Assignment 2
due on Tuesday, November 7, 2017

Name:

For three partitions λ, μ, ν of d let $k(\lambda, \mu, \nu)$ denote the *Kronecker coefficient*, i.e., the dimension of the $\mathfrak{S}_d$-invariant space $\dim([\lambda] \otimes [\mu] \otimes [\nu])^{\mathfrak{S}_d}$.

Exercise 1 (10 points)**.**
Determine $k((2,1),(2,1),(2,1))$.

Exercise 2 (10 points)**.**
Let $\lambda \vdash d$. Let λ^t denote the transpose Young diagram of λ. Let $d \times 1$ denote the column partition. Prove that $k(d \times 1, \lambda, \lambda^t) \leq 1$.

20 bonus points if you also prove $k(d \times 1, \lambda, \lambda^t) = 1$.

Exercise 3 (20 points)**.**
Let λ, μ, ν be partitions of d and let $\tilde{\lambda}, \tilde{\mu}, \tilde{\nu}$ be partitions of $\tilde{d}$. Prove the "semigroup property": If $k(\lambda, \mu, \nu) > 0$ and $k(\tilde{\lambda}, \tilde{\mu}, \tilde{\nu}) > 0$, then $k(\lambda + \tilde{\lambda}, \mu + \tilde{\mu}, \nu + \tilde{\nu}) \geq \max\{k(\lambda, \mu, \nu), k(\tilde{\lambda}, \tilde{\mu}, \tilde{\nu})\}$, where the addition of partitions is defined as adding row-lengths of the corresponding Young diagrams.

Hint: Look at the analogous proof for plethysm coefficients.

Exercise 4 (20 points)**.**
Let (i) denote the partition that only has a single row. Let λ, μ, ν be partitions of d. As a preliminary task, prove that the sequence

$$K_i := k(\lambda + (i), \mu + (i), \mu + (i))$$

is monotonously nondecreasing. Then prove that the sequence K_i stabilizes, i.e.,
$\exists i_0 \ \forall i \geq i_0 \ : \ K_i = K_{i_0}$.