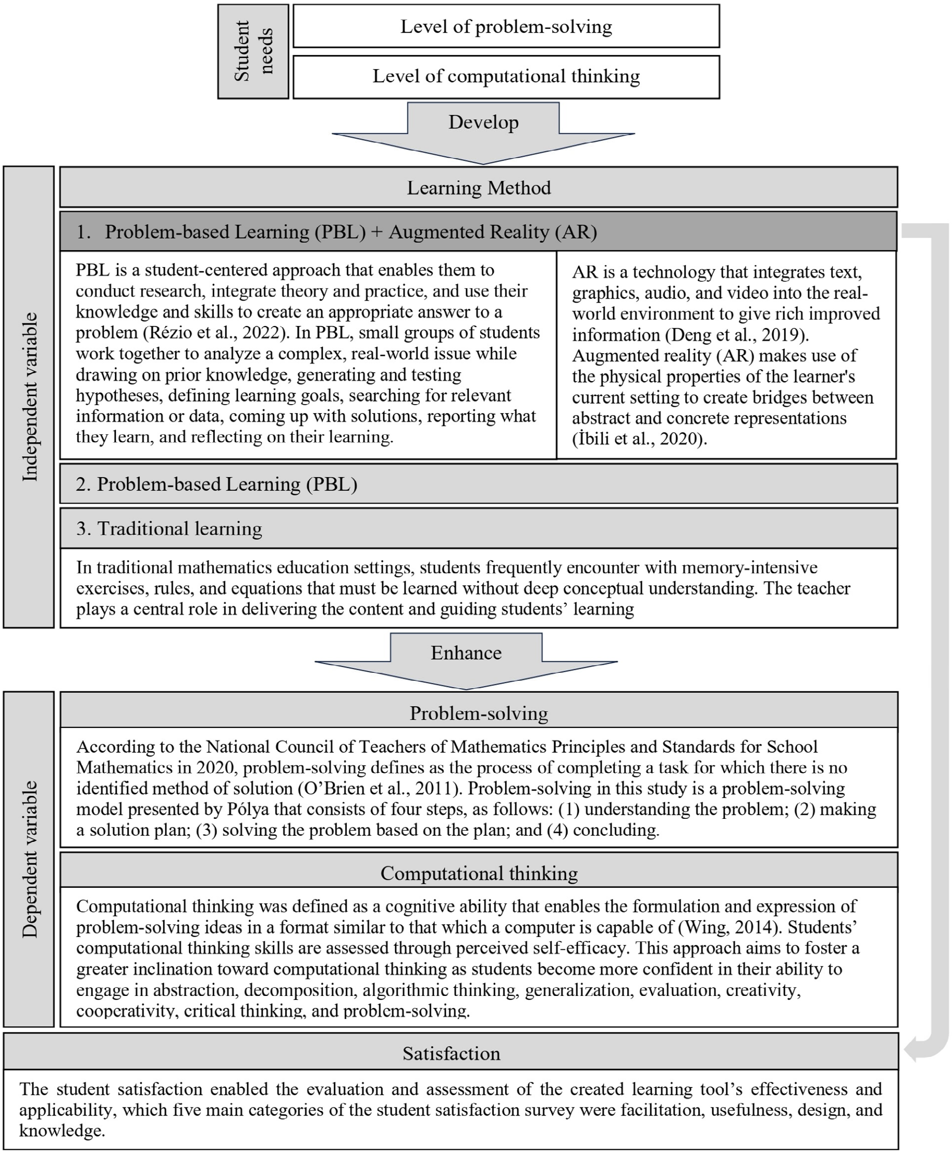
Additionally, Angeli et al. (2016) stated briefly that the framework's goal is to get students thinking and solving problems by having them come up with a solution, automate it through algorithmic thinking, and then generalise it to other problems when similar patterns are found or recognised.

CT is easiest to comprehend as an umbrella term that refers to a certain set of related cognitive skills required for computational tasks and activities (Doleck et al., 2017). CT skills are frequently illustrated by the following: abstraction, generalisation, decomposition, algorithmic thinking, debugging, cooperativity, creativity, critical thinking, and problem-solving (Alyahya and Alotaibi, 2019; Angeli et al., 2016; Doleck et al., 2017; Korkmaz et al., 2017; Wing, 2006).

3 Objective of the study

The integration of AR into PBL was driven by the need to enhance students' comprehension of complex and abstract concepts, particularly in subjects like geometry. Traditional teaching approaches often rely on static images or textbook diagrams, which can hinder students' ability to mentally visualise and manipulate 3D objects. This limitation frequently results in disengagement and a lack of conceptual clarity. By incorporating AR, educators aimed to bridge this gap with an interactive and immersive tool that transforms abstract geometric concepts into more accessible and engaging learning experiences. Research supports this approach, highlighting AR's role in presenting virtual content that is otherwise difficult to conceptualise in the physical world (Cai et al., 2019). For instance, AR has been found to help students visualise abstract geometric concepts from textbooks, such as points, lines, angles, and shapes, which are often challenging to grasp through traditional methods (Pahmi et al., 2023; Wang et al., 2024). AR-enabled devices, when paired with markers or other targets, allow students to view dynamic 3D representations of these concepts in real-time, offering a low-cost yet effective solution to improve spatial understanding (Ibili et al., 2019; İslim et al., 2024). Through the AR application, students can actively manipulate 3D objects by rotating, zooming, and exploring them from multiple perspectives, fostering a stronger connection between theoretical knowledge and visual representation.

Figure 1 Conceptual framework



PBL is a student-centered approach that enables them to conduct research, integrate theory and practice, and use their knowledge and skills to create an appropriate answer to a problem (Rézio et al., 2022). In PBL, small groups of students work together to analyze a complex, real-world issue while drawing on prior knowledge, generating and testing hypotheses, defining learning goals, searching for relevant information or data, coming up with solutions, reporting what they learn, and reflecting on their learning.

AR is a technology that integrates text, graphics, audio, and video into the real-world environment to give rich improved information (Deng et al., 2019). Augmented reality (AR) makes use of the physical properties of the learner's current setting to create bridges between abstract and concrete representations (İbili et al., 2020).

In traditional mathematics education settings, students frequently encounter with memory-intensive exercises, rules, and equations that must be learned without deep conceptual understanding. The teacher plays a central role in delivering the content and guiding students' learning

According to the National Council of Teachers of Mathematics Principles and Standards for School Mathematics in 2020, problem-solving defines as the process of completing a task for which there is no identified method of solution (O'Brien et al., 2011). Problem-solving in this study is a problem-solving model presented by Pólya that consists of four steps, as follows: (1) understanding the problem; (2) making a solution plan; (3) solving the problem based on the plan; and (4) concluding.

