Exercise sheet 3

CS242 Formal Specification and Verification

Autumn term 2006

2.1.1 Use the predicates

$$\begin{aligned} A(x,y): &\quad x \text{ admires } y \\ B(x,y): &\quad x \text{ attended } y \\ P(x): &\quad x \text{ is a professor} \\ S(x): &\quad x \text{ is a student} \\ L(x): &\quad x \text{ is a lecture} \end{aligned}$$

and the nullary function symbol (constant)

$$m: \quad \text{Mary}$$

to translate the following into predicate logic:

(a) Mary admires every professor.

(b) Some professor admires Mary.

(c) Mary admires herself.

(d) No student attended every lecture.

(e) No lecture was attended by every student.

(f) No lecture was attended by any student.

2.2.4 Let ϕ be $\exists x(P(y,z) \land (\forall y(\neg Q(y,x) \lor P(y,z))))$, where P and Q are predicate symbols with two arguments.

(a) Draw the parse tree of ϕ.

(b) Identify all free and bound variable leaves in ϕ.

(c) Is there a variable in ϕ which has free and bound occurences?

(d) Consider the terms w (w is a variable), $f(x)$ and $g(y,z)$, where f and g are function symbols with arity 1 and 2, respectively.

i. Compute $\phi[w/x]$, $\phi[w/y]$, $\phi[f(x)/y]$ and $\phi[g(y,z)/z]$.

ii. Which of w, $f(x)$ and $g(y,z)$ are free for x in ϕ?

iii. Which of w, $f(x)$ and $g(y,z)$ are free for y in ϕ?

(e) What is the scope of $\exists x$ in ϕ?

(f) Suppose that we change ϕ to $\exists x(P(y,z) \land (\forall x(\neg Q(x,x) \lor P(x,z))))$. What is the scope of $\exists x$ now?

2.4.5 Let ϕ be the sentence $\forall x \forall y \exists z (R(x,y) \to R(y,z))$, where R is a predicate symbol of two arguments.

(a) Let $A \stackrel{\text{def}}{=} \{a,b,c,d\}$ and $R^{\mathcal{M}} \stackrel{\text{def}}{=} \{(b,c),(b,b),(b,a)\}$. Do we have $\mathcal{M} \models \phi$? Justify your answer, whatever it is.

(b) Let $A' \stackrel{\text{def}}{=} \{a,b,c\}$ and $R^{\mathcal{M}'} \stackrel{\text{def}}{=} \{(b,c),(a,b),(c,b)\}$. Do we have $\mathcal{M}' \models \phi$? Justify your answer, whatever it is.

2.4.11 For each set of formulas below, show that they are consistent, i.e. that for some model and environment, each formula in the set is true:

(b) $\forall x \neg S(x,x)$, $\forall x \exists y S(x,y)$, $\forall x \forall y \forall z((S(x,y) \wedge S(y,z)) \to S(x,z))$;

(d) $S(x,x)$, $\forall x \forall y (S(x,y) \to (x = y))$.

2.5.1 Show that the following semantic entailments are not valid:

(b) $\forall x(P(x) \to R(x)), \forall x(Q(x) \to R(x)) \models \exists x(P(x) \wedge Q(x))$

(d) $\forall x \exists y S(x,y) \models \exists y \forall x S(x,y)$

(f) $\exists x(\neg P(x) \wedge Q(x)) \models \forall x(P(x) \to Q(x))$