Assignment 5
due on Wednesday, May 16, 2018

Name:

**Exercise 1** (10 points)**.**
Given a polynomial map $\varphi : \mathbb{C}^a \to \mathbb{C}^b$. Prove that the preimage $\varphi^{-1}(V) \subseteq \mathbb{C}^a$ of a Zariski-closed set $V \subseteq \mathbb{C}^b$ is Zariski-closed.

**Exercise 2** (10 points)**.**
Find $a, b$ and a polynomial map $\varphi : \mathbb{C}^a \to \mathbb{C}^b$ such that there exists a $\mathbb{C}$-closed set $W \subseteq \mathbb{C}^a$ for which $\varphi(W) \subseteq \mathbb{C}^b$ is not $\mathbb{C}$-closed.

**Exercise 3** (10 points)**.**
Find $a, b$ and a polynomial map $\varphi : \mathbb{C}^a \to \mathbb{C}^b$ such that there exists a Zariski-closed set $W \subseteq \mathbb{C}^a$ for which $\varphi(W) \subseteq \mathbb{C}^b$ is not Zariski-closed.

**Exercise 4** (10 points)**.**
Consider the set $S \subseteq \mathrm{End}_n$ of symmetric matrices. Consider the action of $\mathrm{End}_n$ on $\mathsf{Sym}^d\mathbb{C}^n$ from the lecture. Prove that for every $p \in \mathsf{Sym}^d\mathbb{C}^n$ the Zariski-closure of $Sp$ coincides with the $\mathbb{C}$-closure of $Sp$.