Computational thinking was defined as a cognitive ability that enables the formulation and expression of problem-solving ideas in a format similar to that which a computer is capable of (Wing, 2014). Students' computational thinking skills are assessed through perceived self-efficacy. This approach aims to foster a greater inclination toward computational thinking as students become more confident in their ability to engage in abstraction, decomposition, algorithmic thinking, generalization, evaluation, creativity, cooperativity, critical thinking, and problem-solving.

The student satisfaction enabled the evaluation and assessment of the created learning tool's effectiveness and applicability, which five main categories of the student satisfaction survey were facilitation, usefulness, design, and knowledge.

Source: Authors

Furthermore, studies have highlighted the cognitive and meta-cognitive benefits of AR in mathematical learning, including improvements in conceptual understanding (Auliya and Munasiah, 2020; Cahyono et al., 2020; Gardeli and Vosinakis, 2019; Pujiastuti et al., 2020; Schutera et al., 2021), spatial thinking (Gecu-Parmaksiz and Delialioğlu, 2020; Guntur and Setyaningrum, 2021; Hanid et al., 2022; Velázquez and Méndez, 2021), retention (Gargrish et al., 2021), problem-solving (Guntur and Setyaningrum, 2021), communication (Batubara et al., 2022), and critical and creative thinking (Velázquez and Méndez, 2021). However, despite these advantages, Cevikbas et al. (2023) analysed 59 studies on the advantages of AR in mathematics education and discovered that only around 4% of these studies specifically investigated the impact of AR on the development of CT skills. This revealed a research gap, indicating that the relationship between AR and CT in mathematics education remains insufficiently explored.

The integration of AR aligns seamlessly with PBL principles, where real-world scenarios are used to drive inquiry and exploration (Hmelo-Silver, 2004). AR enhances PBL by enabling students to interact with realistic, immersive visualisations, making abstract concepts more tangible, and fostering hands-on, collaborative learning. This integration has the potential to significantly improve educational outcomes by blending real-world problem-solving with interactive technology. While AR provides dynamic visualisations and experiences, PBL's focus on real-world problem contexts ensures that students develop critical thinking and problem-solving skills (Darhim et al., 2020). Together, these approaches encourage comprehension, participation, and cooperation among learners. By leveraging AR and 3D models, students can develop a deeper understanding of problem states, investigate underlying causes, and enhance their problem-solving and CT skills in mathematics. Moreover, the AR learning experience extends beyond traditional classroom settings, as AR smartphone applications enabled self-directed learning (Iqbal et al., 2022). Similarly, PBL encourages students to become independent, self-directed learners with the ability to think creatively (Tick, 2007).

According to Arici and Yilmaz (2023), combining PBL with AR is more effective than traditional PBL and conventional methods in maintaining academic achievement and improving reflective thinking skills related to problem-solving and decision-making abilities. Aligned with Fidan and Tuncel (2019), who found students who used AR in the PBL process performed significantly better learning performance than students who only engaged in PBL or traditional teacher-led instruction in the classroom. However, the study of PBL and AR combined is still relatively new and has not been thoroughly investigated (Arici and Yilmaz, 2023), particularly at the middle or junior high school level (Fidan and Tuncel, 2019).

Therefore, this study stands out by addressing the combination of AR with the PBL model in teaching mathematics at the secondary school level. The outcomes of this study are anticipated to serve as a valuable point of reference for identifying alternative and innovative strategies in mathematics education, with the aim of enhancing mathematical problem-solving and CT skills among secondary-level students. The purpose of this study was to design a learning environment that integrates PBL with AR for mathematics and to assess its effect on problem-solving and CT skills. This approach considers the importance of enhancing these abilities, as well as the engaging and interactive learning experiences that can be created through the combination of PBL and AR technologies. The conceptual framework is presented in Figure 1. The research questions are as follows:

RQ 1 What is the effect of PBL integrated with AR on students' problem-solving and CT skills?

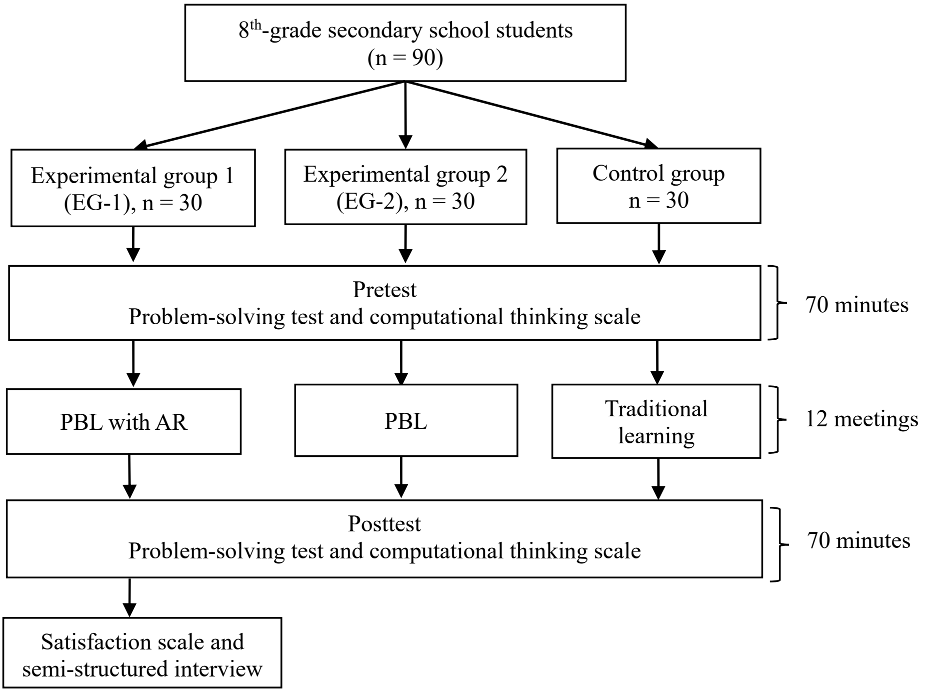
RQ 2 What is the level of student satisfaction with learning using the GLARE application?

4 Methods

4.1 Study design

This study applied a quasi-experimental design, which was used to evaluate the effectiveness of PBL integrated with AR on students' mathematical problem-solving and CT skills. During the research, one experimental group used PBL with AR while the other used PBL, and a third control group used neither. To prevent any unintentional influences, the researcher acted to be a teacher delivering instructions to each group. Meanwhile, the classroom teacher observed the implementation of learning strategies to ensure instructional fidelity. Following the study, several feedback meetings were held to address any deviations and reaffirm the appropriate teaching strategies.

