Assignment 5
due on Tuesday, Nov 28, 2017

Name:

Exercise 1 (15 points)**.**
Let $A = M \otimes N^*$, $B = N \otimes L^*$, and $C = L \otimes M^*$ with $\dim M = m$, $\dim N = n$, and $\dim L = \ell$, that is, A can be viewed as the vector space of $m \times n$-matrices, etc. Let $\langle m, n, \ell \rangle \in A \otimes B \otimes C$ be the tensor of the multiplication of $m \times n$-matrices with $n \times \ell$-matrices. Prove that

$$\langle m, n, \ell \rangle \cong \mathrm{Id}_M \otimes \mathrm{Id}_N \otimes \mathrm{Id}_L .$$

Exercise 2 (15 points)**.**
Let V be an irreducible GL_n-representation. Prove that V is also irreducible as an SL_n-representation, where SL_n is the group of matrices with determinant equal to 1.

Exercise 3 (10 points)**.**
Let W be a vector space of dimension 2 and let $\ell, \ell_1 \ldots, \ell_{n-1} \in W$ be pairwise distinct. Prove that there is a $g \in S^{n-1}W^*$ that vanishes on $\ell_1, \ldots, \ell_{n-1}$, but not on ℓ.