A first introduction to geometric complexity theory

 ${\rm Summer}~2018$ 

## Assignment 1 due on Wednesday, April 18, 2018

Name:

## Exercise 1 (10 points).

For every fixed field  $\mathbb{F}$ , show that every nonzero univariate polynomial in  $\mathbb{F}[x]$  of degree n can have at most n zeros.

## **Exercise 2** (10 points).

Fix the field  $\mathbb{F} = \mathbb{F}_2$ . For a multivariate polynomial *h*, the *arithmetic complexity* L(h) is the size (number of addition and multiplication gates) of the smallest arithmetic circuit computing *h*.

Show that there exists  $m \in \mathbb{N}$  and two multivariate polynomials h and h', both on m variables  $x_1, \ldots, x_m$ , such that

- $\deg(h) = \deg(h')$ , and
- $h(x_1,\ldots,x_m) = h'(x_1,\ldots,x_m)$  for all  $x_1,\ldots,x_m \in \mathbb{F}$ , and
- L(h) < L(h').