Geometric complexity theory 2 Winter 2017/18

Assignment 9
due on Tuesday, Jan 9, 2018

Name:

Exercise 1 (10 points)**.**
Let G be a finite group and let V and W be two G-representations (in particular, V and W are finite dimensional). Then the tensor product $V \otimes W$ of vector spaces is a G-representation via

$$g(v \otimes w) := gv \otimes gw$$

and linear continuation. Prove that the character $\chi_{V\otimes W}$ satisfies $\chi_{V\otimes W}(g) = \chi_V(g) \cdot \chi_W(g)$.

Exercise 2 (10 points)**.**
The tensor product of Specht modules $[(2,1)] \otimes [(2,1)]$ is a 4-dimensional $\mathfrak{S}_3$-representation. Compute its character and decompose it as a linear combination of characters of irreducible $\mathfrak{S}_3$-representations.

Exercise 3 (10 points)**.**
The tensor power of Specht modules $W := [(2,1)]^{\otimes n}$ is a 2^n-dimensional $\mathfrak{S}_3$-representation via

$$g(v_1 \otimes v_2 \otimes \cdots \otimes v_n) = gv_1 \otimes gv_2 \otimes \cdots \otimes gv_n$$

and linear continuation. Determine the multiplicities of the irreducible $\mathfrak{S}_3$-representations in W.

Exercise 4 (10 points)**.**
Let $H \leq G$ be a subgroup of a (not necessarily finite) group. For each $g \in G$ we define the *coset* as its orbit under the right multiplication:

$$gH := \{gh \mid h \in H\}.$$

Prove that distinct cosets have empty intersection. Also prove that all cosets have the same cardinality.