Figure 2 Experimental procedure



Source: Authors

Quantitative data collection involved problem-solving skills test scores and responses from a CT skills questionnaire. Pre- and post-test results were systematically analysed to measure changes in students' problem-solving and CT abilities, using established measurement tools and statistical methods to determine the significance of the findings. This rigorous approach enhanced the reliability and broader applicability of the study's outcomes. Additionally, after completing the learning experiment, students in the PBL + AR group responded to a questionnaire assessing their satisfaction with the AR-enhanced learning experience. To gain deeper insights, they also participated in semi-structured interviews, providing qualitative data on their experiences with the AR application. The research procedure is presented in Figure 2.

4.2 Participants

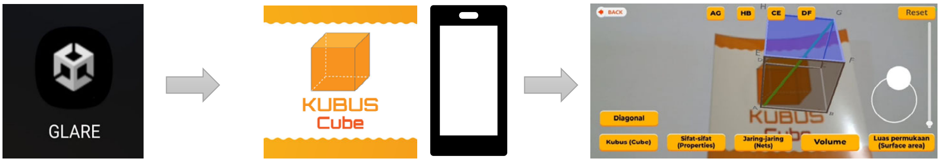
The study used a purposeful sampling method, and its subject was a public secondary school in Indonesia with which the researchers were collaborating. It involved 90 eighth-grade students, who were divided into three groups based on their existing class assignments, as determined by the school. The allocation was non-random to ensure students remained in their regular classroom environments, minimising disruption to their learning routines. The three groups were as follows: PBL + AR (16 men and 14 women), PBL (15 men and 15 women), and the control group (13 men and 17 women). The researcher selected one class out of the three for the PBL + AR group, considering most students there can afford Internet data plans and have smartphones. Furthermore, researchers chose the PBL and control group at random. Then, 15 students from the PBL + AR group volunteered for the interview following the end of the learning experiment.

4.3 Design of PBL activities and AR learning environment

Using PBL as a model, the worksheets were created with pen and paper to facilitate both group (EG-1 and EG-2) work and individual study. To complement these activities, the AR application called geometry learning using augmented reality experience (GLARE) was developed using the Unity 3D platform and Vuforia SDK. This marker-based AR system, compatible with Android devices, was specifically designed to assist students in visualising geometric objects more clearly. The application featured specific interactive buttons, such as rotate, zoom in, and zoom out, allowing students to explore 3D geometry objects (including cube, cuboid, prisms, pyramids, cone, cylinder, and sphere) from different perspectives. To access the GLARE application, a download link for the GLARE '.apk' file was shared with students via WhatsApp. Students in EG-1 installed the application on their smartphones and used special tracker cards called 'markers', to display virtual objects. By pointing their smartphone cameras at the markers, students activated the AR content, which allowed them to view and manipulate the 3D objects in detail by zooming in and rotating them (as illustrated in Figure 3).

The integration of AR into the mathematics learning activity was focused on engaging students with real-world mathematical problems. Each problem was associated with a specific marker card that served as a trigger for the AR content. Once the marker was scanned, an interactive scenario related to the problem was displayed on the screen. These immersive, dynamic scenarios brought abstract mathematical concepts to life, allowing students to interact with them in a meaningful way. Using the interactive features of the AR application, students worked collaboratively to analyse and solve the problems, fostering deeper engagement and comprehension.

Figure 3 Marker-based AR presented the space diagonal of a cube (see online version for colours)



Install the GLARE application

Marker detection by a mobile device's camera.

3D visualisation

Source: Authors

In terms of learning objectives, all PBL activities were the same for EG-1 and EG-2 in this study. In contrast to EG-2, these activities in EG-1 were supported by the GLARE application. The EG-1 students first read the problem scenario and then used the 3D visualisation of the problem provided by the GLARE application. After determining the problem's extent, the EG-1 students gathered the necessary information regarding 3D geometry concepts. The students then discussed their problems with the GLARE application. In opposition to EG-1, pencil and paper activities were used in the PBL process for EG-2. Additionally, they talked about the problems in their groups and tried to determine the best solution.

First, a week before the experiment, the EG-1 students used the GLARE application related to geometric concepts. This application was used as a familiarisation activity to lessen the novelty effect of AR on students. The teacher then explained both the GLARE application and PBL activities to EG-1. However, only PBL actions were informed to EG-2. Students played the role of active learners, with the teacher assisting as a facilitator. The lessons in the experimental process were carried out by combining the phases of technology and PBL, following the strategy described by Fidan and Tuncel (2019), with some of these stages included applications for AR (see Figure 4).

1 Presentation of problem

   In EG-1, students utilised AR applications to engage with the problem. They started by launching the GLARE application using their smartphone cameras focused on marker cards, followed by reading the scenarios displayed on the screen. They then examined the displayed object in detail by zooming in and exploring the problem from different perspectives. Meanwhile, in EG-2, the problem was presented on a worksheet.

2 Definition of the problem

   Each student endeavoured to define the problem clearly through individual analysis and group collaboration. During group projects, methods such as discussion and brainstorming were employed.

3 Determining the (un)knowns

   The students completed the worksheets in the relevant sections by using AR to evaluate the problems and identify their known and unknown components. They then determined the requirements for solving the problem.

4 Data gathering and sharing

Students searched for unfamiliar concepts within the GLARE content, mathematics textbooks, or other sources. Students in EG-1 might also use their smartphones to browse the internet and search for information (refer to Figure 5).

5 Generating solutions

Students engaged in discussions with their group members to explore potential solutions. Each group then selected the best answers and presented them to other groups in the classroom.

6 Reflection and evaluation

Students provided examples from real life that related to the problem. Students in EG-1 used the relevant AR application to examine questions and then write their answers on worksheets. Conversely, EG-2 students only reviewed the questions' solutions using their workbooks.

Figure 4 PBL assisted by AR (see online version for colours)

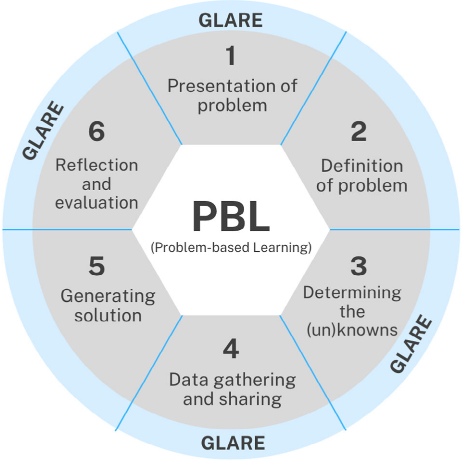


Figure 5 Students were gathering and sharing information with their group members (see online version for colours)

Source: Authors

Meanwhile, the CG was taught through teacher-led methods like presentations and direct instruction, using standard course materials such as slides and textbooks, without any technological tools, focusing on individual tasks that met the same learning goals. Five mathematics professionals and the classroom teacher examined assignments for three groups to ensure they met the same learning objectives.

