Assignment 7
due on Wednesday, May 30, 2018

Name:

Exercise 1 (20 points)**.**
Given a directed acyclic graph G on n vertices with two distinct vertices s and t such that each path from s to t has the same length. Assign a formal label ℓ_e to each edge e in G. Construct a graph G' on $n-1$ vertices from G by identifying s and t.

Let A be the $(n-1) \times (n-1)$ adjacency matrix of G', i.e., $A_{i,j} = \ell_{(i,j)}$. Here we assume that the row and the column corresponding to s are the first row and the first column. Let I denote the $(n-1) \times (n-1)$ identity matrix and let E denote the $(n-1) \times (n-1)$ matrix that has a single 1 in the top left cell and zeroes everywhere else.

Prove that

$$\sum_{s\text{-}t\text{-path } p \text{ in } G} \left(\prod_{\text{edge } e \in p} \ell_e \right) = \sum_{\text{cycle } c \text{ through } s \text{ in } G'} \left(\prod_{\text{edge } e \in c} \ell_e \right) = \pm \det(A + I - E),$$

for either $\pm = 1$ or $\pm = -1$.

Exercise 2 (20 points)**.**
Prove that $\det_n$ is VP_e-hard.

Hint: Use the previous exercise.