Assignment 1
due on Wednesday, April 18, 2018

Name: $\square$

Exercise 1 (10 points).
For every fixed field $\mathbb{F}$, show that every nonzero univariate polynomial in $\mathbb{F}[x]$ of degree $n$ can have at most $n$ zeros.

Exercise 2 (10 points).
Fix the field $\mathbb{F}=\mathbb{F}_{2}$. For a multivariate polynomial $h$, the arithmetic complexity $L(h)$ is the size (number of addition and multiplication gates) of the smallest arithmetic circuit computing $h$. Show that there exists $m \in \mathbb{N}$ and two multivariate polynomials $h$ and $h^{\prime}$, both on $m$ variables $x_{1}, \ldots, x_{m}$, such that

- $\operatorname{deg}(h)=\operatorname{deg}\left(h^{\prime}\right)$, and
- $h\left(x_{1}, \ldots, x_{m}\right)=h^{\prime}\left(x_{1}, \ldots, x_{m}\right)$ for all $x_{1}, \ldots, x_{m} \in \mathbb{F}$, and
- $L(h)<L\left(h^{\prime}\right)$.