4.4 Data collection

4.4.1 Problem-solving test

Student problem-solving skills were evaluated using eighth essay questions about 3D geometry topics. The problem-solving skills rubric created by Ahdhianto et al. (2020) was used to evaluate the essay responses. Two teachers independently used the scoring rubric to evaluate the problem-solving skills of 15 eighth-grade students. The results demonstrated an Intraclass Correlation Coefficient (ICC) and Cronbach's alpha coefficient both exceeding 0.9, indicating that the rubric was well-designed with clear indicators that were consistently interpreted by the raters. This consistency reflects the high reliability of the teachers' assessments of students' problem-solving abilities.

Additionally, the questions' content was reviewed by five mathematics experts. The results presented eight questions that could be used to evaluate problem-solving skills. Then, the pilot application of the test was conducted with 32 ninth graders because they had already studied geometry in eighth grade. The characteristics of essay questions were as follows: reliability test (r) was 0.784; item validity (t) was $0.736 \leq t \leq 0.822$; discrimination power (DP) was $0.145 \leq DP \leq 0.311$; and difficulty index (DI) was $0.391 \leq DI \leq 0.509$. As a result, six questions could be used in the research.

4.4.2 CT scale

The CT skill scale (CTSS) to examine students' CT was developed by Tsai et al. (2021) and Korkmaz et al. (2017). Five mathematics experts analysed the content validity, and reported that there were 44 items that could be used in the questionnaire on CT. Then, a pilot application of the test was conducted with 32 eighth graders. The resulting Cronbach's alpha score was 0.865. Additionally, the instrument's validity was assessed by analysing the item correlation values and the corrected item-total correlation for each construct in the study. The findings indicated that 32 items within the CT scale were validated.

As a result, the final questionnaire used in the study consisted of 32 items spread across 9 distinct subdimensions: dimension 1 (abstraction) with 2 items, dimension 2 (decomposition) with 3 items, dimension 3 (algorithmic thinking) with 7 items, dimension 4 (generalisation) with 3 items, dimension 5 (evaluation) with 3 items, dimension 6 (creativity) with 4 items, dimension 7 (cooperation) with 3 items, dimension 8 (critical thinking) with 4 items, and dimension 9 (problem-solving) with three items.

4.4.3 Student's satisfaction

The student satisfaction scale, developed by Abdullah et al. (2019), enables the evaluation and assessment of the created learning tool's effectiveness and applicability by assisting in the collection of detailed feedback from students on a variety of its components. The student satisfaction survey covered five primary categories: facilitation, usefulness, design, and knowledge, with responses ranging from 'strongly disagree' to 'strongly agree' on a 1 to 5 Likert scale. Five mathematics experts evaluated the content validity and identified 17 items that could be included in the student satisfaction questionnaire.

Subsequently, a semi-structured interview was used to gain deeper insights into participants' experiences with the GLARE application during the learning process. These interviews were held individually during the week following the learning experiment, each lasting approximately 10 minutes. The interviews took place in a quiet room at the school to ensure students felt comfortable sharing their experiences. For analysis, each interview was audio-recorded with the student's consent and transcribed. A simple thematic analysis was used to analyse the data, where recurring ideas were identified and grouped into themes. One researcher manually reviewed the transcripts to identify patterns and categorised responses into relevant themes, such as engagement, comprehension, and user difficulty. A translator was involved to accurately interpret any responses that required translation, ensuring the clarity of the data. Furthermore, the interview results were compared with the students' satisfaction results.

4.5 Data analysis

The tests, questionnaires, and interview responses were evaluated by both researchers and the classroom teacher to minimise potential bias in scoring and ensure that the evaluations were focused solely on student performance. Then, the scores were analysed using multivariate analysis of variance (MANOVA) to assess the effectiveness of integrating PBL with AR on students' problem-solving and CT skills. Meanwhile, the qualitative data from the semi-structured interviews were analysed by evaluating the frequency of students' responses to each question.

5 Results

Before conducting the MANOVA analysis, the prerequisites for this test and the consistency of the research data with these prerequisites were as follows.

The Saphiro-Wilk test was used to analyse the normality of the dependent variables (problem-solving and CT), resulting in the variables exhibiting normal distribution ($p > 0.05$). Pearson's correlation was utilised to assess multicollinearity, revealing a significant pattern of correlations among all the dependent variables ($r_{pretest[90]} = 0.460$, $r_{posttest[90]} = 0.696$, all correlations were positive; $p < 0.001$). The absolute values were less than 0.8, indicating that collinearity was significantly less likely to exist. Box's test was conducted to assess the homogeneity of variance-covariance matrices, indicating that the variance-covariance matrices were statistically equivalent ($Box's\ M = 5.240$, $F_{pretest\ [6,188642.769]} = 0.844$, $p = 0.535$; $Box's\ M = 3.795$, $F_{posttest[6,188642.769]} = 0.611$, $p = 0.721$). Additionally, Levene's test indicated that the error variances for both dependent variables were equal ($F_{PS\ pretest\ [2,87]} = 2.401$, $p = 0.097$; $F_{PSposttest[2,\ 87]} = 2.374$, $p = 0.099$; $F_{CTpretest[2,\ 87]} = 1.983$, $p = 0.144$; $F_{CTposttest[2,87]} = 0.025$, $p = 0.976$). Since the requirements for MANOVA were met, MANOVA was conducted in the study.

RQ 1 What is the effect of PBL when integrated with AR on students' problem-solving and CT skills?

Before intervention

In this study, the null hypothesis (H_0) was that there was no significant difference between the problem-solving and CT skills of students using PBL + AR, PBL, and students from the control group. The confidence level was set at 95%. According to the MANOVA results in Table 2, H_0 was accepted for the pretest score for problem-solving and CT based on the learning method, $F_{[4,172]} = 0.504, p = 0.733$; *Wilk's lambda* = 0.977, $\eta_p^2 = 0.012$.

Table 2 MANOVA result for pretest scores

Effect		*Value*	*F*	*Hypothesis df*	*Error df*	*Sig.*	*Partial Eta squared*
Group	Wilks' lambda	0.977	0.504^b	4.000	172.000	0.733	0.012

Note: ^{b}Exact statistics.

Source: Authors

Table 3 demonstrated that the learning method had no significant effect on problem-solving ($F_{[2,87]} = 0.612$, $p = 0.545$, $\eta_p^2 = 0.014$), and on CT ($F_{[2,87]} = 0.744$, $p = 0.478$, $\eta_p^2 = 0.017$). Overall, it was concluded that there was no significant difference between the learning methods (PBL + AR, PBL, and control group) on problem-solving and CT. Therefore, three different groups (PBL + AR, PBL, and the control group) have the same initial problem-solving and CT skills.

