Assignment 13
due on Wednesday, July 26, 2017

Name:

Exercise 1 (10 points)**.**
Let $V = \mathbb{C}^M$. Determine the multiplicities of the irreducible $\mathsf{GL}(V)$-representations in $\mathrm{Sym}^2 \mathrm{Sym}^3 V$.

Exercise 2 (10 points)**.**
Let $V = \mathbb{C}^M$. Let $n, d \in \mathbb{N}$. Let λ be a partition of nd. Prove that if $\ell(\lambda) > d$, then the irreducible $\mathsf{GL}(V)$-representation of type λ does not occur in $\mathrm{Sym}^d \mathrm{Sym}^n V$.

Exercise 3 (10 points)**.**
Prove that $a_{4,4,4}(4, 3) = 1$.

Exercise 4 (10 points)**.**
Let $n, d \in \mathbb{N}$. Let $\lambda = (k, 2^i, 1^j)$ be a partition of nd such that $j \geq i+3$. Prove that $a_\lambda(d, n) = 0$.

For example, for $k = 13$, $i = 2$, $j = 5$ we have

$\lambda =$