Assignment 3
due on Wednesday, May 17, 2017

Name:

Exercise 1 (10 points).
Let $S_{n,i,j}$, $P_{n,i,j}$, and C_n, $n, i, j \in \mathbb{N}$, be indeterminates. Let

$$\begin{aligned} \mathrm{gen}_1 &= C_1 \\ \mathrm{gen}_n &= \sum_{i,j=1}^{n-1} S_{n,i,j}(\mathrm{gen}_i + \mathrm{gen}_j) \\ &\quad + \sum_{i,j=1}^{n-1} P_{n,i,j}\, \mathrm{gen}_i \cdot \mathrm{gen}_j \\ &\quad + C_n \end{aligned}$$

Prove that (gen_n) is VP-complete.

Exercise 2 (10 points).
Let $(f_n) \in \mathrm{VP}$ and let $p(n)$ be minimal such that that $f_n \in \mathbb{F}[X_1, \dots, X_{p(n)}]$. Let $g_n \in \mathrm{End}_{p(n)}$. Prove that $(g_n f_n) \in \mathrm{VP}$.

Exercise 3 (10 points).
The characteristic polynomial of a matrix A is defined as $c_A(X) = \det(A - X \cdot I)$ where I is the identity matrix. Let $c_A(X) = s_{A,0}X^n + s_{A,1}X^{n-1} + \cdots + s_{A,n}$.

1. Show that

$$\begin{aligned} s_{A,0} &= (-1)^n \\ s_{A,k} &= \frac{1}{k} \sum_{\kappa=1}^{k} (-1)^{\kappa-1} s_{A,k-\kappa}\, \mathrm{tr}(A^\kappa), \qquad 1 \le k \le n. \end{aligned}$$

2. Show that $s_{A,n} = \det A$.

3. Show that there is a family of weakly skew circuits of polynomial size computing $(\det_n)$.