Assignment 12
due on Wednesday, July 19, 2017

Name:

Exercise 1 (10 points)**.**
Let $\lambda = (2,2)$ and let $[\lambda]$ denote the corresponding Specht module. The cyclic group $\mathbb{Z}_4$ naturally embeds into $\mathfrak{S}_4$ and thus $[\lambda]$ becomes a $\mathbb{Z}_4$-representation. Determine the dimension of the space of $\mathbb{Z}_4$-invariants in $[\lambda]$.

Exercise 2 (10 points)**.**
We look at the vanishing ideal of orbit closures. Assume that there exists a function $f \in I(\overline{\mathsf{GL}_{n^2} \det_n})_d$ such that f is a weight vector of a weight λ. If n divides d, then moreover we assume that λ is not the rectangle $(n^2 \times \frac{d}{n})$. Prove that there exists a function $f' \in I(\overline{\mathsf{GL}_{n^2} \det_n})_d$ such that f and f' are linearly independent.

Exercise 3 (10 points)**.**
Let V denote the irreducible GL_3-representation of type $(3,1,0)$. We embed $\mathsf{GL}_2 \hookrightarrow \mathsf{GL}_3$ via

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mapsto \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

In this way, V is a GL_2-representation. Determine the multiplicities of the decomposition of V into irreducibles.

Exercise 4 (10 points)**.**
Let $G = \mathsf{GL}_2$. Let V denote the irreducible G-representation of type $(3,1)$ and let W denote the irreducible G-representation of type $(2,0)$. Then $V \otimes W$ is a G-representation via

$$g(v \otimes w) := (gv) \otimes (gw) \quad \text{for all } g \in G,\ v \in V,\ w \in W$$

and linear continuation. Embedding $G \hookrightarrow G \times G$, $g \mapsto (g,g)$, we can interpret $V \otimes W$ as a G-representation.

What dimension does $V \otimes W$ have? Determine the multiplicities in the decomposition of $V \otimes W$ into irreducible G-representations.