Assignment 9
due on Wednesday, June 28, 2017

Name:

Exercise 1 (15 points)**.**
Let $V := \mathbb{C}^2$ and $W := \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$. The group $G := \mathsf{GL}_2$ acts on V via matrix-vector multiplication. Then G also acts on W via

$$g(v_1 \otimes v_2 \otimes v_3) := (gv_1) \otimes (gv_2) \otimes (gv_3).$$

Determine the multiplicities $\mathrm{mult}_\lambda(W)$.

Exercise 2 (15 points)**.**
We have seen in the lecture that for the cyclic group $\mathbb{Z}_3$ there are exactly three isomorphism types of irreducible representations. Let $V = \mathbb{C}^n$. Let $\mathbb{Z}_3$ be generated by π. The group $\mathbb{Z}_3$ acts on the vector space $V \otimes V \otimes V$ via

$$(\pi)(u \otimes v \otimes w) = (v \otimes w \otimes u)$$

and linear continuation. Determine the three multiplicities of $\mathbb{Z}_3$-irreducibles in $V \otimes V \otimes V$.

Exercise 3 (10 points)**.**
Prove that the border Waring rank of X_1X_2 exceeds 1 by using multiplicities in the coordinate rings of orbit closures as follows. Let $G := \mathsf{GL}_2$. Find a partition λ such that

$$\mathrm{mult}_\lambda \mathbb{C}[\overline{G(X_1X_2)}] > \mathrm{mult}_\lambda \mathbb{C}[\overline{G(X_1^2)}].$$