Table 3 Tests of between-subjects effects

Source	*Dependent variable*	*Type III sum of squares*	*df*	*Mean square*	*F*	*Sig.*	*Partial Eta squared*
Group	CT pretest	145.489	2	72.744	0.744	0.478	0.017
	PS pretest	8.156	2	4.078	0.612	0.545	0.014
Error	CT pretest	8.500.733	87	97.710			
	PS pretest	579.633	87	6.662			

Source: Authors

After intervention

The MANOVA results (see Table 4) revealed a significant difference in problem-solving and CT based on the learning method (PBL + AR, PBL, and control group), $F_{[4,172]} = 9.811, p < 0.001$; *Wilks' lambda* = 0.663, $\eta_p^2 = 0.186$.

Table 4 MANOVA results for posttest scores

Effect		*Value*	*F*	*Hypothesis df*	*Error df*	*Sig.*	*Partial Eta squared*
Group	Wilks' lambda	0.663	9.811^b	4.000	172.000	< 0.001	0.186

Note: ^{b}Exact statistics.

Source: Authors

The findings in Table 5 indicated a significant effect of the learning method on CT ($F_{[2,87]} = 13.675$, $p < 0.001$, $\eta_p^2 = 0.239$). Additionally, the learning method also significantly influenced problem-solving ($F_{[2,87]} = 20.054$, $p < 0.001$, $\eta_p^2 = 0.316$). Overall, it was concluded that there were significant differences in problem-solving and CT skills among the learning methods (PBL + AR, PBL, and the control group).

Subsequently, Post hoc test indicated learning methods significantly differ in problem-solving and CT (see Table 6). Specifically, students' problem-solving abilities in the PBL + AR and PBL groups were significantly different from those in the control group, with significance levels of < 0.05. Additionally, the problem-solving between PBL + AR and PBL groups were significantly different from each other (p = 0.043, < 0.05). Moreover, students' CT in the PBL + AR and PBL groups significantly differed from the control group, with significance levels of < 0.001 and 0.039, respectively. When applying the same analysis to compare the PBL + AR and PBL groups regarding CT skills, the results indicated a statistically significant difference (p = 0.036, < 0.05).

Table 5 Tests of between-subjects effects for posttest scores

Source	*Dependent variable*	*Type III sum of squares*	*df*	*Mean square*	*F*	*Sig.*	*Partial Eta squared*
Group	CT posttest	1,837.089	2	918.544	13.675	< 0.001	0.239
	PS posttest	1,350.422	2	675.211	20.054	< 0.001	0.316
Error	CT posttest	5,843.900	87	67.171			
	PS posttest	2,929.233	87	33.669			

Source: Authors

Table 6 Post hoc test

Dependent variable	*(I) Group*	*(J) Group*	*Mean difference (I-J)*	*Std. error*	*Sig.*
CT posttest	PBL + AR	PBL	5.57*	2.116	0.036
		Control	11.07*	2.116	< 0.001
	PBL	PBL + AR	–5.57*	2.116	0.036
		Control	5.50*	2.116	0.039
	Control	PBL + AR	–11.07*	2.116	< 0.001
		PBL	–5.50*	2.116	0.039
PS posttest	PBL + AR	PBL	3.83*	1.498	0.043
		Control	9.43*	1.498	< 0.001
	PBL	PBL + AR	–3.83*	1.498	0.043
		Control	5.60*	1.498	0.002
	Control	PBL + AR	–9.43*	1.498	< 0.001
		PBL	–5.60*	1.498	0.002

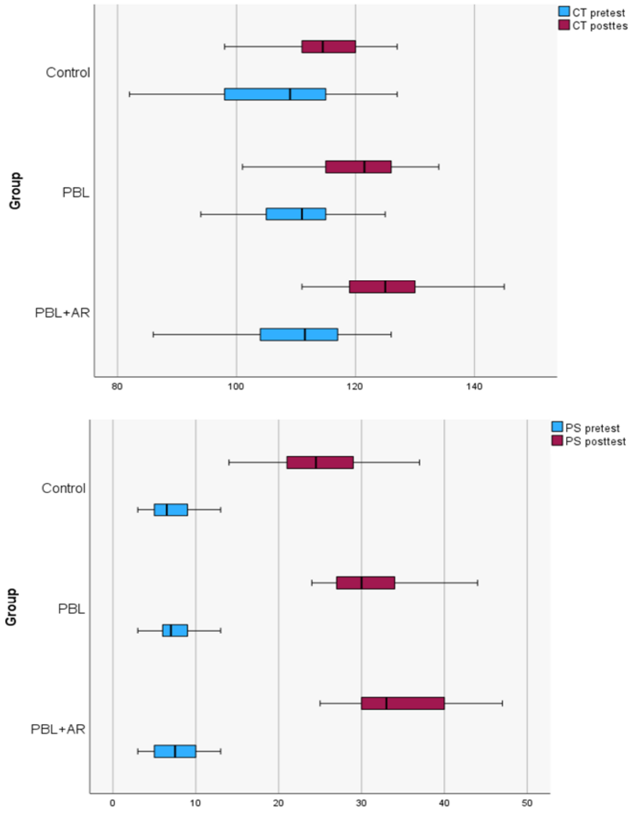
Note: *p = 0.05.

Source: Authors

The findings also indicated that the PBL + AR group ($\bar{x}$ = 34.30, SD = 6.374) demonstrated better performance in problem-solving skills compared to the PBL group

($\bar{x}$ = 30.47, SD = 4.614) and control group ($\bar{x}$ = 24.87, SD = 6.252), with $\Delta\bar{x}_{pretest\text{-}posttestPBL + AR}$ = 26.50. Furthermore, the PBL + AR group ($\bar{x}$ = 125.33, SD = 8.318) also performed better in the CT skills than PBL ($\bar{x}$ = 119.77, SD = 8.084) and control groups ($\bar{x}$ = 114.27, SD = 8.183), with $\Delta\bar{x}_{pretest\text{-}posttestPBL + AR}$ = 14.76. Thus, a holistic analysis in Figure 6 illustrated that learning through PBL + AR had a significant positive effect on students' problem-solving and CT skills compared to the PBL and control groups.

