Assignment 11
due on Wednesday, August 4, 2018

Name:

Exercise 1 (20 points).
Simultaneously diagonalize the following three matrices:

$$\begin{pmatrix} 5 & -30 & -8 \\ -18 & 182 & 48 \\ 69 & -690 & -182 \end{pmatrix}, \quad \begin{pmatrix} -21 & 88 & 24 \\ 156 & -569 & -156 \\ -594 & 2178 & 597 \end{pmatrix}, \quad \begin{pmatrix} -42 & 70 & 20 \\ 300 & -527 & -150 \\ -1140 & 1995 & 568 \end{pmatrix}.$$

Exercise 2 (10 points).
Let $\mathbb{A} = \mathbb{C}[x,y]_2$ and let $V = \mathbb{C}[\mathbb{A}]_2$ be the polynomial GL_2-representation from the lecture. For $\alpha_1, \alpha_2 \in \mathbb{C}^\times$, determine the element $\varrho(\mathrm{diag}(\alpha_1, \alpha_2)) \in \mathsf{GL}(V)$.

Exercise 3 (10 points).
Let $\mathbb{A} = \mathbb{C}[x,y]_2$. Determine a weight decomposition of the 6-dimensional T_2-representation $\mathbb{C}[\mathbb{A}]_2$.

Exercise 4 (10 points).
Consider the group homomorphism $\varphi : T_2 \to T_2$ given by $\varphi(t_1, t_2) = (t_1^2, t_2)$. For $\mathbb{A} = \mathbb{C}[x,y]_2$ let $V = \mathbb{C}[\mathbb{A}]_2$ with the action $\varrho : \mathsf{GL}_2 \to \mathsf{GL}(V)$ from the lecture. Consider the action ϱ' of T_2 on $\mathbb{C}[\mathbb{A}]_2$ given by $\varrho'(t) = \varrho(\varphi(t))$ for $t \in T_2$. Determine a weight decomposition of this 6-dimensional T_2-representation.

Exercise 5 (10 points).
Prove that there are exactly k pairwise non-isomorphic irreducible representations of the cyclic group $\mathbb{Z}/k\mathbb{Z}$.

Exercise 6 (20 points).
Prove that every irreducible representation of the product group $(\mathbb{Z}/k\mathbb{Z}) \times (\mathbb{Z}/\ell\mathbb{Z})$ is 1-dimensional.