Relational Algebra for Query Optimisation

The axioms of relational algebra

LAWS OF RELATIONAL ALGEBRA 1

1. Commutativity of joins and product

If $*$ denotes any type of join or product, then $E*F \equiv F*E$.

2. Associativity for joins and products

If $*$ denotes any type of join or product, then $(E * F) * G \equiv E * (F * G)$.

3. Cascade of projections/selections

$\Pi_{A1 \dots An} (\Pi_{B1 \dots Bm} (E)) \equiv \Pi_{A1 \dots An} (E)$ whenever $\{ B1,\dots, Bm \} \supseteq \{ A1,\dots, An \}$

$\sigma_{F1 \wedge F2} (E) \equiv \sigma_{F1} (\sigma_{F2} (E))$.

Consequence: Selection is commutative.

LAWS OF RELATIONAL ALGEBRA 2

4. Commuting selections and projections

$\Pi_{A1 \dots An} (\sigma_F (E)) \equiv \sigma_F (\Pi_{A1 \dots An} (E))$

provided that condition F depends only on A1 ,..., An.

More generally

$\Pi_{A1 \dots An} (\sigma_F (E)) \equiv \Pi_{A1 \dots An} (\sigma_F (\Pi_{A1 \dots An\, B1 \dots Bm} (E)))$

5. Commuting selection with Cartesian product

$\sigma_F(E1 \times E2) \equiv \sigma_F(E1) \times E2$

if F involves only attributes of E1

LAWS OF RELATIONAL ALGEBRA 3

5. Commuting selection with Cartesian product

$\sigma_F(E1 \times E2) \equiv \sigma_F(E1) \times E2$

if F involves only attributes of E1

Corollary to law 5:

If $F = F1 \wedge F2$, where F1 and F2 involve attributes of E1 and E2, then $\sigma_F(E1 \times E2) \equiv \sigma_{F1}(E1) \times \sigma_{F2}(E2)$

If F1 involves only attributes of E1 and F2 involves both attributes of E1 and E2 then

$\sigma_F(E1 \times E2) \equiv \sigma_{F2}(\sigma_{F1}(E1) \times E2)$

LAWS OF RELATIONAL ALGEBRA 4

6. Commuting selection with a union.
   Assume (for well-defined union) that E1, E2 and E1 $\cup$ E2 have the same attributes. Then:

$$\sigma_F(E1 \cup E2) \equiv \sigma_F(E1) \cup \sigma_F(E2)$$

7. Commuting selection with a difference

$$\sigma_F(E1 - E2) \equiv \sigma_F(E1) - \sigma_F(E2)$$

Rules above suffice for describing relationship between selections and joins, since a join is expressible in terms of Cartesian product, selection and (if it is a natural join) a projection.

LAWS OF RELATIONAL ALGEBRA 5

8. Commuting projection with Cartesian product

$$\Pi_{A1 \ldots An} (E1 \times E2) \equiv \Pi_{B1 \ldots Bm} (E1) \times \Pi_{C1 \ldots Ck} (E2)$$

where $A_* = B_* \cup C_*$ is a disjoint union, and B_* are attributes of E1, and C_* attributes of E2.

9. Commuting projection with union

$$\Pi_{A1 \ldots An}(E1 \cup E2) \equiv \Pi_{A1 \ldots An} (E1) \cup \Pi_{A1 \ldots An} (E2)$$