CS242

UNIVERSITY OF WARWICK

TRAILER FOR THE CLASS TEST ON MONDAY MARCH 1st *2010*

Formal Specification and Verification

Time allowed: 50 minutes

Answer all questions.

Write your answers straight onto the test paper in the space provided below each question. In the unlikely event that you require additional space to answer any question, write on the reverse side of the paper, indicating clearly that you have done this.

Unless otherwise indicated, rough work on the reverse side of all questions will be ignored by the marker. At the end of the allowed time, return the completed test intact. Do not separate any pages.

Enter your University Registration Number (library card number), usercode, and course and year of study in the spaces below.

University Registration Number _____   Usercode _____

Course and year of study _____

Do not write below this line

0	1	2	3	4	5
5	25	10	25	10	25

Total	

Some of the questions in this paper will refer to properties of your university registration number - the seven-digit code you specified on the previous page. For the purpose of specifying these properties, the following definitions will apply:

D is the set of digits $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

I is the subset $\{1, 2, 3, 4, 5, 6, 7\}$ of digits in D

E is the subset of even digits in D

O is the subset of odd digits in D

X is the subset of digits in D that appear in your seven-digit university code

M is the set of all positive integers whose decimal representations contain only digits in X

$d : I \rightarrow X$ is a function mapping i to the i-th digit in your university code

s is the sum of all seven digits in your university code

Note that, following a standard convention, a set S can be interpreted as a predicate such that $S(x)$ is true if and only if $x \in S$.

By way of illustration, my registration number is 7570320, so that in my case $X = \{7, 5, 0, 3, 2\}$, the sum $s=24$, and $d(1)=7$, $d(2)=5$, $d(3)=7$ etc. The set M contains (e.g) 20, 533 and 5573207 but not 0 and 628. Hence $M(533)$ is true whilst $M(628)$ is false.

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0. Write down the values of X, $d(4)$, s, $M(533)$ and $M(628)$ in your case.
[5 marks]

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1. The properties of your usercode as specified on the previous page supply a model $\mathcal{M}$ and an environment for the interpretation of predicate logic formulae. My usercode supplies a different model $\mathcal{M}_0$.

Draw an expression tree for the predicate logic formula ϕ below, and determine whether or not it is satisfied (a) in the model $\mathcal{M}$ and (b) in the model $\mathcal{M}_0$, justifying your answers:

$$\phi \equiv (X(5) \rightarrow O(d(3))) \vee (((\forall s)(I(s) \rightarrow s \leq 8)) \wedge M(2 * s))$$

[25 marks]

Continued

Several other questions at this point ...

2. Testing your familiarity with some basic logical concepts

3. An exercise in interpreting predicate logic assertions based on the model $\mathcal{M}$

4. Testing your basic understanding of key definitions

5. Write down the natural deduction rules for introducing and eliminating $\wedge$ and $\rightarrow$. Use them to prove that $p \rightarrow q \vdash p \wedge r \rightarrow q \wedge r$.   [25 marks]