Figure 6 Distribution of problem-solving and CT mean scores by groups (see online version for colours)



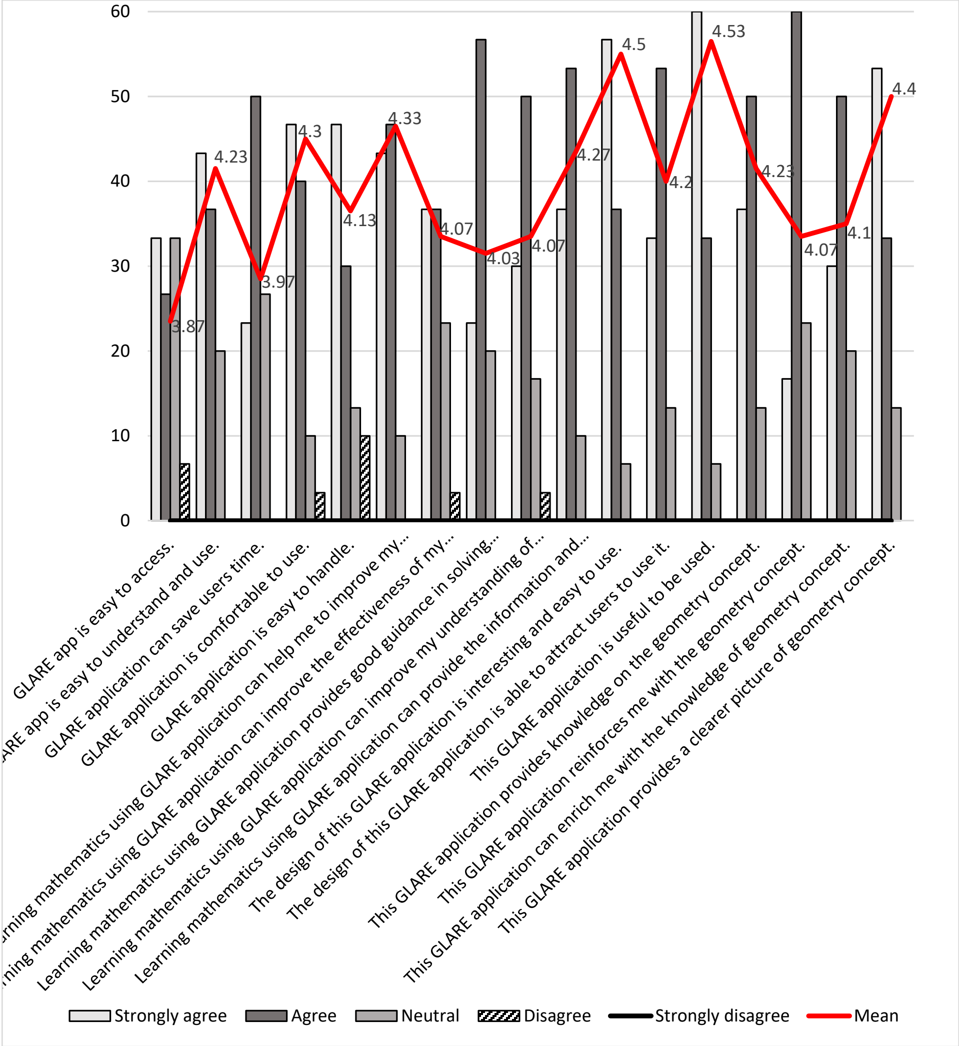
Source: Authors

RQ 2 What is the level of student satisfaction with learning using the GLARE application?

Participants were provided with a questionnaire featuring a five-point Likert scale, and their responses were used to gauge satisfaction with using GLARE in learning activities. Overall, most students gave positive feedback on the GLARE application with a mean score of around 4, as illustrated in Figure 7. Over 90% of students found GLARE easy to understand and handle, comfortable to use, and time-saving. Moreover, around 60% of the students reported that the application was easy to access. However, fewer than 10% of students mentioned they could not access the application due to not being Android users or facing restricted access from their parents.

Furthermore, approximately 90% of students believed GLARE improved their comprehension of mathematical concepts by providing the necessary knowledge and data. About 80% of students agreed that using the GLARE application to learn mathematics offered valuable guidance for solving mathematical problems and enhanced their understanding of geometry concepts. Similarly, slightly over 70% of them stated that they could learn mathematics more effectively with the GLARE application, especially in geometry topics, as it reinforced their understanding of these concepts. Additionally, more than 80% of students found the design of the GLARE application interesting and engaging, claiming it attracted users and provided a clearer picture of geometry concepts.

Figure 7 Students' satisfaction of the GLARE application (see online version for colours)



Source: Authors

Then, 15 students from the PBL + AR class participated in semi-structured interviews to see how they perceived using the GLARE application to learn geometry concepts. The following three questions were asked during the interviews.

- Question 1: Do you like the GLARE application? Why?
- Question 2: What do you think are the positive impacts of using the GLARE application in classroom learning?
- Question 3: What difficulties did you encounter when using the GLARE application?

Table 7 Semi-structured interview results

Question	*Representative comments*	*Frequency of mention*
Q1	Easy to use and understand.	12
	Assisting in understanding the problems.	10
	Increasing comprehension of nets, properties, formulae, and other topics related to geometry.	8
	Offering information and knowledge about mathematical content.	5
	Facilitating learning 3D geometry objects more easily.	3
Q2	Making the lesson more fun and interesting.	10
	Rising enthusiasm for the subject.	7
	Understanding complex ideas more easily	5
	Fostering learning motivation	4
	Encouraging self-directed learning	2
Q3	No difficulties.	11
	The application was difficult to install.	4
	Requiring more time to install the application.	2
	Not compatible with iOS.	1
	Lack of internet connectivity.	1

Source: Authors

Table 7 summarises 15 students' responses. In response to question 1, students noted that the GLARE application was user-friendly and comprehensible because the teacher had given instructions before the experiment. Students also highlighted the application as a valuable resource for learning mathematical topics, especially geometry. GLARE streamlined the learning experience, improving students' understanding of geometry concepts such as properties of 3D objects, nets, and formulae of surface area and volume as well. Furthermore, it assisted students in grasping problem scenarios. For example, students offered the following comments:

> "This application simplifies the process of learning about geometric shapes. I can learn formulae, methods, edges, nets, and more. The lesson also becomes more exciting and enjoyable."
>
> "I experienced no trouble using the application because the teacher provided me clear instructions on how to use it before the class."
>
> "I like this application because it's easy to use for learning mathematics. Awesome!"

Responding to question 2, the GLARE application, according to most students, made learning activities more interesting and enjoyable, which increased their enthusiasm for studying. The application motivated them to study intensely and promoted independent learning. Additionally, it made students understand complicated ideas easily. The following are comments made by students:

> "Thanks to this application, I can understand difficult content better. It also encourages independent learning."
>
> "I'm motivated to study more since I find the process of learning to be so enjoyable. I really like it."

Among the students who responded to question 3, there were no reported difficulties during the learning process with GLARE. However, some students encountered challenges during the installation phase, such as requiring extra time for installation and internet connectivity issues. In addition, another student was unable to access the application due to not having an Android device. Some examples of student statements follow.

