Assignment 8
due on Wednesday, June 6, 2018

Name:

Exercise 1 (20 points)**.**
Define the *continuant polynomial*

$$K_n := \operatorname{tr}\left(\begin{pmatrix} x_1 & T \\ T & 0 \end{pmatrix} \cdots \begin{pmatrix} x_n & T \\ T & 0 \end{pmatrix}\right) \in \mathbb{C}[x_1, \ldots, x_n, T]_n.$$

Prove that if a p-family (h_n) has formulas whose size is polynomially bounded, whose depth is logarithmically bounded, and where at each gate a homogeneous polynomial is computed, then there exists a polynomially bounded function p such that $T^{p(n)-n} h_n \in \overline{\mathsf{GL}_{n+1} K_n}$.

Hint: Use the construction from the lecture and additionally that

$$\begin{pmatrix} f & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} f & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ f & 1 \end{pmatrix}$$

Exercise 2 (20 points)**.**
Prove that the permanent (per_n) is an element of the complexity class VNP.

Hint: Write an arithmetic formula f in n^2 variables such that $f(A) = 1$ if $A \in \{0,1\}^{n^2}$ is a permutation matrix and $f(A) = 0$ if $A \in \{0,1\}^{n^2}$ is not a permutation matrix.