Assignment 2
due on Wednesday, May 10, 2017

Name:

Exercise 1 (10 points)**.**
Prove that the Waring rank of a polynomial is always finite by proving that

$$X_1X_2\cdots X_d = \frac{1}{d!\,2^{d-1}} \sum_{\alpha\in\{-1,1\}^{d-1}} \left(\prod_{i=1}^{d-1}\alpha_i\right)(\alpha_1X_1+\alpha_2X_2+\cdots+\alpha_{d-1}X_{d-1}+X_d)^d.$$

Exercise 2 (10 points)**.**
Let $\mathbb{A} = \mathbb{C}[x,y]_2$. Let $X \subseteq \mathbb{A}$ denote the set of Waring rank 1 polynomials. Determine the homogeneous part $I(X)_2$ of the vanishing ideal $I(X)$.

Exercise 3 (10 points)**.**
Let $\mathbb{A} = \mathbb{C}[x,y,z]_2$, so $xy \in \mathbb{A}$. Let $X := \overline{\mathsf{GL}_3(xy)} \subseteq \mathbb{A}$. Determine the homogeneous part $I(X)_2$ of the vanishing ideal $I(X)$.

Exercise 4 (10 points)**.**
Let V and W be metric spaces and let $f : V \to W$ be a continuous map. Prove that for all subsets $A \subseteq V$ we have $f(\overline{A}) \subseteq \overline{f(A)}$ and that $\overline{f(A)} = \overline{f(\overline{A})}$.