Exercise sheet 2

CS242 Formal Specification and Verification

Autumn term 2006

1.3.1 Given the following formulas, draw their corresponding parse tree:

(d) $p \wedge (\neg q \rightarrow \neg p)$

(f) $\neg((\neg q \wedge (p \rightarrow r)) \wedge (r \rightarrow q))$

1.3.2 For the formula below, list all its subformulas:

(a) $p \rightarrow (\neg p \vee (\neg\neg q \rightarrow (p \wedge q)))$

1.3.7 Draw a parse tree that represents an ill-formed formula such that

(a) one can extend it by adding one or several subtrees to obtain a tree that represents a well-formed formula;

(b) it is inherently ill-formed; i.e. any extension of it could not correspond to a well-formed formula.

1.4.2 Compute the complete truth table of the formula

(c) $p \vee (\neg(q \wedge (r \rightarrow q)))$

1.4.12 Show that the following sequent is not valid:

(c) $p \rightarrow (q \rightarrow r) \vdash p \rightarrow (r \rightarrow q)$

1.5.2 Which of these formulas are semantically equivalent to $p \to (q \vee r)$?

(b) $q \wedge \neg r \to p$

(d) $\neg q \wedge \neg r \to \neg p$

1.5.7 Construct a formula in CNF based on the following truth table:

(b)

p	q	r	ϕ_2
T	T	T	T
T	T	F	F
T	F	T	F
F	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T
F	F	F	F

1.5.8 Write a recursive function `IMPL_FREE` which requires a (parse tree of a) propositional formula as input and produces an equivalent implication-free formula as output. How many clauses does your case statement need? Recall the definition of a well-formed formula.

1.5.9 Compute `CNF(NNF(IMPL_FREE`$\neg(p \to (\neg(q \wedge (\neg p \to q))))))$.

1.5.15 Apply algorithm `HORN` to each of these Horn formulas:

(a) $(p \wedge q \wedge w \to \bot) \wedge (t \to \bot) \wedge (r \to p) \wedge (\top \to r) \wedge (\top \to q) \wedge (u \to s) \wedge (\top \to u)$

(g) $(\top \to q) \wedge (\top \to s) \wedge (w \to \bot) \wedge (p \wedge q \wedge s \to v) \wedge (v \to s) \wedge (\top \to r) \wedge (r \to p)$