Assignment 4
due on Tuesday, Nov 21, 2017

Name: $\square$

Exercise 1 (10 points).
Let $X \in K^{m \times m}, Y \in K^{m, n}, Z \in K^{n \times m}$, and $W \in K^{n \times n}$ with $\operatorname{det} W \neq 0$. Prove that

$$
\operatorname{det}\left(\begin{array}{cc}
X & Y \\
Z & W
\end{array}\right)=\operatorname{det}(W) \operatorname{det}\left(X-Y W^{-1} Z\right),
$$

Exercise 2 (20 points).
In the proof of Strassen's lower bound for 3 -slice tensors, we used the tensor $T_{A}^{\wedge}$. What happens if we use $T_{A}^{S}$ (defined by projecting onto $S^{2} A$ ) instead?

Exercise 3 (10 points).
Generalize Strassen's lower bound: If $T=(I, X, Y)$ is a 3 -slice tensor with slices in $K^{b \times b}$, then

$$
\underline{R}(T) \geq b+\frac{1}{2} \operatorname{rk}(X Y-Y X) .
$$

