Assignment 7
due on Wednesday, June 14, 2017

Name:

Exercise 1 (10 points)**.**
Let U_n denote the group of $n \times n$ upper triangular matrices with 1s on the main diagonal. For $1 \leq i < j \leq n$, $\alpha \in \mathbb{C}$, let $x_{ij}(\alpha) \in U_n$ denote the identity matrix with an entry α in row i and column j. Prove that U_n is generated as a group by the set $\{x_{ij}(\alpha) \mid 1 \leq i < j \leq n, \alpha \in \mathbb{C}\}$.

Exercise 2 (10 points)**.**
Let $V := \mathbb{C}^n$. The group GL_n acts on V by matrix-vector multiplication. Decompose V into a direct sum of irreducibles and determine their types λ.

Exercise 3 (10 points)**.**
Let $V := \mathbb{C}^{n \times n}$ denote the vector space of $n \times n$ matrices. The group GL_n acts on V by left multiplication. Decompose V into a direct sum of irreducibles and determine their types λ.

Exercise 4 (10 points)**.**
Let V be a polynomial GL_n-representation. For $i < j$ let $x_{ij}(\alpha)$ be defined as in Exercise 1. The *raising operator* $E_{ij} : V \to V$ is a linear map defined via

$$E_{ij}(v) = \lim_{\varepsilon \to 0} \tfrac{1}{\varepsilon}((x_{ij}(\varepsilon)v) - v).$$

Prove that E_{ij} is well-defined, i.e., that the limit exists.