Assignment 6
due on Wednesday, June 7, 2017

Name:

Exercise 1 (10 points)**.**
Let $\mathbb{A} = \mathbb{C}[X_1, X_2, \ldots, X_M]_2$. Show that for $M \geq 2$ the GL_M-representation $\mathbb{C}[\mathbb{A}]_2$ is not irreducible.

Exercise 2 (10 points)**.**
Choose some N and give a representation V of GL_N and a vector $v \in V$ such that the orbit span $\langle \mathsf{GL}_N v \rangle$ is not irreducible.

Exercise 3 (10 points)**.**
Let $i \in \mathbb{C}$ denote the imaginary unit, i.e., $i^2 = -1$. Let $C := \{e^{\alpha i} \mid 0 \leq \alpha < 2\pi\} \subseteq \mathbb{C}$ denote the circle group. Prove that C is linearly reductive by showing that for every C-representation V there exists a C-invariant inner product.

Exercise 4 (10 points)**.**
If $H \leq G$ is a subgroup, then every G-representation is also an H-representation in the natural way. Construct an example of a group G, an irreducible G-representation V, and a subgroup $H \leq G$ such that V is not irreducible as an H-representation.