Assignment 2
due on Wednesday, April 25, 2018

Name:

Exercise 1 (10 points)**.**
Use the discriminant polynomial to show that the Waring rank of $X^2 + XY + Y^2$ is at least 2.

Exercise 2 (10 points)**.**
We have seen polynomials whose Waring rank exceeds their border Waring rank. In contrast to this observation, prove that there is no polynomial h that satisfies $\mathrm{WR}(h) > 1 = \underline{\mathrm{WR}}(h)$.

Exercise 3 (20 points)**.**
Consider the 3-dimensional vector space $\mathbb{A} = \mathbb{C}[X, Y]_2$ with basis $\{x^2, xy, y^2\}$, so every polynomial in $\mathbb{A}$ has a unique expression as $ax^2 + bxy + cy^2$. Consider homogeneous degree 2 polynomials in a, b, c to obtain the 6-dimensional vector space $\mathbb{C}[\mathbb{A}]_2$. For example, the discriminant $b^2 - 4ac$ is an element of $\mathbb{C}[\mathbb{A}]_2$. Prove that the discriminant is the only polynomial (up to scale) in $\mathbb{C}[\mathbb{A}]_2$ that vanishes on polynomials of Waring rank 1.