Assignment 6

due on Tuesday, Dec 12, 2017

Name:

Exercise 1 (10 points)**.**
Prove that the unit tensor $\sum_{i=1}^{n} e_i \otimes e_i \otimes e_i \in \mathbb{C}^n \otimes \mathbb{C}^n \otimes \mathbb{C}^n$ is characterized by its stabilizer (which is a subgroup in $\mathsf{GL}_n \times \mathsf{GL}_n \times \mathsf{GL}_n$). You do not have to determine the stabilizer.

Exercise 2 (20 points)**.**
Let $\lambda \vdash n$. We know from Gay's theorem that the $(n \times 1)$-weight space in the irreducible GL_n representation $\{\lambda\}$ is isomorphic to the Specht module $[\lambda]$ as an $\mathfrak{S}_n$-representation.

For some n of your choice, find a partition $\mu \vdash 2n$ such that the $(n \times 2)$-weight space of $\{\mu\}$ is not irreducible as an $\mathfrak{S}_n$-representation.

Exercise 3 (10 points)**.**
Let $v := x_1^3 + x_2^3 \in \mathrm{Sym}^3\mathbb{C}^2$. Determine the multiplicities

$$\mathrm{mult}_{(5,4)}\mathbb{C}[\mathsf{GL}_2 v]_3$$

and

$$\mathrm{mult}_{(6,3)}\mathbb{C}[\mathsf{GL}_2 v]_3.$$