Assignment 1
due on Wednesday, May 3, 2017

Name:

Exercise 1 (10 points)**.**
Use the discriminant polynomial to show that the Waring rank of $X^2 + XY + Y^2$ is at least 2.

Exercise 2 (10 points)**.**
We have seen polynomials whose Waring rank exceeds their border Waring rank. In contrast to this observation prove that the set of Waring rank 1 polynomials is $\mathbb{C}$-closed.

Exercise 3 (10 points)**.**
Consider the action of $\mathbb{C}^{N \times N}$ on $\mathbb{C}[X_1, \ldots, X_N]_d$ defined in the lecture. Compute the following polynomial in the standard monomial basis:

$$\begin{pmatrix} 2 & 3 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} (X_1 X_2^2 + X_3)$$

Exercise 4 (10 points)**.**
Let GL_n denote the group of invertible complex $n \times n$ matrices. Let $G = \mathsf{GL}_n \times \mathsf{GL}_n$ and let $V = \mathbb{C}^{n \times n}$. Define an action of G on V by

$$(g_1, g_2)v := g_1 \cdot v \cdot g_2^t,$$

where "$\cdot$" is the product of matrices. Let $v \in V$ have rank exactly k. Prove that

$$Gv = \{w \in V \mid \mathrm{rk}(w) = k\}.$$