> "The GLARE application helps me solve math problems, but it takes a long time to download."
>
> "The application was running slowly, maybe because there was lacking internet connectivity."
>
> "The application seems interesting; however, since I use an iOS mobile, I am unable to install it. The application is limited to Android devices."

6 Discussion

The current study aimed to investigate the effectiveness of integrating AR with PBL on students' problem-solving and CT skills. This study consisted of two experiment groups (PBL + AR and PBL) and one control group, with findings revealing significant differences in problem-solving and CT skills across these learning methods. Notably, the PBL + AR group demonstrated the highest improvements in both problem-solving and CT skills, underscoring the benefits of combining the PBL approach with AR technology. This aligns with Fidan and Tuncel (2019), who found that integrating PBL activities with AR technology significantly enhanced student achievement by stimulating cognitive processes and leveraging advanced technologies like AR. PBL supported by AR technology represents an innovative pedagogical approach that fosters knowledge development, promotes student engagement, and cultivates positive learning attitudes (Lee, 2022). Moreover, AR simulation enhances the inquiry process by providing more interactive and engaging learning experiences, ultimately leading to improved academic performance (Saltan and Arslan, 2016). By creating immersive and realistic learning environments, AR supports the development of cognitive skills and enables students to apply learned concepts in real-world situations (Fidan and Tuncel, 2019), making it especially useful for engaging with mathematics concepts and procedures (DeJarnette and González, 2016).

AR technology could improve geometry education by promoting an emphasis on visualisation, interaction, and real-world applications, all of which contribute to improved problem-solving skills (Koparan et al., 2023). Through AR simulation, students can recall previous learning experiences, revisit challenging concepts, and strengthen their understanding through interactive, detailed activities that support them in solving problems (Poçan et al., 2023). This technology promotes greater independence and autonomy in students' mathematics learning processes (Cevikbas et al., 2023). Furthermore, Huang et al. (2023) found that integrating AR into board games effectively supports students' understanding of CT concepts, while Hanid et al. (2022) reported significant improvements in students' CT, visualisation skills, and achievement in geometry topics when AR-based learning approach was used. AR's real-time simulation encourages consistent practice, reinforcing CT skills (Huang et al., 2023).

AR not only impacts skill development but also enhances students' engagement through visually interactive approaches that keep students motivated while they solve problems (Saraiva et al., 2021). Its immersive nature benefits students who thrive in active learning environments, strengthening their visualisation skills (Herpich et al., 2019). Learning to visualise space is essential, as it enables students to imagine objects and understand how to approach them, with the ability to conceptualise figures in various planes and dimensions helping them develop a clear vision of the problems (González, 2015). By allowing students to interact and explore geometric shapes and structures in 3D space, AR effectively overcomes the limitations of traditional two-dimensional teaching materials (Pahmi et al., 2023). This aligns with educational theories that support AR's role in fostering problem-solving and CT. Piaget's constructivist theory emphasises the role of hands-on experiences and interactions in knowledge construction, while Vygotsky's social constructivist perspective highlights the influence of social interactions and cultural contexts on learning (Adorni et al., 2024). Both theories highlight that engaging students in algorithmic thinking and problem-solving activities can significantly enhance cognitive development (Adorni et al., 2024).

Additionally, PBL helps students develop problem-solving proficiency by engaging them in small-group collaboration to solve simulated, real-world challenges (Vasiliou et al., 2013). This approach encourages both individual and group activities, where students take greater responsibility for their learning by developing knowledge and skills, while peer collaboration fosters a sense of connectedness that further enhances their CT skills, learning motivation, and problem-solving strategies (Gong et al., 2020). Through structured discussions, argumentation, and the exchange of diverse perspectives, students deepen their comprehension and refine their problem-solving strategies (Grabinger and Dunlap, 2002). As a result, combining PBL with emerging technologies like AR offers a more effective approach compared to traditional paper-and-pen activities by providing a flexible learning paradigm that addresses the limitations of both AR and traditional PBL approaches (Fidan and Tuncel, 2019).

Research consistently supports PBL as an effective framework for improving problem-solving skills (Amin et al., 2021; Darma, 2018; Hendriana et al., 2018), and within this framework, CT plays a crucial role in problem-solving by integrating critical thinking, computer science, and digital tools to solve complex problems (Rodríguez del Rey et al., 2021). Beyond improving problem-solving skills, CT also builds students' confidence, empowering them to resolve challenging tasks independently (Lin et al., 2024). Student-centred, inquiry-based, and hands-on learning approaches have been shown particularly effective for developing strong CT skills, as these approaches actively engage students, increasing their motivation and receptiveness to problem-solving activities, ultimately improving CT development (Saad, 2020). In PBL settings, students' CT skills are further reinforced through meaningful, inquiry-based content that reflects real-world challenges they may encounter in their future careers (Gilchrist et al., 2021). Considering the importance of CT in generating high-quality solutions to complex problems, fostering these skills through interactive, technology-enhanced learning experiences ensures that students are well-equipped to navigate academic and professional challenges (Saad, 2020).

One key finding from the semi-structured interviews and satisfaction questionnaires was that students had a positive perception of using the GLARE program in their learning process. They found the application easy to use and understand, particularly due to the teacher's guidance before the intervention. Consistent with these findings, Lin et al. (2015) reported that most students expressed high satisfaction with AR-based learning and encountered no usability issues. Additionally, the GLARE application facilitated a better understanding of geometry concepts, including 3D object formulae, nets, and properties, even for complicated topics. It also served as a valuable resource for content knowledge and information that assisted in effective problem-solving. This aligns with Bacca Acosta et al. (2014), who found that AR is particularly effective for teaching abstract or complex concepts. AR allows students to directly interact with digital objects, an experience that traditional teaching tools cannot easily replicate (Lin et al., 2021). The rich visual representations provided by AR enable students to grasp abstract concepts more quickly (Lee, 2022), by seamlessly integrating immersive experiences into their real-world setting, reducing reliance on imagination for conceptual understanding (Bacca Acosta et al., 2014; Pellas et al., 2019). Furthermore, Fidan and Tuncel (2019) highlighted that AR designs featuring 3D modelling offered more realistic learning experiences than two-dimensional models, allowing students to explore learning materials in detail from various perspectives and visualise abstract concepts, which enhances their ability to understand, analyse, and solve problems. Particularly for abstract scientific concepts, AR has been shown to boost students' confidence in learning (Lin et al., 2021), as its interactive experiences promote self-sufficiency in understanding and applying mathematical concepts (İslim et al., 2024).

Beyond improving comprehension, the GLARE application also made learning activities more engaging and enjoyable, fostering students' enthusiasm and motivation to explore complex geometry concepts. The application supported independent learning and contributed to increased student engagement, a benefit widely recognised in AR-based learning environments (Childs et al., 2023; Flores-Bascuñana et al., 2019; Koparan et al., 2023; Lin et al., 2015; Saltan and Arslan, 2016). By combining virtual worlds and real environments, AR enhances user engagement, creating an immersive experience where digital objects feel part of the surroundings (Rebollo et al., 2022). AR applications provide additional information and enhance focus, helping students engage more deeply with mathematical content and ultimately improving their understanding (Koparan et al., 2023). The innovative and interactive nature of AR attracts students' attention and fosters interest in mathematics (İslim et al., 2024), addressing the lack of intrinsic motivation commonly observed in traditional mathematics instruction, where mathematical concepts are frequently presented as abstract formulae to memorise rather than as practical tools for understanding and interpreting the real world (Medina Herrera et al., 2019). Furthermore, integrating tangible AR tools in classrooms transformed the role of teachers from a knowledge transmitter to a facilitator, guiding students toward greater autonomy while encouraging critical and creative thinking (Koparan et al., 2023; Poçan et al., 2023). When implemented appropriately within a well-structured pedagogical approach, AR could significantly enhance teaching and learning processes by fostering interactivity, motivation, and sustained student interest (Ciloglu and Ustun, 2023; Koparan et al., 2023).

In this context, our findings highlight key distinctions from previous studies. While Fidan and Tuncel (2019) demonstrated that integrating AR with PBL enhances cognitive skills and student achievement, our study found that the combination of AR and PBL resulted in significantly greater improvements in problem-solving and CT skills. This disparity could be attributed to the immersive and interactive nature of AR technology, which provided students with more engaging, real-world problem-solving scenarios compared to the more traditional AR applications used in previous research. Additionally, whereas prior studies have predominantly focused on higher education settings, our research explores the effect of AR and PBL at the secondary school level, addressing a relatively underexplored area. According to Piaget (1976), secondary school students' cognitive development transitions from concrete to abstract thinking, and presenting concepts in a concrete form allowing them to learn more effectively. The use of AR technology in mathematics instruction effectively facilitates the retention of abstract concepts by concretising them, reinforcing learned material, and enabling the interactive exploration of geometric objects, ultimately enhancing academic achievement, particularly in subjects requiring strong spatial visualisation skills, such as geometry (Cetintav and Yilmaz, 2023). Furthermore, AR systems create an engaging, game-like learning experience, which can be particularly beneficial for secondary school students, as they possess greater technical ability and a longer attention span compared to elementary school students (İslim et al., 2024).

On the other hand, several challenges emerged during the GLARE application's implementation. One major issue was the additional time required for installation due to the lack of internet access, which delayed the intervention process. Additionally, device incompatibility across different operating systems created technical difficulties that researchers needed to address. These challenges align with findings by Koparan et al. (2023) and Ciloglu and Ustun (2023), who reported that students frequently encountered challenges in AR-based learning due to limited access to necessary equipment tools. Not all students might have reliable internet connections or/and smartphones capable of running AR applications, which can hinder the seamless integration of AR into the learning process. One potential solution is integrating AR into collaborative learning environments like PBL, where students can share devices, fostering teamwork, expanding access to technology, and encouraging peer learning and collaboration (Voulgari et al., 2024).

7 Implications, limitations, and conclusions

7.1 Implications

The findings have significant research implications as they indicate that, in contrast to conventional teaching approaches, PBL integrated with AR was successful in attaining higher problem-solving and CT skills scores. Particularly, students who are digital natives have shown interest in emerging technologies such as AR. The present study's findings indicate that integrating PBL with modern technologies may improve its efficacy for enhancing learning. In theory, AR can be integrated into PBL procedures as they relate to authentic learning, such as understanding the problem, data gathering, and evaluation. Furthermore, students were able to more clearly visualise 3D objects and thoroughly examine course materials from various perspectives because of AR's ability to convert abstract geometry concepts into real-life representations. Using the application in learning activities encouraged autonomous student learning as well.

7.2 Limitations

There are certain limitations associated with AR technology that need to be acknowledged, such as device compatibility issues since it currently operates exclusively on Android devices. Therefore, researchers and developers should consider creating applications that are compatible with other devices, such as iOS. Additionally, students face challenges in installing the application due to unreliable internet connections. Both issues could be addressed by collaborating with schools or relevant institutions to ensure access to learning tools. It is recognised that integrating technology is not only innovative and creative but also offers the advantage of assisting teachers in teaching complex subjects like geometry. Furthermore, during the activity, some students can use other applications, which could cause problems. Consequently, it is essential that teachers keep a careful eye on their students to stay focused during the learning process.

7.3 Conclusions

This study focused on comparing students' problem-solving and CT skills among three groups and learning what students' satisfaction was with using the GLARE application for learning mathematics. Data from the mathematical problem-solving tests, the CT scale, semi-structured interviews, and the satisfaction scale were gathered. According to the findings, student problem-solving and CT skills were positively impacted by the PBL integrated with instructional activities. The results further demonstrated that students supported the use of the GLARE application in the classroom as a learning tool. In terms of student problem-solving and CT skills, the post hoc analysis revealed that the PBL + AR instructional activities were significantly different from those engaged with the PBL and control groups. Furthermore, when compared to PBL and control groups using mean scores, the PBL + AR group proved to have a greater impact on student problem-solving and CT skills.

The findings from the semi-structured interviews and satisfaction scale revealed that the students' preferred options for the GLARE characteristics were its fun and user-friendly features. Students indicated that the application helped improve their comprehension of geometry concepts, including 3D object formulae, nets, and even more complex properties. They viewed the application as a valuable tool for gaining content knowledge and information that supported efficient problem-solving. Additionally, the GLARE application claimed to have the potential to enhance students' motivation to learn by facilitating independent learning. On the other hand, a few students encountered problems installing the application because of incompatibilities with their devices and poor internet connectivity.

AR technology has shown potential as an effective educational tool by enhancing the PBL process. However, a primary challenge lies in the difficulty teachers face in creating professional AR content, particularly with 3D objects. It is necessary to provide platforms that allow teachers to create and distribute their own AR lesson plans. Additionally, creating a centralised data repository accessible to teachers through a common educational platform could be beneficial. Socio-economic differences are a cause for concern as well, since some students may not have access to mobile phones, monitors, or special glasses, which are necessary for displaying technology. Furthermore, instructional technologies may currently be insufficient for integrating AR systems effectively in classrooms. It is essential to facilitate all public schools with the required equipment to ensure equal opportunities for students with related instructional technologies.

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Data availability

The datasets generated or/and analysed during the current study are available from the corresponding author on reasonable request.

Declarations

This study protocol was reviewed and approved by The Research Ethic Committee Institute for Research and Community Service University of Bengkulu, approval number 21/KER-LPPM/EC/2023.

All authors declare that they have no conflicts of interest.

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